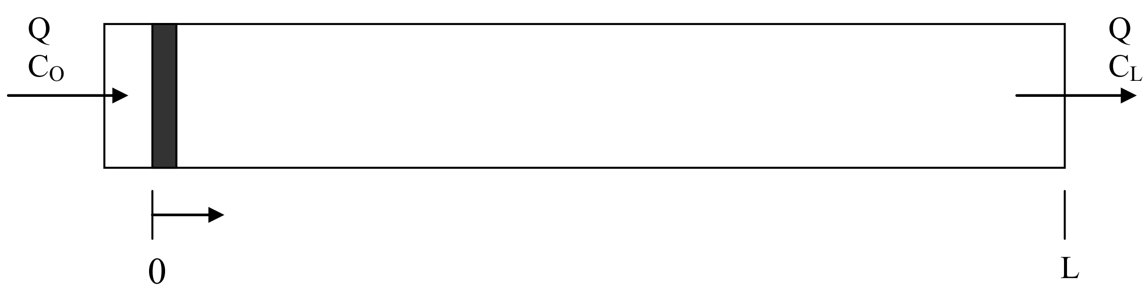
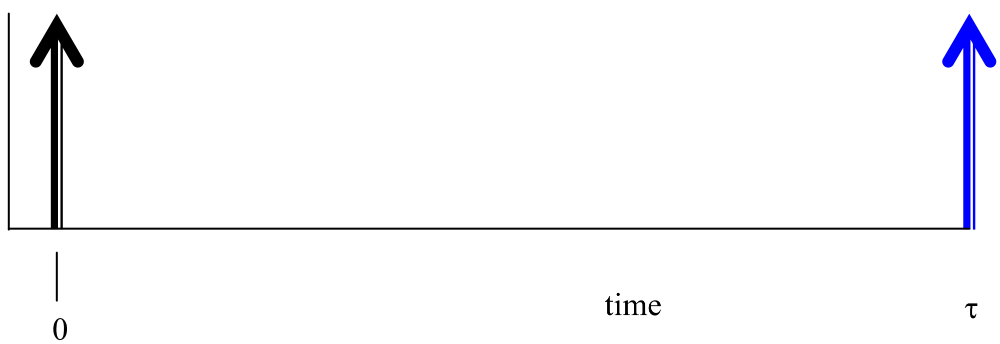
IDEAL PLUG FLOW REACTOR

Characteristics of ideal plug flow

- PERFECT MIXING IN THE RADIAL DIMENSION (UNIFORM CROSS SECTION CONCENTRATION)

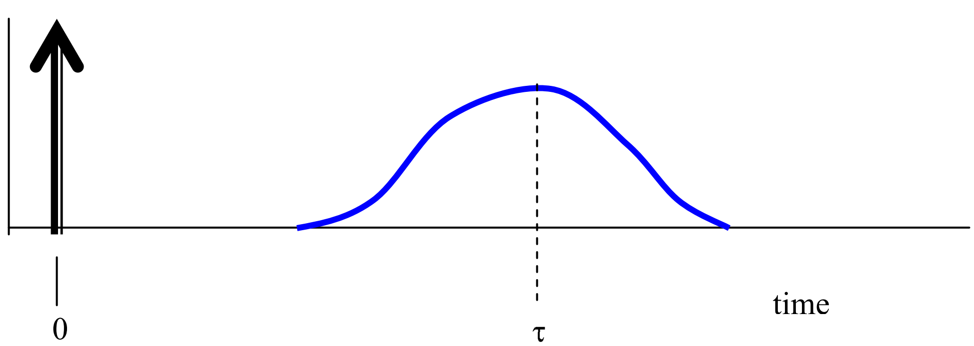
- NO MIXING IN THE AXIAL DIRECTION, OR NO AXIAL DISPERSION (SEGREGATED FLOW)





TRACER PULSE INPUT AT $t = 0$ TRANSLATED TO EQUAL PULSE OUTPUT AT $t = \tau$, □□□ $\tau = L/v$ (L = PFR length, v = average velocity)

