CVEN 6511: Nonlinear Finite Element Analysis of Solids and Porous Media

Meeting time and zoom link: TuTh 8-9:15am, `https://cuboulder.zoom.us/j/97558795230`. Recorded lectures posted after class here.

Instructor: Professor Richard Regueiro, 303.492.8026, `richard.regueiro@colorado.edu`; online office hours:
Su, 9-10pm, `https://cuboulder.zoom.us/j/92346548353`
Tu, 9-10pm, `https://cuboulder.zoom.us/j/93089633057`

Course Assistant: Mr. Thomas Allard, `thomas.allard@colorado.edu`; online office hours:
M, 4-5pm, `https://cuboulder.zoom.us/j/98301256223`
W, 5-6pm, `https://cuboulder.zoom.us/j/98803686383`

Course Description: Nonlinear finite element (FE) analysis of solids and porous materials involving applications in structural engineering, geotechnical and geological engineering, mechanical and aerospace engineering, bioengineering, chemical engineering, and other modern engineering disciplines (see Fig.1) has become more popular as computational modeling has advanced and computers have become faster and can handle larger amounts of data.

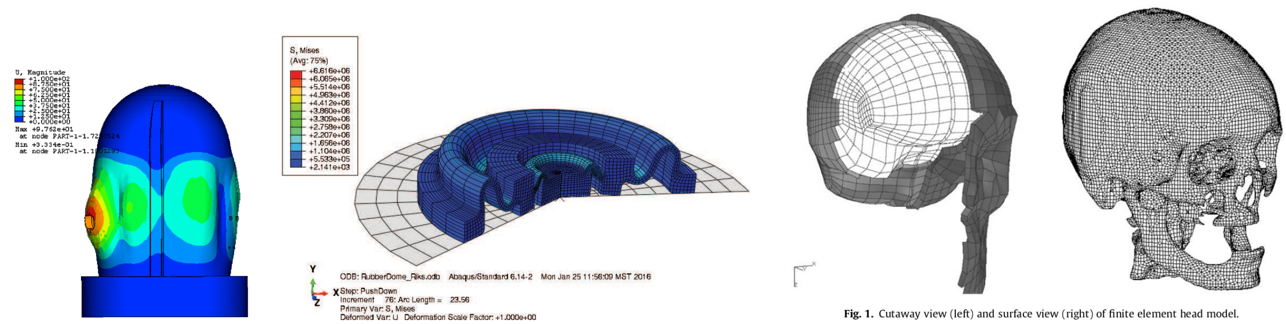

Figure 1. (left) Nonlinear FE analysis of failure in nuclear power plant reinforced concrete containment vessel (`www.simulia.com`). (center) Nonlinear FE analysis of post-buckled rubber keyboard key mechanism (courtesy of Ian Stevenson, Simulia). (right) Nonlinear FE analysis of brain poromechanics and skull mechanics (Motherway et al. *Legal Medicine* 2009).

The complexity of such problems stems from the *inherent material nonlinearity of the solid, or solid skeleton* of an inelastic porous material (such as soil, rock, concrete, porous metal, foam, or extracellular matrix of a soft biological tissue), the coupled mechanical behavior of the solid and fluid phases, and the possibility of large strains and motions encountered in the boundary value problem. As a **first course on nonlinear finite element analysis of solids and porous materials**, attention will be focussed on small deformations while providing a basis for advanced computational studies in large deformation inelasticity, failure mechanics, additional multiphysics modeling (aside from poromechanics), etc., of solid-like materials.

The course will cover nonlinear inelastic constitutive modeling for solids (elasto-plasticity, visco-plasticity, visco-elasticity, elasto-damage, thermo-elasticity, ...), Finite Element (FE) implementation of constitutive models, and an overview (time permitting) of FE implementation of coupled solid-fluid mechanical governing equations for inelastic porous materials. Specifics of the numerical integration and FE implementation will be taught in the context of a commercially-available FE software program (see below) as well as short `Python` codes. Steady state and transient conditions will be considered.

Course Objective: *To develop the mathematical language, numerical skills, and thought process to formulate, numerically implement, and use nonlinear inelastic constitutive models (and potentially poromechanics) for nonlinear finite element analysis of solids and porous materials.*

Suggested Prerequisites: CVEN 5511 (or any introductory linear Finite Element Method (FEM) course) and CVEN 5131 (recommended Co-Req, or any introductory continuum mechanics course), or their equivalents, some knowledge of Python, and C or Fortran programming; or instructor consent.

