

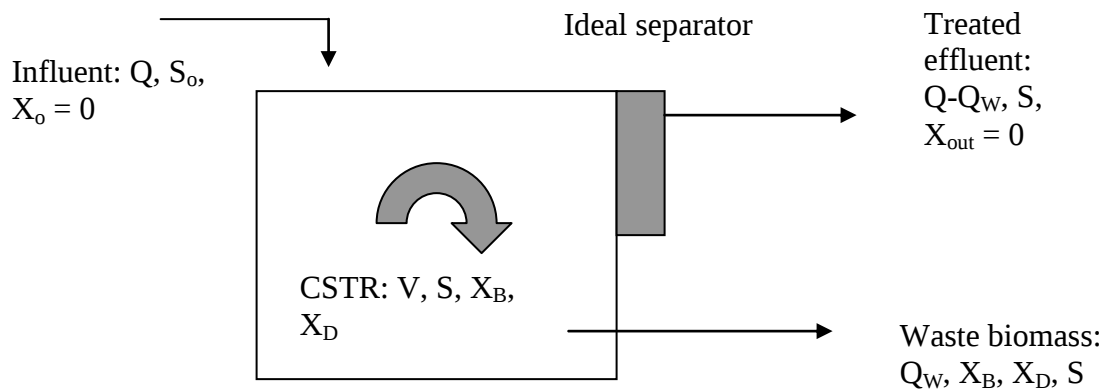
## CSTR WITH SOLIDS RECYCLE

### Components:

- Ideal CSTR, Volume =  $V$
- Ideal solids separator, volume = 0

### Flows

- Inflow ( $Q$ ) and treated outflow ( $Q - Q_w$ )
- Waste biomass ( $Q_w$ ) (wasting from CSTR mixed contents) to maintain steady-state biomass concentration
- $Q_w \ll Q$



### Characteristics:

Hydraulic residence time:  $\tau$  or HRT

$$\tau = \frac{V}{Q}$$

Solids residence time"  $\Theta$  or SRT = mass solids in CSTR/solids mass wasting rate

$$\Theta = \frac{VX}{Q_w X} = \frac{V}{Q_w}$$

Note that since  $Q_w \ll Q$ ,  $\Theta \gg \tau$

General Mass Balance Expression for one reactive component, S:

$$QS_o - (Q - Q_w)S - Q_w S + V r_s = V \frac{dS}{dt}$$

At steady state condition for S:

$$QS_o - (Q - Q_w)S - Q_w S + V r_s = 0$$

Rearranging:

$$Q(S_o - S) + V r_s = 0$$

$$r_s = \frac{(S_o - S)}{\tau}$$

For heterotrophic growth and decay in a CSTR, four components and two reactions are of interest: SUBSTRATE COD ( $S_s$ ), OXYGEN ( $S_o$ ), CELL-COD ( $X_{BH}$ ), and DEBRIS-COD ( $X_D$ )

|                                                | Components                         |                                             |                                                  |                             | Rates                |
|------------------------------------------------|------------------------------------|---------------------------------------------|--------------------------------------------------|-----------------------------|----------------------|
| Process                                        | Soluble COD<br>$S_s$<br>(mg/L COD) | Dissolved $O_2$ ,<br>$S_o$<br>(mg/L $O_2$ ) | Heterotrophic<br>biomass, $X_{BH}$<br>(mg/L COD) | Debris, $X_D$<br>(mg/L COD) | $\rho_j$             |
| <b>Aerobic<br/>Heterotrophic<br/>Growth</b>    | $-1/Y_H$                           | $-\frac{(1 - Y_H)}{Y_H^{(1)}}$              | 1                                                | na                          | $\mu_H X_{BH}^{(2)}$ |
| <b>Decay and<br/>Lysis of<br/>Heterotrophs</b> |                                    | $1 - f_D^{(3)}$                             | -1                                               | $f_D^{(6)}$                 | $b_H X_{BH}^{(4)}$   |

<sup>(1)</sup> heterotrophic cell yield =  $Y_H$  = g-heterotroph cell-COD/g-COD consumed

<sup>(2)</sup> heterotrophic growth rate ( $\rho_1$ ) =  $X_{BH} \mu_H = \hat{\mu}_H \left( \frac{S_s}{K_s + S_s} \right) X_{BH}$  where  $\mu_H$  is specific growth rate ( $d^{-1}$ )

<sup>(3)</sup>  $f_D$  = g debris-COD produced/g biomass-COD decayed. Assumes direct exertion of decayed biomass COD

<sup>(4)</sup> heterotrophic decay rate ( $\rho_2$ ) =  $X_{BH}b_H$  where  $b_H$  = specific decay rate ( $d^{-1}$ )  
net heterotrophic biomass growth rate =  $r_{XB} = (\mu_H - b_H)X_{BH}$

debris generation rate =  $r_D = f_D b_H X_{BH}$

COD consumption rate =  $r_S = -\left(\frac{\mu_H}{Y_H}\right)X_{BH}$  (growth only)