COMPARE WITH CSTR RESPONSE TO TRACER PULSE DISPERSION

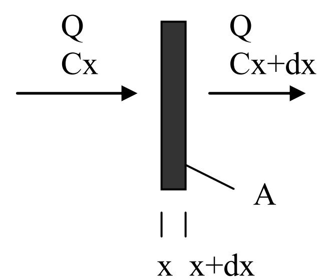


In an ideal PFR, concentration is a function of both distance along the flow path, x, and time, t:

$$C = C(x,t)$$

For a mass balance on a reacting compound, take mass balance on differential axial element with uniform reaction potential (concentration), where

dV = differential volume
A = cross sectional area
dx = differential distance



and

$$dV = Adx$$

Mass balance over differential element on a reactant, C

In = QCx
Out = QCx+dx
Generation = $dVr_C = Adxr_C$

Accumulation = $dV\frac{\delta Cx}{\delta t} = Adx\frac{\delta Cx}{\delta t}$

$$QCx - QCx{+}dx + dVr_C = dV\frac{\delta Cx}{\delta t}$$

$$Cx{+}dx = Cx + dCx$$

$$Q(Cx - Cx - dCx) + dVr_C = dV\frac{\delta Cx}{\delta t}$$

$$-Q\frac{\delta Cx}{\delta V} + r_C = \frac{\delta Cx}{\delta t} = -\frac{\delta Cx}{\delta\left(V/Q\right)} + r_C$$ since Q is constant

□□□ $\delta(V/Q) = \delta\tau$

$$-\frac{\delta Cx}{\delta\tau} + r_C = \frac{\delta Cx}{\delta t}$$

is the non-steady state ideal PFR mass balance for a reactant.

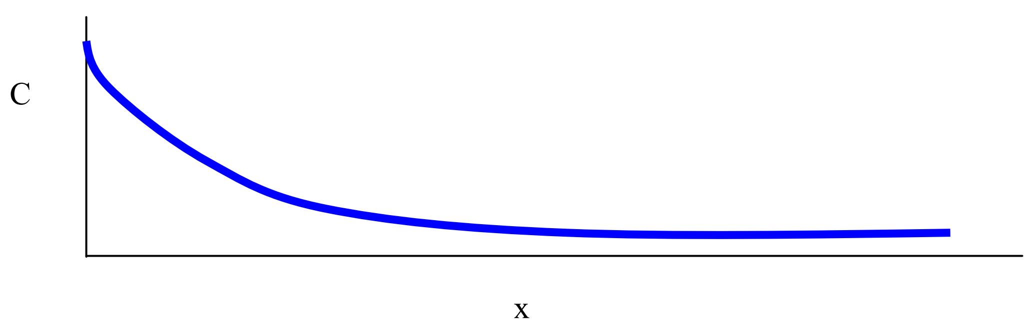
At steady state, $\frac{\delta Cx}{\delta t} = 0$

And the ordinary differential can be substituted for the partial differential

$$\boxed{\frac{dCx}{d\tau} = r_C}$$

Comments

1. At steady-state, the concentration of a reactant at any single point along the PFR is constant at Cx. Overall a stable concentration profile is obtained at steady state, with the concentration varying in space as the reaction occurs along the flow path.



2. In an ideal PFR, τ is the absolute residence time for mass flowing through the reactor, not the average residence time as in a CSTR.

3. Compare ideal batch and ideal PFR mass balances:

$$\text{Ideal PFR}: \quad \frac{dC}{d\tau} = r_C$$

$$\text{Ideal batch}: \quad \frac{dC}{dt} = r_C$$

Position in a PFR is equivalent to time in a batch reactor

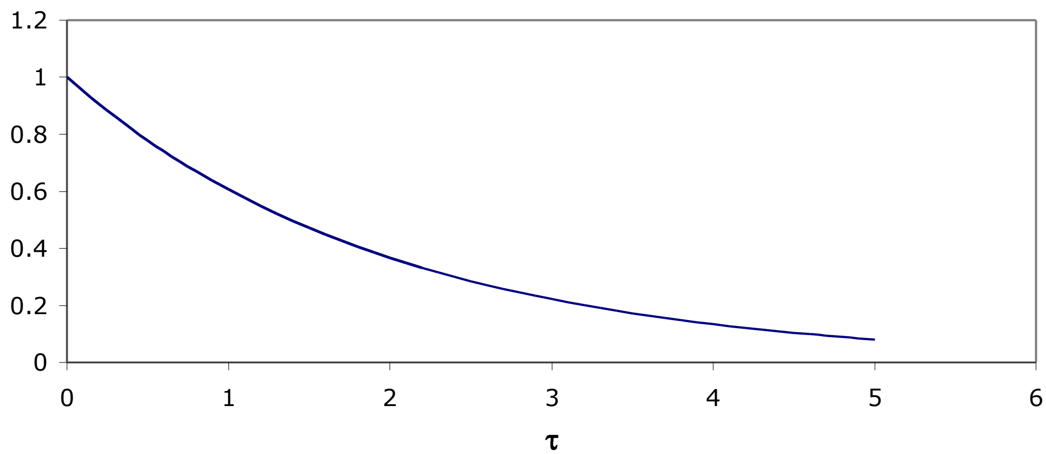
For a 1st order reaction, r = -kC, in a PFR at steady state