Grading: problem sets 60% (normally due as a pdf file on canvas by 11:59pm on a Wednesday, but not every week; **no late problem sets will be accepted**; ask questions early and often!), take-home mid-term exam 15% (most likely given in the first half of April; it will be timed, such as 24 or 48 hours), final project 20% (report due by 11:59pm, May 8, last day of the final exam period; presentation during the CVEN 6511 course final exam period (May 4, 7:30-10pm) or as scheduled), class participation 5%*.

*The *class participation* grade is based on in-class attendance (you may miss up to 2 class sessions without an excuse), and you must participate in "in-class" discussion through zoom breakout sessions when they happen. The reasoning is as follows:

(i) Based on my experience, students who rely solely on watching recordings of lectures find themselves re-watching the lectures multiple times to understand the lecture content coverage, as opposed to watching once to refresh what they learned when attending "in-class" participating synchronously.

(ii) Since this course is fully online, students have commented in the past on how they don't meet other students online, and I will try to address this feedback by having more frequent zoom breakout sessions during class time for you all to interact with each on zoom (and then hopefully meet up in person on campus).

Also, **when asking a question during class, please show your video**. I know it's early in the morning, but this demonstration of participation will be greatly appreciated!

Grading of coding assignments: Most Problem Sets (starting with Problem Set 2) have code-writing portions. Template Python code will be provided for those who are not proficient programmers and thus need a start to their codes. When problem sets are graded, I will show my solution Python code in the recording, but not provide the working code files themselves. You may receive back up to 50% of points deducted due to non-working code after graded problem sets are returned and my working Python code is shown on zoom (with recording available after each class with URL link on canvas), assuming you get your code to work.

References: course notes provided as pdf file.

- A. Anandarajah, *Computational Methods in Elasticity and Plasticity: Solids and Porous Media*, Springer, 2010. `https://libcat.colorado.edu/Record/b6622570`
- J. Bear, *Dynamics of Fluids in Porous Media*, American Elsevier, NY, 1972. `https://libcat.colorado.edu/Record/b7451104`
- T. Belytschko, W.-K. Liu, B. Moran, *Nonlinear Finite Elements for Continua and Structures*, John Wiley, NY, 2000. `https://libcat.colorado.edu/Record/in00000089213`
- R. de Boer, *Theory of Porous Media: Highlights in the Historical Development and Current State*, Springer, 2000. `https://libcat.colorado.edu/Record/b8007516`
- R. de Boer, *Trends in Continuum Mechanics of Porous Media*, Springer, 2005. `https://libcat.colorado.edu/Record/b5495120`
- O. Coussy, *Poromechanics*, Wiley, 2004. `https://libcat.colorado.edu/Record/b12203408`

- M.A. Crisfield, *Non-linear Finite Element Analysis of Solids and Structures*, Vols. 1 (1991) and 2 (1997), Wiley. `https://libcat.colorado.edu/Record/b7609123`
- F. Dunne, N. Petrinic, *Introduction to Computational Plasticity*, Oxford University Press, 2005. `https://libcat.colorado.edu/Record/b9605827`
- R.M. Ferencz, T.J.R. Hughes, "Iterative Finite Element Solutions in Nonlinear Solid Mechanics," *Handbook of Numerical Analysis*, Vol.VI, eds. P.G. Ciarlet and J.L. Lions, Elsevier Science, 1998, pg.3-178.
- T.J.R. Hughes, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*, Prentice-Hall, 1987. `https://libcat.colorado.edu/Record/b7459745`
- J. Lubliner, *Plasticity Theory*, Macmillan Pub., NY, 1990. `https://libcat.colorado.edu/Record/b10848188`
- S. Nemat-Nasser, *Plasticity: a treatise on the finite deformation of heterogeneous inelastic materials*, Cambridge University Press, 2004.
- N.S. Ottosen, M. Ristinmaa, *The Mechanics of Constitutive Modeling*, Elsevier, 2005. `https://libcat.colorado.edu/Record/b9607672`
- J.C. Simo, T.J.R. Hughes, *Computational Inelasticity*, Springer-Verlag, 1998. `https://libcat.colorado.edu/Record/b7982161`
- J.C. Simo, "Numerical Analysis and Simulation of Plasticity," *Handbook of Numerical Analysis*, Vol. VI, eds. P.G. Ciarlet and J.L. Lions, Elsevier Science, 1998, pg.183-499. `https://libcat.colorado.edu/Record/b10328465`
- E.A. de Souza Neto, D. Peric, D.R.J. Owen, *Computational Methods for Plasticity: Theory and Applications*, Wiley, 2008. `https://libcat.colorado.edu/Record/b12208558`

