CVEN 7511 syllabus - Fall 2022

Instructor: Prof. Richard Regueiro, 303-492-8026. office hrs: MW 11:10am-12pm, Tu 10-11am, Th 11am-12pm, hybrid on Zoom (https://cuboulder.zoom.us/j/7462640502) and in ECOT 421, richard.regueiro@colorado.edu

Course Description: The course will cover kinematics, thermodynamics, constitutive modeling, numerical time integration, and finite element implementation of large deformation hyperelasticity and inelasticity. Kinematics of the multiplicative decomposition (see Fig.1) and resulting objective stress rates in the current configuration will be covered. Linearization for formulation and finite element implementation of algorithmic (consistent) tangent moduli, local Newton-Raphson iteration for solution of nonlinear constitutive models, and mixed formulations for nearly-incompressible elasticity will also be covered. The emphasis is on being able to formulate and numerically implement within a nonlinear finite element program, your own finite strain inelastic constitutive model for a solid material of interest to you, as well as the nonlinear governing equations themselves.

For the Final Project, students will be asked to choose a solid material for which to develop a finite strain constitutive model, and then implement the model using UMAT (or VUMAT for high strain rate problems, UHYPER, or user element UEL) in ABAQUS (www.3ds.com/products-services/simulia/products/abaqus), or in full source code in Matlab or Python, or another nonlinear finite element or finite difference or meshfree program of your choosing. Various model types include hyper-elasticity, hyper-viscoelasticity, hyper-elasto-plasticity, hyper-elasto-plastic-damage, etc.

Proper formulation of finite strain constitutive models (multiplicative decomposition of the deformation gradient with isochoric-volumetric split, damage (d), plastic (p), and thermal (θ) parts: \( \mathbf{F} = \Theta \mathbf{F}^\varepsilon \mathbf{F}^d \mathbf{F}^\theta \), \( F_{ij} = \Theta_{ij} \mathbf{F}_{ij}^\varepsilon \mathbf{F}_{ij}^d \mathbf{F}_{ij}^\theta \) will be emphasized, as well as the numerical details of implementing such models using ABAQUS UMAT, UHYPER, UANISOHYPER_INV, UMAT, UEL, ..., and also constitutive model implementation in full Matlab and Python codes.

Recommended prerequisites: CVEN 5511 (intro to linear FEM), CVEN 5131 (nonlinear continuum mechanics), CVEN 6511 (small strain elastoplasticity, mixture theory, and nonlinear FEM), or equivalents, some knowledge of Matlab or Python and Fortran or C/C++ programming; or instructor consent. Introductory finite element method (FEM) and continuum mechanics courses are required.

Grading: problem sets 60%, take-home mid-term exam and programming assignment 15%, final project 25%.

Primary References:
-course notes written on board; journal articles and book chapters provided as pdf
Secondary References: (books in Engineering library or online as e-book), selected journal articles and book chapters provided as pdf on https://canvas.colorado.edu/


*recommended (but it mainly depends on reader preference based upon writing and presentation style of the authors)

**selected chapters provided as pdf
Course Outline: (tentative; subject to change by Instructor)

1. **Review of Nonlinear Continuum Mechanics (5 weeks):** (a) motion (material and spatial descriptions), curvilinear coordinates, base vectors, metric tensors, strain; (b) material time derivative, deformation rate, spin; (c) stress, objectivity, isotropy; (d) balance of momenta (material and spatial descriptions), thermodynamics: balance of energy, second law of thermodynamics, Clausius-Duhem inequality, Helmholtz free energy function, constitutive forms; (e) hyperelasticity, isochoic-volumetric split.