$$\frac{dC}{d\tau} = -kC$$

$$\int_{Co}^{C_L} \frac{dC}{C} = \int_0^{\tau} -k d\tau$$

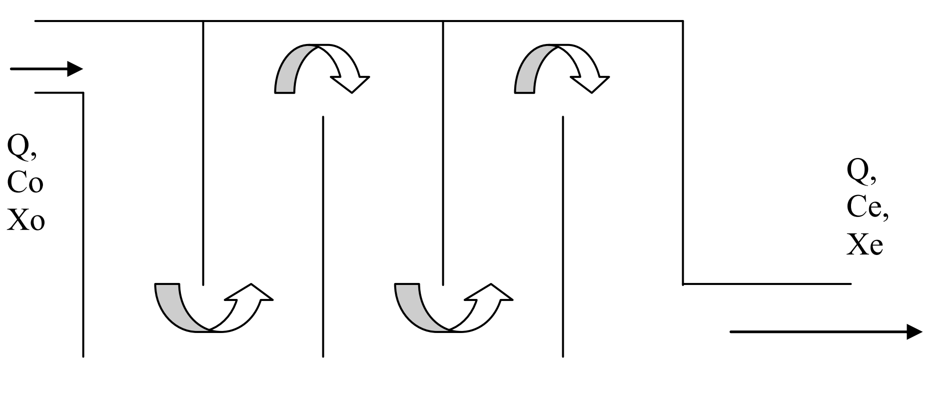
$$C_L = C_0 \exp(-k\tau)$$

Ideal PFR, steady-state 1st order reaction profile



Example:

Chlorine contact basin for disinfection



Where
$Q = 0.25\ m^3/s$
A = channel cross section between baffles = 18 m^2
r_d = rate of microorganism kill in presence of chlorine = $-k_dX$
X = concentration of microorganisms at any point in contact reactor
Xo = influent concentration of microorganisms = 10^6 E. coli/100 ml
$k_d = 5\ hr^{-1}$
r_c = rate of chlorine decay (from microorganism Cl-demand) = $-k_cX$
$k_c = 10^{-5}$ (mg-chlorine/L)(#/100mL)$^{-1}$hr^{-1}

2 rate expressions, 2 constituents, 2 coupled mass balances

find:

1. reactor volume and flow path length, L, such that $X_L < 10^3$ cells/100 ml
2. chlorine concentration which must be added to insure that there is detectable chlorine at PFR exit (detection level = C_L = 0.05 mg/L)

1. Steady-state mass balance on cells

$$X_L = X_o \exp(-k_d\tau)$$

$$\tau = (1/k_d)\ln(X_o/X_L) = (1/5)(hr)\ln(10^6/10^3) = 1.4\ hr$$

$$V = Q\tau = 0.25\ m^3/s * 3600\ s/hr * 1.4\ hr = 1{,}260\ m^3$$

$$L = V/A = 1{,}260\ m^3/18\ m^2 = 70\ m$$

3. Steady state mass balance on chlorine

$$\frac{dC_c}{d\tau} = -k_c X = -k_c X_o \exp(-k_d d\tau)$$

$$\int_{Cco}^{C_L} dC_c = -k_c X_o \int_0^{\tau} \exp(-k_d \tau) d\tau$$

$$C_L = C_{co} - \frac{(k_c X_o)}{k_d} + \frac{k_c X_o \exp(-k_d \tau)}{k_d}$$

$$C_L = C_{co} - \frac{(k_c X_o)}{k_d}(1 - \exp(-k_d \tau))$$

$C_{CO} = 0.05 + (10^{-5}(10^{6})/5)(1\text{-exp}(-5(1.4)) =$ **2.05 mg/L**