CVEN 5534, Fall 2010
Homework 3 Solutions

1. Assume influent $COD_S = 400\ g/m^3$

ΔCOD_S consumed $= 0.95(400) = 380\ g/m^3$

$$Q(\Delta COD_S) = 380\frac{g}{m^3}\ 70\frac{m^3}{min}\left(1440\frac{min}{d}\right)10^{-3}\frac{kg}{g} = 3.83\times10^4\ \frac{kg\,COD_S}{d}$$

$$R_S = -3.83\times10^4\ \frac{kg\,COD_S}{d}$$

a. $$R_{O_2} = (1-Y)R_S = 0.65(3.83\times10^4) = \boxed{2.49\times10^4\ \frac{kg\,O_2}{day}}$$

(given 1 kg COD = 1 kg O_2)

$$R_X = Y R_S = 0.65(3.83\times10^4) = 1.34\times10^4\ \frac{kg\,COD_X}{d}$$

$$R_X = 1.34\times10^4\frac{kg\,COD_X}{d}\cdot\frac{1\,kg\,cells}{(1.42\,kg\,COD)}\cdot\frac{365\,d}{yr}$$

b. $$R_X = \boxed{3.45\times10^6\ \frac{kg\,cells}{yr}}$$

2. compare O_2 and NO_3^- as electron acceptors in respiration of 1 mole of CH_2O

O_2: $$1CH_2O + 1O_2 \rightarrow 1CO_2 + 1H_2O$$

NO_3^-: $$1CH_2O + 0.8NO_3^- + 0.8H^+ \rightarrow 1CO_2 + 1.4H_2O + 0.4N_2$$

ratio $$\frac{1\,mole\,(32\,g/mole)\,O_2}{0.8\,mole\,(14\,g/mole)\,NO_3\text{-}N} = \boxed{2.86\ \frac{g\,O_2}{g\,NO_3\text{-}N}\ \left(\text{or } -2.86\ \frac{g\,COD}{g\,NO_3\text{-}N}\right)}$$

3. Bad problem statement – credit to all

What I meant:

$$Y = 0.4 \frac{g\,COD_X}{g\,COD_S} \quad \& \quad Y_{obs} = 0.32 \frac{g\,COD_X}{g\,COD_S}$$

a. meant to say 30% of lost cells ($Y - Y_{obs}$) are lysed (and available as substrate)

$$\text{Lost cells} = \frac{0.4 - 0.32}{g\,COD_S} g\,COD_X = 0.08 \frac{g\,COD_X}{g\,COD_S}$$

$$\text{Lysed cells recycled} = 0.3(0.08) = \boxed{0.024 \frac{g\,COD_\ell}{g\,COD_S}}$$

b. then COD_d (debris) is

$$COD_d = 0.7(0.08) \frac{g\,COD_d}{g\,COD_S} = \boxed{0.056 \frac{g\,COD_d}{g\,COD_S}}$$

So for biodegradation of 1 gram COD_S you get

$0.32\ g\,COD_X$ (viable cells)
$0.024\ g\,COD_\ell$ (lysed COD recycled)
$0.056\ g\,COD_d$ (debris COD)

neglecting further metabolism of COD_ℓ etc, etc ...

5. $\Psi_1 = 1$, $\Psi_2 = \frac{0.03(44)}{1(59)} = 0.0224$, $\Psi_3 = \frac{0.02(18)}{1(59)} = 0.006$,

$\Psi_4 = \frac{0.92(18)}{1(59)} = 0.281$, $\Psi_5 = \frac{0.95(16)}{1(59)} = 0.258$, $\Psi_6 = \frac{0.02(18)}{1(59)} = 0.038$

$\Psi_7 = \frac{0.98(61)}{1(59)} = 1.01$ check $\sum_1^4 \Psi_r = 1.31$, $\sum_5^7 \Psi_p = 1.31$ ✓

a) (mass) $1\,CH_3COO^- + 0.022\,CO_2 + 0.006\,NH_4^+ + 0.281\,H_2O \rightarrow$

$\rightarrow 0.258\,CH_4 + 0.038\,C_5H_7NO_2 + 1.01\,HCO_3^-$

b) (COD) $Y_1 = 1$, $Y_3 = \frac{0.258(4)}{1(1.08)} = 0.956$ $Y_4 = \frac{0.038(1.42)}{1(1.08)} = 0.05$

