CVEN 6511: Nonlinear Finite Element Analysis of Solids and Porous Media

Meeting time and place: TuTh 8-9:15am, ECCE 1B41, [https://cuboulder.zoom.us/j/95931876178](https://cuboulder.zoom.us/j/95931876178)

Instructor: Prof. Rich Regueiro, 303.492.8026, richard.regueiro@colorado.edu, ECOT 421, hybrid office hours: Tu 9:30-11am, Th 1-2pm, [https://cuboulder.zoom.us/j/95931876178](https://cuboulder.zoom.us/j/95931876178)

Course Description: Nonlinear finite element analysis of solids and porous materials involving applications in structural engineering, geotechnical and geological engineering, mechanical and aerospace engineering, bioengineering, chemical engineering, and other modern engineering disciplines (see Fig[1]) has become more popular as computational modeling has advanced and computers have become faster and can handle larger amounts of data.

Figure 1. (left) Nonlinear FE analysis of failure in nuclear power plant concrete containment vessel ([www.simulia.com](http://www.simulia.com)). (center) Nonlinear coupled solid-fluid mechanical FE analysis of oil reservoir mechanics (Sandia). (right) Nonlinear FE brain poromechanics and skull mechanics (Motherway et al. Legal Medicine 2009).

The complexity of such problems stems from the inherent material nonlinearity of the solid, or solid phase of an inelastic porous material (such as soil, rock, or concrete skeleton, porous metal, foam, or extracellular matrix of a soft biological tissue), the coupled mechanical behavior of the solid and fluid phases, and the possibility of large strains and motions encountered in the boundary value problem. As a first course on nonlinear finite element analysis of solids and porous materials, attention will be focussed on small deformations while providing a basis for advanced computational studies in large deformation inelasticity, failure mechanics, additional multiphysics modeling (aside from poromechanics), etc., of solid-like materials.

The course will cover nonlinear constitutive modeling for solids (plasticity, visco-plasticity, visco-elasticity, ...), Finite Element (FE) implementation of constitutive models, FE implementation of coupled solid-fluid mechanical governing equations for inelastic porous materials, and nonlinear FE analysis of applications in structural, geotechnical, geological, mechanical, biomechanical, and other related modern engineering problems. Specifics of the numerical integration and FE implementation will be taught in the context of a commercially-available FE software program (see below). Governing equations for biphasic porous media (solid skeleton and pore fluid), will be derived and expressed in weak form for implementation by the FE method. Steady state and transient conditions will be considered.

Course Objective: To develop the mathematical language, numerical skills, and thought process to formulate, numerically implement, and use nonlinear constitutive models and/or poromechanics for nonlinear finite element analysis of solids and porous materials.

Prerequisites: CVEN 5511 (Intro Linear FEM) and CVEN 5131 (Co-Req, Continuum Mechanics), or their equivalents, some knowledge of Matlab or Python, and C or Fortran programming; or instructor consent.

Grading: Problem sets 60%, in-class midterm exam 20%, final project 20%.

References: course notes provided as pdf file.

Course Outline: (tentative)

1. Overview of nonlinear FE method and Newton solution methods (5 weeks): (a) Newton-Raphson and Quasi-Newton methods, Line-search, Arc-length methods; (b) nonlinear elastostatics of axially-loaded bar at small strain, and nonlinear FEM.

2. 1D and 3D elasto-plasticity for solids (6 weeks): (a) uniaxial stress 1D elasto-plasticity, 3D deviatoric (isochoric) plasticity: deviatoric J2 plasticity (thermodynamics, yield criteria, Kuhn-Tucker conditions, evolution of internal variables for isotropic and kinematic hardening/softening); numerical time integration (return mapping algorithm) and consistent tangent operator for FE implementation; reduction to 1D elasto-plasticity; (b) 3D pressure-sensitive plasticity for soil, rock, concrete: Mohr-Coulomb, Drucker-Prager, cap, and three-invariant models; return mapping/numerical integration; consistent tangent (time permitting); (c) other constitutive models for solid phase: viscoelasticity, thermoelasticity, elasto-damage, ... student interest (time permitting).

3. Balance laws and FE implementation for bi-phasic (solid-fluid) porous media (4 weeks): (a) bridging pore-scale to continuum scale through concept of volume fraction and mixture theory; (b) conservation laws; (c) Darcy’s law; (d) effective stress principle; (e) thermodynamics of mixtures (1st and 2nd laws); (f) coupled strong and weak forms; (g) coupled matrix FE equations; (h) time integration for transient analysis (generalized trapezoidal rule); (i) consideration of inertia terms (time permitting).

FE software and Final Project: The commercially-available FE software program ABAQUS www.simulia.com as well as Matlab or Python will be used throughout the course to learn how to implement a nonlinear finite element program for small deformation elasto-plasticity, poromechanics, as well as analyze engineering problems of interest. The project will have two options:

(1) use FE software to solve an engineering problem of your choosing that exercises a nonlinear constitutive model for solids or porous materials and calibrate/compare your simulation results against experimental data or a field case study, or
(2) implement in ABAQUS and/or Matlab or Python (or other programming language) a new elastoplastic constitutive model (or from a journal paper), and/or poromechanical finite element, and verify your implementation against an analytical, or a separate numerical, solution.

To learn more about the methodology of verification and validation (V&V) refer to the following:


These are assigned as self-reading for you to become familiar with the terms “verification” and “validation,” and the difference between “calibration” and “validation.” They have very different meanings.

Special considerations: If you have a disability and require special accommodations, provide Prof. Regueiro with a letter from Disability Services outlining your needs. Refer to the webpage http://www.colorado.edu/disabilityservices If you have a conflict as a result of religious observances, please notify Prof. Regueiro at least 2 weeks in advance of the exam or assignment due date, https://www.colorado.edu/policies/observance-religious-holidays-absences-classes-or-exams

Bechtel Computing Laboratory: To gain access to the Bechtel Lab in ECCE 157, 161, follow instructions posted on the door. Once inside, you can login to the machines with your identikey username and password.