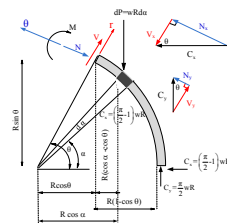

The Four Books of Structural Analysis

AUGUST 9, 2022

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University of Colorado, Boulder
2022



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NOTICES

1. Intentionally, this book can not be printed. It is best read on a computer to easily follow the multiple hyperlinks and bookmarks.
2. It is particularly important that you start with the Preface, as this is an atypical book.
3. This book is free, feel free to share it.

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Dedication

To my grandfather



whom I never met.

And to all future Structural Engineers.

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Preface

Genesis

This book, like so many others, had its genesis in notes of three courses taught over the span of over thirty years. But not only notes, but also a multitude of documents collected over the years in anticipation of this book. This resulted in a big puzzle where all the pieces had to smoothly fit together.

Hence, at the dusk of my academic career, and with a shade of vanity, I thought that I could share my 35+ years of teaching Structural Analysis with intrepid readers through a *magnum opus*.

Coverage

Broadly speaking the book is divided into four parts:

Book I is an extensive history of structural analysis. It does not pretend to be exhaustive, but was probably the most interesting part for me to write. Unconstrained, I have selected key events at first, and then when Galileo Galilei came, it had to follow a more disciplined path.

Book II is what one would expect students to be exposed in a first course in structural analysis (following Statics and Mechanics of Deformable Bodies (a.k.a. Materials). I have greatly expanded the coverage of some topics insufficiently covered in most books such as Cables, Arches, 3D structures. This book ends with a chapter containing numerous examples of preliminary design as it is important for the structural analyst to also have a sense of design.

Book III is what many institutions refer to as *Matrix Structural Analysis*. It is entirely devoted to the finite element method of framework members at first, but then rapidly expand into continuum elements. Along the way extensive coverage is given to variational methods as the foundation of the finite element method.

Book IV is based on a new course I had introduced and which has only few counterparts in academia. It is devoted to the nonlinear analysis of framed structures, but also addresses plasticity, stability dynamics, and last but not least Performance Based Earthquake Engineering.

Hence, the pertinacious reader will be rewarded with an encyclopedic knowledge of structural engineering.

Yet another book?

The casual reader would wonder why is there yet another book on Structural Analysis? I have found that most textbooks on structural analysis) are really variation on a theme, all practically identical (and some have had as many of 15 editions and counting).

Many of them provide a rudimentary coverage of the underlying theory, and most importantly limit the examples to simple structures.

Finally, throughout the book I have attempted to correlate the various procedures of structural analysis with the principles of applied mechanics and mathematics on which they are based.

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Audience

Students: This book is appropriate for three consecutive courses: Structural Analysis, Intermediate Structural Analysis, and Nonlinear Structural Analysis, combining in a single volume what has traditionally has required in at least two books. It further benefits from consistent notation throughout the coverage and includes illustrative examples prepared intentionally be challenging to the student.

However, only “mature audience” should consult it. By that, I mean those students who do not necessarily look for a simple and verbose coverage of the basics¹, of students motivated enough to explore sub-topics traditionally not covered, and students who aim to be structural engineers.

This book will also be of great values to those students who would like to see a *unified* (notation, philosophy) coverage of structural engineering with smooth transitions from fundamentals to intermediary and into advanced.

Engineers: This book is also addressed to structural engineers, wise enough to take a pause from computer programs, and explore the beauty of analytical solutions that can be of much greater value than thought of. Indeed too often many of them run to the computer before any attempt to obtain an exact or approximate analytical solution which could thne be validated by a program.

Historians: The first of the four books exhaustively covers the history of structural analysis. Aside from the great classical books that addressed this them, this is by far the most exhaustive coverage that can be found in a structural analysis book.

Style

A book is characterized by its content and its form. The form (or style) is utterly and blatantly personal, it reflects the teaching style, the focus of interest, ultimately, it reflects the delivery system of the author. As such, I have at times peppered this book with personal comment, and the depth and breadth of the coverage reflect my personal take on the topic.

