# A FINITE DEFORMATION COUPLED ISOTROPIC DAMAGE ANISOTROPIC PLASTICITY MODEL AND ITS NUMERICAL IMPLEMENTATION IN EXPLICIT FINITE ELEMENT AND FINITE DIFFERENCE CODES

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**ABSTRACT:** A coupled isotropic damage and anisotropic plasticity model is formulated for finite deformations. A multiplicative decomposition of the deformation gradient and volumetric/deviatoric split of the elastic stretching tensor separate some of the different deformation mechanisms that a polycrystalline metal under high strain rates would experience. Standard thermodynamic arguments result in constitutive equations and constraints on the evolution of the internal state variables. Texture effects on plastic flow are represented in the inner product space of effective stress and structure tensors. Associated and nonassociated inelastic flow rules are formulated and compared. Dislocation drag effects on flow stress are included. A semi-implicit numerical integration of the model's system of ordinary differential equations (ODEs) is carried out for implementation in explicit finite element and finite difference codes.

**INTRODUCTION:** Polycrystalline metals that undergo high strain rates—possibly nearing the shock regime ($10^4$ 1/s)—experience a complex combination of deformation mechanisms, such as thermally-activated dislocation motion, dislocation drag, texture effects, void nucleation, growth, and coalescence, small deviatoric elastic stretching, potentially large volumetric elastic stretching, and large rotations. For simplicity, no higher gradients of deformation leading to a system of partial differential equations (PDEs) for the constitutive model are included. The damage and plasticity model described in this paper is a phenomenological description of the physical deformation mechanisms observed at the void and dislocation length scales. More complex constitutive models are presently being developed that include spatial gradients of internal state variables and free surface creation due to crack propagation, among other deformation mechanisms and material processes experienced by polycrystalline metals within high strain rate environments. Ultimately, the intent is to implement these constitutive models within finite element and finite difference codes to help engineers better design and understand devices that would be subjected to extreme environments.

**PROCEDURE, RESULTS, AND DISCUSSIONS:** The formulation of the model begins with a description of the kinematics, followed by thermodynamic considerations to place constraints on the constitutive equations, and ends with constitutive assumptions for the Helmholtz free energy and evolution equations for damage, plasticity variables, and internal state variables.

**Kinematics:** The multiplicative decomposition of the deformation gradient is shown in

Fig.1 and written here as [Davison *et al.* 1977]

$$\boldsymbol{F} \;=\; \boldsymbol{F}^e\boldsymbol{F}^d\boldsymbol{F}^p \;, \quad F_{iI} \;=\; F^e_{i\hat{I}}F^d_{\hat{I}\bar{I}}F^p_{\bar{I}I} \tag{1}$$

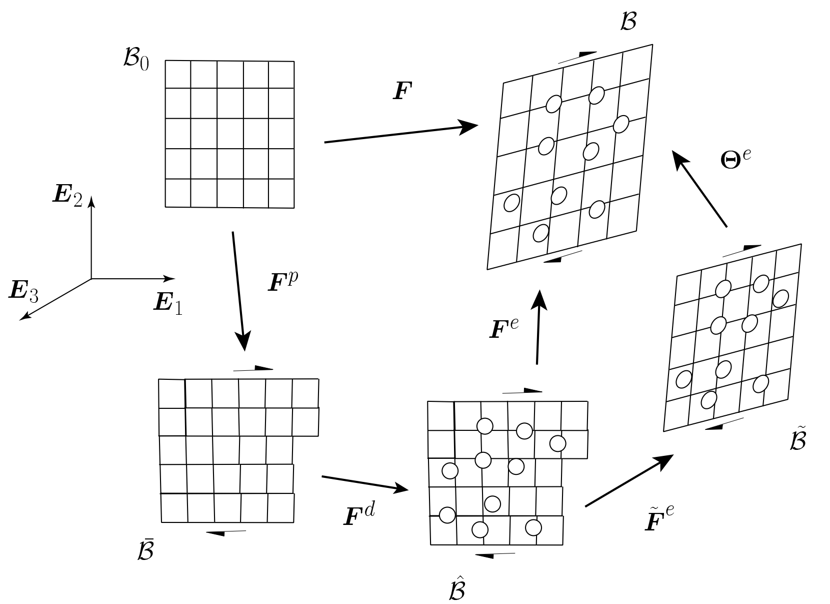


Figure 1. Multiplicative decomposition of the deformation gradient $\boldsymbol{F} \;=\; \boldsymbol{\Theta}^e\tilde{\boldsymbol{F}}^e\boldsymbol{F}^d\boldsymbol{F}^p$. Note that $\bar{\mathcal{B}}$ and $\hat{\mathcal{B}}$ are incompatible configurations. This will be discussed in another paper.

where $e$ stands for elastic, $d$ for damage, and $p$ for plastic. One of two possible polar decompositions of the elastic part of the deformation gradient $\boldsymbol{F}^e$ is $\boldsymbol{F}^e = \boldsymbol{V}^e\boldsymbol{R}^e$, where $\boldsymbol{V}^e$ is an elastic stretching tensor and $\boldsymbol{R}^e$ is an elastic proper orthogonal tensor [Bammann and Johnson 1987]. The elastic stretching tensor may be split into volumetric and deviatoric parts as [Simo *et al*. 1985]

$$\boldsymbol{V}^e \;=\; \boldsymbol{\Theta}^e\tilde{\boldsymbol{V}}^e \;, \quad \boldsymbol{\Theta}^e \;=\; J^{e1/3}\tilde{\mathbf{1}} \;, \quad \tilde{\boldsymbol{V}}^e \;=\; J^{e-1/3}\boldsymbol{V}^e \;. \tag{2}$$