O<sub>2</sub> consumption rate =  $r_O = -\left(\frac{(1-Y_H)}{Y_H}\mu_H + (1-f_D)b_H\right)X_{BH}$

Apply rate expressions to steady-state mass balances on CSTR with solids retention:

Viable heterotrophic cells:

$$0 - 0 - Q_W X_{BH} + V X_{BH} (\mu_H - b_H) = 0$$

simplify and note that  $\Theta = V/Q_W$

$$\frac{1}{\Theta} = \mu_H - b_H \quad (1)$$

for CSTR with biomass retention (recycling), the growth rate is a function of  $\Theta$ , not  $\tau$ .

Assuming Monod kinetics with COD the limiting substrate:

$$\mu_H = \frac{\hat{\mu}_H S_S}{(K_S + S_S)} \quad (2)$$

Substituting (2)  $\rightarrow$  (1):

$$S_s = \frac{K_s \left( \frac{1}{\Theta} + b_H \right)}{\hat{\mu}_H - \left( \frac{1}{\Theta} + b_H \right)} \quad (\text{I})$$

Steady-state mass balance on COD substrate, S:

$$QS_{SO} - Q_w S_s - (Q - Q_w) S_s - (\mu_H / Y_H) X_{BH} V = 0$$

$$(\mu_H / Y_H) X_{BH} = (S_{SO} - S_s) / \tau$$

$$X_{BH} = \left( \frac{1}{\tau} \right) \frac{Y_H (S_{SO} - S_s)}{\mu_H} \quad (3)$$

Substituting (1) into (3) for  $\mu_H$ :

$$X_{BH} = \left( \frac{\Theta}{\tau} \right) \frac{Y_H (S_{SO} - S_s)}{(1 + b_H \Theta)} \quad (\text{II})$$

Heterotrophic cell concentration is proportional to  $\Theta$  and inversely proportional to  $\tau$

Can substitute into (II) for  $S_s$  from (I)

$$X_{BH} = \left( \frac{\Theta}{\tau} \right) \frac{Y_H \left( S_{SO} - \left( \frac{K_s (1 + b_H \Theta)}{\hat{\mu}_H \Theta - (1 + b_H \Theta)} \right) \right)}{(1 + b_H \Theta)} \quad (\text{IIa})$$

Steady-state mass balance of debris:

$$-Q_W X_D + V f_D b_H X_{BH} V = 0$$

$$X_D = f_D b_H X_{BH} \Theta$$

Substituting for  $X_{BH}$  from (II):

$$X_D = f_D b_H \Theta \left( \frac{\Theta}{\tau} \right) \frac{Y_H (S_{SO} - S_S)}{(1 + b_H \Theta)} \quad (\text{III})$$

Total biomass,  $X_T = X_{BH} + X_D$

$$X_T = (1 + f_D b_H \Theta) \left( \frac{\Theta}{\tau} \right) \frac{Y_H (S_{SO} - S_S)}{(1 + b_H \Theta)} \quad (\text{IV})$$

Define active fraction of biomass,  $f_A = \frac{X_{BH}}{X_T}$

$$f_A = \frac{1}{(1 + f_D b_H \Theta)}$$

Define observed yield as the net yield, considering biomass decay (different from growth yield) =  $Y_{Hobs}$

To maintain system in steady-state with respect to biomass components, net growth and accumulated debris must be wasted:

$$\text{Net growth} = Y_{Hobs} Q (S_{SO} - S_S)$$

$$\text{Wasted biomass} = Q_W X_T$$

Steady-state condition:

$$Q_W X_T = Y_{Hobs} Q (S_{SO} - S_S)$$

$$Y_{\text{Hobs}} = \frac{Q_W X_T}{Q(S_{\text{SO}} - S_S)}$$

Note that  $Q_W/Q = (V/\Theta)/(V/\tau) = \tau/\Theta$

And substituting for  $X_T$  from (IV):

$$Y_{\text{Hobs}} = \frac{Y_H (1 + f_D b_H \Theta)}{(1 + b_H \Theta)} \quad (\text{V})$$

Since  $f_D < 1$ ,  $\frac{(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} < 1$  and  $Y_{\text{Hobs}}$  always  $< Y_H$

Waste biomass production:

$W_T = Q_W X_T$  (rate of total biomass wasting)

