CSTR with cell recycle

Oxygen Constrained Design Example

Physical limit of conventional aeration equipment: $\frac{R_O}{V} \leq 1{,}500 \frac{g\ O_2}{m^3 d}$

Operating information

$S_{S0} = 250 \frac{g}{m^3}$ COD $\quad S_S \leq 10 \frac{g}{m^3}$ COD (permit)

$Q = 1\ m^3/s$

$S_{NH0} = 40 \frac{g}{m^3}$ NH_4-N

Bioprocess Kinetic & Stoichiometric data

$\hat{\mu}_H = 4 d^{-1} \quad \hat{\mu}_A = 1 d^{-1}$

$K_S = 25 \frac{g\,COD}{m^3} \quad K_{NH} = 3 \frac{g\text{-}NH_4\text{-}N}{m^3}$

$b = 0.05 d^{-1}$ (both H & A)

$Y_H = 0.45 \frac{g\,MLVSS}{g\,COD(S)} \times 1.42 \frac{g\,COD}{g\,MLVSS} = 0.64 \frac{g\,COD(X)}{g\,COD(S)}$

$f_D = 0.2$

$i_{N/XB} = 0.09 \frac{g\,NH_4\text{-}N}{g\,COD(X)}$

Assume steady-state CSTR with cell recycle

$$\mu_H = \frac{\hat{\mu}_H S_S}{K_S + S_S} \quad \text{and} \quad \mu_A = \frac{\hat{\mu}_A S_{NH}}{K_{NH} + S_{NH}}$$

X_{BA} is negligible compared with X_{BH}

Show relation between θ_c and:

S_S, S_{NH}

find associated values of τ_{min} and

X_B, X_D, $X_T = X_B + X_D$

find biomass waste production rate, W_T

Select suitable values for θ_c (τ_{min}, $X_{T\,max}$)

for ① COD oxidation

② COD and NH_4 oxidation

Relations: $S_S = \dfrac{K_S(1+b\theta_c)}{\hat{\mu}_H\theta_c - (1+b\theta_c)}$, $S_{NH} = \dfrac{K_{NH}(1+b\theta_c)}{\hat{\mu}_A\theta_c - (1+b\theta_c)}$

$X_B\tau = \dfrac{Y_H\,\theta_c(S_{S0} - S_S)}{\hat{\mu}_H\theta_c - (1+b\theta_c)}$ $\quad NR = i_{NXB}\,Y_{Hobs}(S_{S0} - S_S)$

$X_D\tau = f_D b\,\theta_c^2\,\dfrac{(Y_H(S_{S0} - S_S))}{(1+b\theta_c)}$

$X_T\tau = (X_B + X_D)\tau$

$Y_{Hobs} = (1 + f_D b\theta_c)Y_H / (1+b\theta_c)$

$W_T = Q(Y_{Hobs}(S_{S0} - S_S))$

$\left(\dfrac{R_O}{V}\right)_① = \dfrac{1}{\tau_①}\left[(S_{S0} - S_S)(1 - Y_{Hobs})\right]$ COD oxidation

$$\left(\frac{R_O}{V}\right)_{②} = \frac{1}{\tau_{①}}\left[(S_{SO}-S_S)(1-Y_{Hobs}) + 4.57\left[(S_{NHO}-S_{NH}) - 0.09Y_{Hobs}(S_{SO}-S_S)\right]\right]$$

See spreadsheet tables for
$S_S(\theta_c)$, $S_{NH}(\theta_c)$, $X_B\tau$, $X_D\tau$, $X_T\tau$, NR,
and W_T for both COD and COD + nitrification

Check S_{min} for COD & NH_4
and $\theta_{c\,min}$ for $\underline{\underline{NH_4}}$ (critical $\theta_{c\,min}$)

$$S_{S\,min} = \frac{K_S b}{\hat{\mu}_H - b} = \frac{25(0.05)}{4-0.05} = 0.3\ g\frac{COD}{m^3}$$

$$S_{NH4\,min} = \frac{K_{NH}b}{\hat{\mu}_A - b} = \frac{3(0.05)}{1-0.05} = 0.16\ g\frac{NH_4\text{-}N}{m^3}$$

Check $\theta_{c\,min}\,|_{autotrophs}$

$$\theta_{c\,min} = \frac{K_{NH} + S'_{NHO}}{S'_{NHO}(\hat{\mu}_A - b) - K_{NH}b} \quad \text{where } S'_{NHO} = S_{NHO} - NR_{avg}$$

$NR_{avg} = 10\,g/m^3\ NH_4\text{-}N$, $S'_{NHO} = 40 - 10 = 30\ \frac{g}{m^3}\ NH_4\text{-}N$

$$\theta_{c\,min} = \frac{3+30}{30(1-0.05) - 3(0.05)} = 1.2\,d$$

want to minimize volume or τ (and maximize X)
to find τ_{min}

$$\frac{R_O}{V} = 1500 \frac{g}{m^3 d} \quad max$$

$$\therefore \tau_{min①} = \frac{1}{1500}\left[(S_{S0}-S_S)(1-Y_{Hobs})\right] \quad \text{(COD only)}$$

$$\tau_{min②} = \frac{1}{1500}\left[(S_{S0}-S_S)(1-Y_{Hobs}) + 4.57\left[(S_{NH0}-S_{NH}) - (0.09\,Y_{Hobs}(S_{S0}-S_S))\right]\right]$$

COD + nitrification

$$X_{Tmax①} = X_T\tau / \tau_{min①}$$

$$X_{Tmax②} = X_T\tau / \tau_{min②}$$

other constraints

$\Theta_c > 3d$ for good flocculation of biomass particles

$X_T > 1000 \frac{g}{m^3}$ MLVSS $\simeq 1400 \frac{g}{m^3}$ CODX

See spreadsheet for

τ_{min} / X_{Tmax} calculations

and graphs