Course Outline: (tentative)

1. **Overview of nonlinear FE method and Newton solution methods (6 weeks)**: (a) Newton-Raphson and Quasi-Newton methods, Line-search, Arc-length methods; (b) nonlinear elastostatics and elastodynamics of axially-loaded bar at small strain, and nonlinear FE method.

2. **1D and 3D inelasticity for solids (8 weeks)**: (a) uniaxial stress 1D elasto-plasticity, 3D deviatoric (isochoric) plasticity: deviatoric J2 plasticity (thermodynamics, yield criteria, Kuhn-Tucker conditions, evolution of internal variables for isotropic and kinematic hardening/softening); numerical time integration (return mapping algorithm) and consistent tangent operator for FE implementation; reduction to 1D elasto-plasticity; (b) other inelastic constitutive models for solids: visco-elasticity, visco-plasticity, elasto-damage, thermo-elasticity, ...

3. **Overview of balance laws and FE implementation for bi-phasic (solid-fluid) porous media (1 week, time permitting)**: (a) concept of volume fraction and mixture theory; (b) conservation laws; (c) Darcy's law; (d) effective stress principle; (e) thermodynamics of mixtures (1st and 2nd laws); (f) coupled strong and weak forms; (g) coupled matrix FE equations; (h) time integration for transient analysis (generalized trapezoidal rule).

FE software and Final Project: The commercially-available FE software program `ABAQUS` (`www.simulia.com`) as well as `Python` will be used throughout the course to learn how to implement a nonlinear finite element program for small deformation nonlinear elasticity, elasto-plasticity, other inelastic constitutive models, poromechanics (time permitting), as well as analyze engineering problems of interest. The final project will have two options:

(1) use FE software to solve an engineering problem of your choosing that exercises a nonlinear inelastic constitutive model for solids or porous materials and calibrate or compare your simulation results against experimental data or a field case study, or
(2) implement in ABAQUS and/or Python (or other programming language) a new elasto-plastic (or other inelastic) constitutive model (or from a journal paper), and/or poromechanical finite element, and verify your implementation against an analytical, or a separate numerical, solution.

To learn more about the methodology of verification and validation (V&V) refer to the following:

- Oberkampf et al. 2004, "Verification, validation, and predictive capability in computational engineering and physics," Appl. Mech. Rev. 57:345-84.
- Babuska & Oden 2004, "Verification and validation in computational engineering and science: basic concepts," Comput. Methods Appl. Mech. Engrg. 193:4057-4066.
- Schwer 2007, "An overview of the PTC 60/V&V 10: guide for verification and validation in computational solid mechanics," Engineering with Computers 23:245-252.
- 2019, "Verification & Validation of Computational Models Associated with the Mechanics of Materials," The Minerals, Metals & Materials Society, `https://www.doi.org/10.7449/VandV_1`.

These are assigned as self-reading for you to become familiar with the terms "verification" and "validation," and the difference between "calibration" and "validation." They have different meanings, although they are related.

Special considerations: If you have a disability and require special accommodations, please provide Prof. Regueiro with a letter from Disability Services outlining your needs. Refer to the webpage `http://www.colorado.edu/disabilityservices`. If you have a conflict as a result of religious observances, please notify Prof. Regueiro at least 2 weeks in advance of the exam or assignment due date.

Syllabus rules: all other syllabus rules are provided here.

Bechtel Computing Laboratory: To gain physical access to the Bechtel Lab in ECCE 157 and 161, follow the instructions posted on the door. Once inside, you can login to the machines with your identikey username and password. For remote access to the computers on weekends and between 6pm and 7am on weekdays, refer to these instructions.