For over 35 years, I have been a big fan of L^AT_EX and felt pity for those who insisted in writing technical documents in a tool originally meant for lawyers: *Word*. As such, with a decent command of L^AT_EX: countless *packages* developed by others, and few macros I wrote myself I always tried to make sure that any manuscript I authored is not only rigorous, complete, but also “looks nice”²

So, I have paid great attention to the layout, have personally drawn all the figures³ and wrote the MATLAB[®] programs found in the appendix.

Oh “my English” is far from perfect, evidently is is not my native language, tried my best, so be kind and try not to be too critical.

¹hence, for most, this book should never be assigned as the primary textbook in a course

²I have been fortunate to collaborate for nine years with the Tokyo Electric Power Company (TEPCO) on the nonlinear seismic analysis of tall arch dams. After about six years, I thought that we had accomplished all the work. *No! no! Prof. Saouma, in Japan, a program has not only to work properly but it must look beautiful.* This simple comment, along with my many visits/stays in Switzerland (where a great value is placed on sobriety), and a certain taste for architecture, influenced me.

³Starting with Xfig on Unix, ending with Visio on Windows

Why is it free?

On the supply side, there are two main reasons books are written. One is it may provide financial reward to its author, the second it may bring self-satisfaction and then possibly fame. In both cases, there is an anticipation that the publisher will provide text-editing, page layouts, and marketing that is unachievable by the author.

In our disciplines, I would venture to say that very few were awarded sufficient royalties to pay for a transatlantic flight. Fame on the other hand (in theory) should not be of concern to true scholars.

As to the demand side, students/readers have seen the price of books sky-rocket, even though nowadays there are clever marketing strategies whereas a reader may rent a book (or even specific chapters) for a limited time for a fixed fee (akin of renting a movie from Netflix).

As to formatting/marketing!. Any author sufficiently familiar with L^AT_EX can quasi-professionally format any scientific book. No need to have a professional accomplish this task (unless one is stuck with Word that is). Marketing is also nowadays made so much easier, suffice it to publish a book through Amazon and it will be instantaneously be within reach of millions.

On the other hand if a book is well written (as this one pretends to be), and is free, then it will be naturally disseminated.

Finally, as a University Professor, our responsibility is to acquire and *share* knowledge. We are semi-decently paid by our institution, and the crumbs given to us by publisher are not worth a Faustian bargain.

Accordingly, this book can be freely downloaded and freely shared.

Books Consulted

In writing my notes and this book, I have consulted numerous books that have lend me some their coverage or examples. The following are the primary (but not only) ones.

- *Indeterminate Structural Analysis*
Kinney, 1957
- *Elementary Structural Analysis*
Norris and Wilson, 1960
- *Theory of Matrix Structural Analysis*
Przemieniecki, 1968
- *Basic Structural Analysis*
Gerstle, 1974
- *Programming the matrix analysis of skeletal structures*
Bhatt, 1986
- *Mechanics of Structures, Variational and Computational Methods*
Pilkey and Wunderlich, 1994

Finally, I have tried in as much as possible to give proper credit within the book. If some were missing, it was certainly not intentional, and apologies are hereby offered.

A major challenge in teaching Structural Analysis is motivation. Hence, one should always keep in mind that structural analysis is not an end by itself, but only an indispensable tool to design or structural safety assessment (or design).

Victor E. Saouma

Boulder, CO 2023

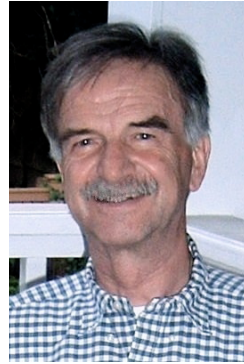
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Acknowledgments

First and foremost, to two great colleagues who, unbeknownst to them, greatly influenced me. The personal character and professional achievement of each was most inspirational. The combination of these characteristics is sadly uncommon in our profession.



Kurt Gerstle



Kaspar Willam

Of course, I have to acknowledge the many questions students raised during my lectures.

... and the institution which hosted me over the years.

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