

14.
$$r_h = K_h \left[\frac{(X_s/X_B)}{K_x + (X_s/X_B)} \right] X_B$$

difference is that r_h does not depend on $\left(\frac{X_s}{K_x + X_s} \right)$ directly although X_s is the substrate

dependence on ratio says that when

X_B large relative to X_s , r_h will be 1st order (X_s -limited)

When X_s large relative to X_B , r_h is zero order ratio reflects surface interaction between particulate substrate & biomass (exo-enzymes)

15. processes

ammonification: biomass-N $\rightarrow$ NH_3 -N + debris N

traditional

i_{NXB} biomass COD $\rightarrow$ NH_3 N + i_{NXD} debris COD

$$r_{\text{SNH}} = (i_{NXB} - i_{NXD} f_D) b X_B$$

lysis-regrowth

$$r_{\text{SNH}} = -r_{\text{SNG}} \quad \text{soluble organic N from } X_s$$

$$r_{\text{SNG}} = -K_a S_{\text{NS}} X_{B1}$$

Synthesis

$$r_{\text{SNH}} = -i_{NXB} / \mu X_B$$

15 nitrification $\left(\frac{-1}{Y_A} \text{NH}_4\text{-N} - \frac{(4.57 - Y_A)}{Y_A} \text{O}_2 \right) = \text{biomass COD}$ 4/6

$(\text{NH}_4^+ \rightarrow \text{NO}_3)$

e donor only

$$r_{\text{NH}} = \mu X_{BA} \left(\frac{-1}{Y_A} \right) = r_{XA}$$

$$\frac{r_{XA}}{X_A} = \mu_A = \hat{\mu}_A \left(\frac{S_{\text{NH}}}{K_{\text{NH}} + S_{\text{NH}}} \right) \left(\frac{S_o}{K_o + S_o} \right)$$

denitrification $-\frac{1}{Y_H} S_S - \frac{(1 - Y_H)}{2.86(Y_H)} S_{\text{NO}} = \text{biomass COD}$

$\uparrow \frac{g_{\text{O}_2}}{g_{\text{NO}_3\text{-N}}}$

$$-\frac{r_{S_{\text{NO}}}}{\left[\frac{(1 - Y_H)}{2.86 Y_H} \right]} = r_{X_{Ba}} = \frac{r_S}{\left(\frac{1}{Y_H} \right)}$$

anoxic

$$r_{X_{Ba}} = \eta_a r_{XB} = \eta_a \mu_H X_{BH}$$

$$= \eta_a \hat{\mu}_H \left(\frac{S_S}{K_S + S_S} \right) \left(\frac{S_{\text{NO}}}{K_{\text{NO}} + S_{\text{NO}}} \right) \left(\frac{K_{\text{DO}}}{K_{\text{DO}} + S_o} \right)$$

$\checkmark$ O_2 inhibition

$$r_{S_{\text{NO}}} = - \left[\frac{(1 - Y_H)}{2.86 Y_H} \right] r_{X_{Ba}}$$

16. PAO : PO₄ removal

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anaerobic (no growth): fermentation

$$\overset{\text{Substrate COD}}{-S_S} + \overset{\text{(acetate COD)}}{S_A} = 0$$

$$r_{S_S} = -r_{S_A}$$

$$\text{PHB formed } (-S_A - \text{COD} + X_{\text{PHB}} - \text{COD} = 0)$$

$$r_{X_{\text{PHB}}} = -r_{S_A}$$

(X_{PP}) Poly-P destroyed, PO₄ released (S_P)

$$r_{S_P} = -r_{X_{PP}}$$

energy from PP used to make PHB

$$-X_{PP} + Y_P X_{\text{PHB}} - \text{COD} = 0 \quad Y_P = \frac{\text{g P used}}{\text{g PHB COD formed}}$$

$$-r_{X_{PP}} = Y_P r_{X_{\text{PHB}}}$$

$$-r_{S_A} = r_{X_{\text{PHB}}} = -\frac{r_{X_{PP}}}{Y_P} = \frac{r_{S_P}}{Y_P}$$

$$-r_{S_A} = \mu_A \left(\frac{S_A}{K_A + S_A} \right) \left(\frac{X_{PP}/X_{BP}}{K_{PP} + (X_{PP}/X_{BP})} \right) X_{BP}$$