## OXYGEN CONSUMPTION

$$r_{o,1} = - \frac{(1 - Y_H)}{Y_H} r_{XBH} \text{ for heterotrophic aerobic growth}$$

incorporate recycling of decayed biomass assuming direct conversion of decayed biodegradable fraction of biomass to COD and exertion of oxygen demand:

| Process                                        | Components                         |                                             |                                                  |                             | Rates                |
|------------------------------------------------|------------------------------------|---------------------------------------------|--------------------------------------------------|-----------------------------|----------------------|
|                                                | Soluble COD<br>$S_S$<br>(mg/L COD) | Dissolved $O_2$ ,<br>$S_O$<br>(mg/L $O_2$ ) | Heterotrophic<br>biomass, $X_{BH}$<br>(mg/L COD) | Debris, $X_D$<br>(mg/L COD) | $\rho_j$             |
| <b>Aerobic<br/>Heterotrophic<br/>Growth</b>    | $-1/Y_H$                           | $-\frac{(1 - Y_H)}{Y_H^{(2)}}$              | 1                                                | na                          | $\mu_H^{(4)} X_{BH}$ |
| <b>Decay and<br/>Lysis of<br/>Heterotrophs</b> |                                    | $1 - f_D$                                   | -1                                               | $f_D^{(6)}$                 | $b_H X_{BH}^{(7)}$   |

For decay stoichiometry that converts recycled COD directly to oxygen consumption:

$$-X_{BH} - (1 - f_D)S_O + f_D X_D = 0$$

$$r_{O,2} = (1 - f_D)r_{XBH}$$

combining

$$r_O = r_{O,1} + r_{O,2}$$

$$r_O = - \left( \frac{(1 - Y_H)}{Y_H} \mu_H + (1 - f_D) b_H \right) X_{BH}$$

substitute  $\mu_H = 1/\Theta + b_H$  and for  $X_{BH}$

$$r_o = \frac{(S_{SO} - S_S)}{\tau} \left[ 1 - \left( \frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} \right) \right] \quad (\text{VIa})$$

but

$$\left( \frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} \right) = Y_{Hobs}$$

so

$$r_o = \frac{(S_{SO} - S_S)}{\tau} [1 - Y_{Hobs}] \quad (\text{mg/L-d}) \quad (\text{VIb})$$

multiply by V to get overall system oxygen uptake rate  $RO = r_o V = r_o Q \tau$  for COD oxidation and heterotrophic biomass decay producing additional oxygen demand

$$RO = Q(S_{SO} - S_S)(1 - Y_{obs}) \quad (\text{mg/day})$$

### NITROGEN: Two processes for removing ammonia:

1. Heterotrophic net growth requirement for nitrogen =  $X_{Ncells}$  is also a function of  $\Theta$ :

$$X_{Ncells} = i_{NXB} Y_{Hobs} (S_{SO} - S_S) \text{ mg-N/l}$$

$$X_{Ncells} = i_{NXB} \left( \frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} \right) * (S_{SO} - S_S) \text{ mg-N/l}$$

### Ammonia nitrogen uptake rate in cell synthesis, $r_{NH}$ :

$$r_{NH} = \frac{X_{Ncells}}{\tau} = \frac{i_{NXB} Y_{Hobs} (S_{SO} - S_S)}{\tau} \quad (\text{mgN/l/d})$$

2. Nitrification (ammonia substrate is rate-determining component)

$$\mu_A = \frac{\hat{\mu}_A S_{NH}}{(K_{NH} + S_{NH})}$$

where  $\mu_A$  is the autotrophic growth rate,  $S_{NH}$  is ammonia nitrogen concentration, and  $K_{NH}$  is the half-saturation constant for nitrifying bacteria for ammonia nitrogen.

And

$$S_{NH} = \frac{K_{NH} \left( \frac{1}{\Theta} + b_A \right)}{\hat{\mu}_A - \left( \frac{1}{\Theta} + b_A \right)}$$

where  $b_A$  is the decay coefficient for nitrifying bacteria (autotrophs).

And

$$X_{BA} = \left( \frac{\Theta}{\tau} \right) \frac{Y_A \left( S_{NHO}^* - \left( \frac{K_{NH} (1 + b_A \Theta)}{\hat{\mu}_A \Theta - (1 + b_A \Theta)} \right) \right)}{(1 + b_A \Theta)}$$

Where  $S_{NHO}^* = S_{NHO} - NR$  and  $S_{NHO}$  = influent ammonia nitrogen.

The rate of oxygen consumption for nitrification,  $r_{ON}$ :

$$r_{ON} = \left( \frac{4.57}{\tau} \right) [S_{NHO} - S_{NH}] - i_{NXB} Y_{Hobs} (S_{SO} - S_S) \quad \text{(VII)}$$

Now calculate the total rate of oxygen consumption for both COD oxidation (growth plus decay) and nitrification,  $r_{OT}$  **by combining VIb and VII:**

$$r_{OT} = \left( \frac{1}{\tau} \right) [S_{SO} - S_S] (1 - Y_{Hobs}) + 4.57 [S_{NHO} - S_{NH}] - i_{NXB} Y_{Hobs} (S_{SO} - S_S)$$