NOTE: NH_4^+ has no COD equivalents in this reaction since NH_4^+ is not oxidized during cell synthesis. CO_2 also assumed to have no COD even though it is e^- acceptor since it is also a product of CH_3COO^- oxidation to CH_4: $CH_3COO^- + H^+ \rightarrow CO_2 + CH_4$

$$\boxed{1\,COD_{CH_3COO^-} \longrightarrow 0.956\,COD_{CH_4} + 0.05\,COD_{C_5H_7NO_2}}$$

$1 - 0.956 - 0.05 = -0.006$ close enough

c) $500\,\frac{kg\,CH_3COO^-}{d}\left(0.258\,\frac{kg\,CH_4}{kg\,CH_3COO^-}\right) = 129\,\frac{kg\,CH_4}{d}$

$\frac{129\,kg\,CH_4}{16\,\frac{kg}{kmole}} = 8.06\,\frac{kmol\,CH_4}{d}$, assume CH_4 is ideal gas

$PV = NRT$

($C_5H_7NO_2 = VSS$)

$V = \frac{8.06\,kmol\left(82.05\,\frac{L \cdot atm}{kmol\,K}\right)(273+35)K}{1\,atm}\,10^{-3}\,\frac{m^3}{L}$ $\boxed{= 204\,\frac{m^3\,CH_4}{d}}$

d) $500\,\frac{kg\,CH_3COO^-}{d}(0.038)\,\frac{kg\,VSS}{kg\,CH_3COO^-}$ $\boxed{= 19\,kg\,VSS/d}$

COD AND NITROGEN STOICHIOMETRIC AND KINETIC MATRIX FOR GROWTH AND DECAY

	Components							Rates
Process	Soluble COD S_S (mg/L COD)	Soluble NH_4-N S_{NH} (mg/L N)	Dissolved O_2, S_O (mg/L O_2)	Heterotrophic biomass, X_{BH} (mg/L COD)	Autotrophic (Nitrifying) Biomass, X_{BA} (mg/L COD)	Debris, X_D (mg/L COD)	Ammonia N in cells X_{NB} (mg/L N)	r_j
Aerobic Heterotrophic Growth	$-\frac{1}{Y_H}$	$-i_{NXB}$	$-\frac{(1-Y_H)}{Y_H}$	1	—	—	i_{NXB}	**1 and 1a** $\mu_H X_{BH}$
Aerobic Growth of Autotrophs (Nitrification)	—	$-\frac{1}{Y_A}$	$-\frac{(4.57-Y_A)}{Y_A}$	—	1	—	—	**2** $\mu_A X_{BA}$
Decay and Lysis of Heterotrophs	$1-f_D$	—	—	-1	—	f_D	—	**3** $b_H X_{BH}$

And reaction rates are

$\mu_H * X_{BH}$ = growth rate for heterotrophs (d^{-1})

$\mu_A * X_{BA}$ = growth rate for autotrophs (d^{-1})

$b_H * X_{BH}$ = decay rate for heterotrophs (d^{-1})

c. **Give the total rate expressions for ammonia nitrogen, heterotrophic bacteria, autotrophic bacteria, debris, soluble COD, and oxygen considering the reactions 1, 1a, 2, and 3**

6c (really b)

$$r_{NH} = -i_{NXB}(\mu_H X_{BH}) - \frac{1}{Y_A}\mu_A X_{BA}$$

$$r_{XBH} = \mu_H X_{BH} - b_H X_{BH}$$

$$r_{XBA} = \mu_A X_{BA}$$ (neglect decay of X_A)

$$r_{XD} = f_D b_H X_{BH}$$

$$r_S = -\frac{1}{Y_H}\mu_H X_{BH} + (1-f_D) b_H X_{BH}$$

$$r_{O_2} = -\left(\frac{1-Y_H}{Y_H}\right)\mu_H X_{BH} - \left(\frac{4.57-Y_A}{Y_A}\right)\mu_A X_{BA}$$