Max rate of acetate consumption is ref. since PAO's NOT growing

rate depends on % PP in PAO's

PAO's

16. aerobic PP synthesis & growth of PAO's

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growth

$$-\left(\frac{1}{Y_{PAO}}\right) X_{PHB} - COD - \frac{(1 - Y_{PAO})}{Y_{PAO}} O_2 + X_{PAO} - COD = 0$$

$$\frac{r_{X_{PHB}}}{\left(-\frac{1}{Y_{PAO}}\right)} = \frac{r_{S_0}}{(-1)\left(\frac{1 - Y_{PAO}}{Y_{PAO}}\right)} = \frac{r_{X_{BP}}}{1} \quad \text{(growth at expense of } X_{PHB}\text{)}$$

$$Y_{PAO} = \frac{X_{PAO} - COD \text{ formed}}{X_{PHB} - COD \text{ consumed}}$$

ref is

$$r_{X_{PAO}} = \hat{\mu} \left(\frac{X_{PHB}/X_{BP}}{K_{PHB} + \frac{X_{PHB}}{X_{BP}}} \right) \left(\frac{S_p}{K_p + S_p} \right) \left(\frac{S_0}{K_0 + S_0} \right) X_{BP}$$

storage of P. P stoichiometry: (X_{PP} is reference)

$$-r_{S_p} + r_{X_{PP}} = 0$$

also - P-stoichiometry:

$$-Y_{PHB} X_{PHB} - COD - S_p - (-Y_{PHB}) S_0 + X_{PP} = 0$$

$$Y_{PHB} = \frac{g - PO_4 - P \text{ stored}}{g \text{ PHB consumed}}$$

16.

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$$r_{X_{PP}} = -r_{S_P} = -\frac{r_{X_{PHB}}}{Y_{PHB}} = \frac{r_{S_0}}{(-1)(-Y_{PHB})}$$

$$r_{X_{PP}} = \hat{q}_{PP} \left[\frac{X_{PHB}/X_{BP}}{K_{PHB} + \frac{X_{PHB}}{X_{BP}}} \right] \left(\frac{S_P}{K_P + S_P} \right) \left(\frac{S_0}{K_0 + S_0} \right) \times \dots$$

max rate of PP synthesis

$$\times \left(\frac{K_{PPmax} - (X_{PP}/X_{BP})}{K_{1PP} + (K_{Pmax} - X_{PP}/X_{BP})} \right) \times X_{BP}$$

note driving force term
only so much of cell mass can be
PP ($K_{Pmax} \approx 0.3 - 0.35 \text{ g PP/g PAO}$)

When X_{PP} approaches K_{Pmax} ,
PP synthesis slows down.

17. a. $Y_{Ho} = 0.7 \frac{g \text{ X-COD}}{g \text{ S-COD}}$ "observed" used considering BOTH lysis & decay 6/6

$$\text{Oxygen requirement} = (1 - 0.7) \frac{g \text{ O}_2}{g \text{ S-COD}} = 0.3 \frac{g \text{ O}_2}{g \text{ S}_s}$$

Assume that $i_{N \times B} = 0.087 \frac{g \text{ N}}{g \text{ X-COD}}$

$$\text{Nitrogen requirement} = 0.087(0.7) = 0.061 \frac{g \text{ NH}_4\text{-N}}{g \text{ S}_s}$$

Assume phosphorus requirement = $0.2 \times \text{N-requirement}$

$$\text{P requirement} = 0.012 \frac{g \text{ P}}{g \text{ S}_s}$$

K 10 mg/g S_s

Ca 10 mg/g S_s

Mg 5 mg/g S_s

S 4 mg/g S_s

Na 2 mg/g S_s

Cl 2 mg/g S_s

Fe 1 mg/g S_s

Zn 0.1 mg/g S_s

Mn 0.05 mg/g S_s

Cu = 0.01 mg/g S_s

M_o = 0.003 mg/g S_s

C_o = < 0.0003 mg/g S_s

Same procedure for b & c
(requirements will be lower
by 18.6% and 49%, resp.)