This decomposition yields $\det\boldsymbol{\Theta}^e \;=\; J^e$ and $\det\tilde{\boldsymbol{V}}^e \;=\; 1$. Since deviatoric elastic strains in metals are small, the isochoric elastic stretching tensor $\tilde{\boldsymbol{V}}^e$ is linearized, whereas the volumetric elastic stretching tensor $\boldsymbol{\Theta}^e$ remains finite. This volumetric/deviatoric split is appropriate for modeling metals that experience strain rates within the shock regime ($>10^4$ 1/s), where pressure and volumetric elastic deformation can be large for an initially undamaged metal ($\phi \approx 0$). The multiplicative decomposition then becomes

$$\boldsymbol{F} \;=\; \boldsymbol{\Theta}^e\tilde{\boldsymbol{F}}^e\boldsymbol{F}^d\boldsymbol{F}^p \;, \quad F_{iI} \;=\; \Theta^e_{i\tilde{I}}\tilde{F}^e_{\tilde{I}\hat{I}}F^d_{\hat{I}\bar{I}}F^p_{\bar{I}I} \tag{3}$$

where $\tilde{\boldsymbol{F}}^e = \tilde{\boldsymbol{V}}^e\boldsymbol{R}^e$. Assuming a statistically random (i.e., isotropic) distribution of spherical voids, Davison *et al.* (1977) chose the damage part of the deformation gradient to be

$$\boldsymbol{F}^d \;=\; \frac{1}{(1-\phi)^{1/3}}\hat{\mathbf{1}} \tag{4}$$

where $\phi$ is the void ratio. The velocity gradient in the current configuration takes the form

$$\boldsymbol{l} \;=\; \dot{\boldsymbol{F}}\boldsymbol{F}^{-1} \;=\; \underbrace{\frac{\dot{J}^e}{3J^e}\mathbf{1}}_{\boldsymbol{l}^{\Theta e}} + \underbrace{\dot{\tilde{\boldsymbol{F}}}^e \tilde{\boldsymbol{F}}^{e-1}}_{\boldsymbol{l}^e} + \underbrace{\frac{\dot{\phi}}{3(1-\phi)}\mathbf{1}}_{\boldsymbol{l}^d} + \underbrace{\tilde{\boldsymbol{F}}^e \hat{\boldsymbol{L}}^p \tilde{\boldsymbol{F}}^{e-1}}_{\boldsymbol{l}^p} \tag{5}$$

The evolution equation for plastic deformation $\boldsymbol{F}^p$ then becomes:

$$\tilde{\boldsymbol{C}}^e \hat{\boldsymbol{L}}^p \;=\; \hat{\boldsymbol{D}}^p + \hat{\boldsymbol{W}}^p \tag{6}$$

where $\hat{\boldsymbol{D}}^p$ and $\hat{\boldsymbol{W}}^p$ are defined constitutively, and $\tilde{\boldsymbol{C}}^e = \tilde{\boldsymbol{F}}^{eT}\tilde{\boldsymbol{F}}^e$. Defining $\hat{\boldsymbol{S}}$ as the second Piola-Kirchhoff stress in the $\hat{\mathcal{B}}$ intermediate configuration, it is mapped to the current configuration by the following equation

$$\boldsymbol{\sigma} \;=\; \frac{1}{J^{e\,1/3}}\tilde{\boldsymbol{F}}^e \hat{\boldsymbol{S}} \tilde{\boldsymbol{F}}^{eT} \tag{7}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress. It becomes apparent that $\hat{\mathcal{B}}$ is the proper intermediate configuration in which to formulate the thermodynamics and define the constitutive model.

**Thermodynamics:** As a result of standard thermodynamic arguments [Coleman and Gurtin 1967], the Clausius-Duhem inequality, constitutive equations, and definitions for the strain-like internal state variables are written in $\hat{\mathcal{B}}$. What remains is to make constitutive assumptions on the Helmholtz free energy $\hat{\psi}$ and evolution equations for the plastic variables $\hat{\boldsymbol{D}}^p$ and $\hat{\boldsymbol{W}}^p$, damage $\phi$, and strain-like internal-state variables $\hat{\epsilon}_{ss}$ and $\hat{\boldsymbol{\beta}}$.

**Constitutive Model:** Since deviatoric elastic strains are assumed to be small and volumetric elastic strains could be large, a quadratic form of the Helmholtz free energy is chosen for the deviatoric variables. The evolution equations for $\hat{\boldsymbol{D}}^p$ and $\hat{\boldsymbol{W}}^p$ account for thermally-activated dislocation motion, dislocation drag at high strain rates, and texture effects. The evolution equation for $\phi$ accounts for void growth. The evolution equations for $\hat{\epsilon}_{ss}$ and $\hat{\boldsymbol{\beta}}$ account for the generation and annihilation of dislocations.

**CONCLUSIONS:** A finite deformation coupled isotropic damage and anisotropic plasticity model was briefly described. Further model details, its numerical implementation, and numerical examples will be discussed in the full length paper.

**ACKNOWLEDGEMENTS:** Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-ACO4-94AL85000.

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