2. **Nonlinear Finite Element Implementation and Solution of Hyperelasticity (4 weeks):** (a) Total Lagrangian Finite Element (FE) formulation (strong form, weak form) with finite strain hyperelasticity; (b) linearization of weak form for solution by Newton-Raphson method, and algorithmic tangent moduli; (c) FE matrix form (1D uniaxial strain, 3D), Matlab and Python code, and ABAQUS User Material and Element subroutines (UHYPER, UMAT, UEL); (d) Total Lagrangian mixed 3D FE formulation and implementation of nearly-incompressible hyperelasticity via isochoic-volumetric split.

3. **Finite Strain Elasto-Plasticity (6 weeks):** (a) multiplicative decomposition of the deformation gradient \( F = F^e \cdot F^p \); (b) strains in reference, intermediate, and current configurations; (c) velocity gradients, deformation rates, and spins; (d) extensions to include thermal deformation, damage, and viscoelasticity; (e) elastic Lie derivative; (f) stress, objectivity, isotropy; (g) thermodynamics and constitutive model forms, reduced dissipation inequality, evolution equation forms; (h) yield and plastic potential functions for various materials (metals, geomaterials, ...); (i) Backward Euler time integration method, exponential map for isotropic materials; semi-implicit schemes; Matlab and Python code, and ABAQUS User Material subroutine (UMAT); (j) # elastic and plastic anisotropy.

4. **Nonlinear Multiphase Continuum Mechanics (1 week overview #):** (a) concept of volume fraction and mixture theory; (b) motion (material and spatial descriptions) and kinematics, material time derivative, deformation rate, spin; (c) balance of mass (material and spatial descriptions); (d) balance of linear momentum (material and spatial descriptions); (e) thermodynamics (first and second laws, constitutive equation forms, including Darcy’s law and effective stress principle at finite strain).

# if there is time

**FE software and Project:** The commercially-available FE software program ABAQUS will be used to learn how to implement finite strain constitutive models via UMAT (implicit, requires consistent tangent, pre-defined rotated configuration, Jaumann objective stress rate), UHYPER (implicit, requires derivatives of free energy with respect to deformation invariants), VUMAT (explicit, no consistent tangent, pre-defined rotated configuration, Green-McGinnis-Naghdi objective stress rate), as well as analyze engineering problems of interest to you using your implemented constitutive model in ABAQUS (with finite strain mixture theory) or another nonlinear finite element, finite difference, or meshfree program of your choosing. Matlab and Python codes will be used to demonstrate full source code finite element implementation of finite strain constitutive models and mixed formulation.
Concept of volume fraction in current configuration $B$: biphasic medium with fluid-filled pores and solid alveolar tissue

$$\text{d}v = \text{d}v_s + \text{d}v_f$$

- The solid skeleton $B = B_{skel}$ (image is lung parenchyma, Lande and Mitzner *J. Appl. Physiol. 2006*);
- (middle) foam (https://www.foambymail.com/);
- (right) sand liquefaction (www.cti.co.jp).

Special considerations: If you have a disability and require special accommodations, please provide Dr. Regueiro with a letter from Disability Services outlining your needs. Refer to the webpage https://www.colorado.edu/disabilityservices. If you have a conflict as a result of religious observances, notify Dr. Regueiro at least 2 weeks in advance of the exam or assignment due date: https://www.colorado.edu/policies/observance-religious-holidays-absences-classes-or-exams.

Review the required syllabus statements: https://www.colorado.edu/academicaffairs/policies-customs-guidelines/required-syllabus-statements.

Bechtel Computing Laboratory: Refer to instructions posted on the door of ECCE 157 or 161, if you do not currently have Buff OneCard swipe access to the Bechtel Lab in ECCE 157, 161. Instructions for remotely accessing Bechtel computers for use of ABAQUS are provided on the webpage.

Zoom link: https://cuboulder.zoom.us/j/98697852078

This course requires the use of the Zoom conferencing tool, which is currently not accessible to users using assistive technology. If you use assistive technology to access the course material, please contact your faculty member immediately to discuss.

For further assistance using Zoom, refer to this webpage: https://oit.colorado.edu/services/conferencing-services/web-conferencing-zoom