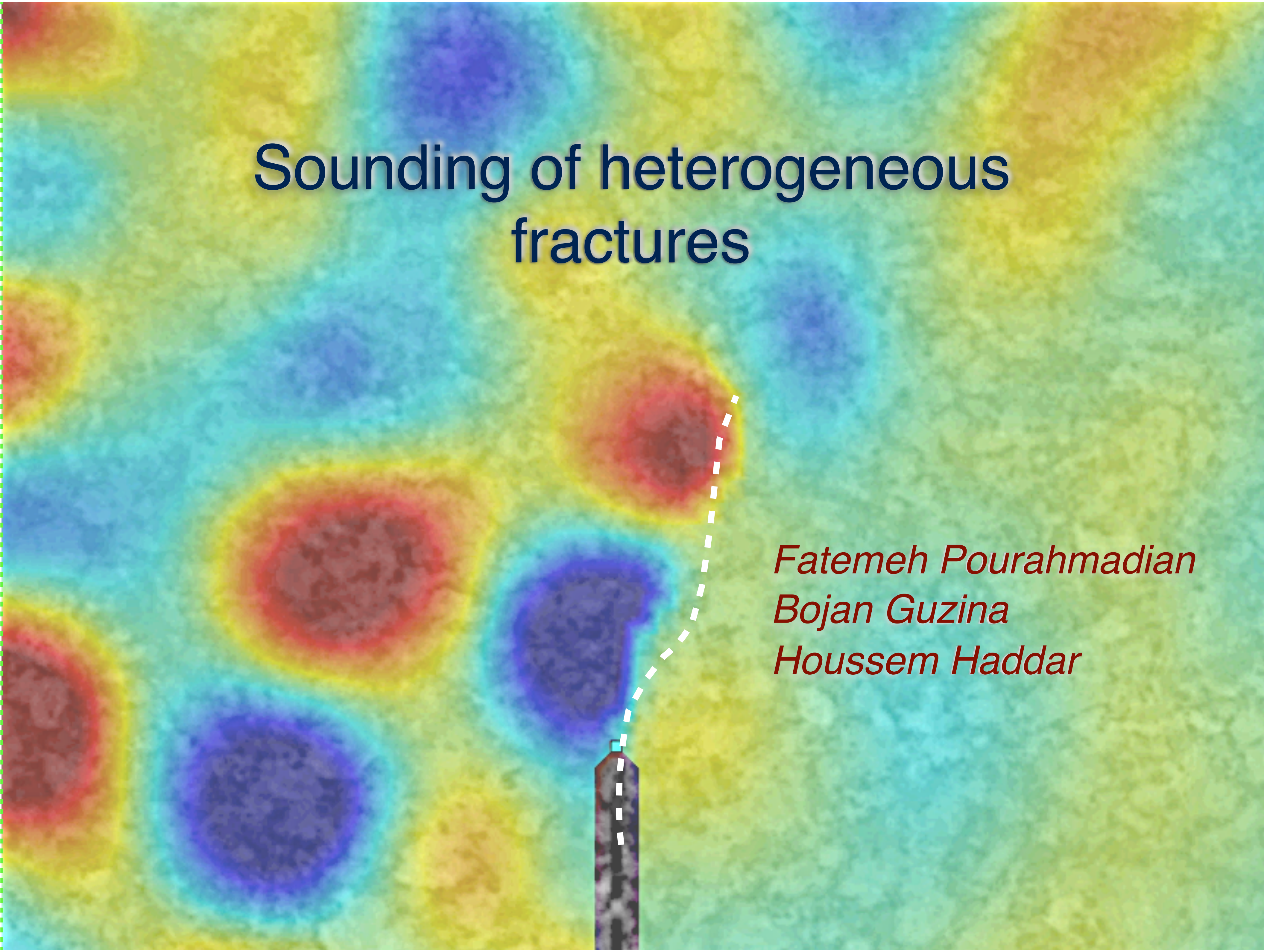
Sounding of heterogeneous fractures

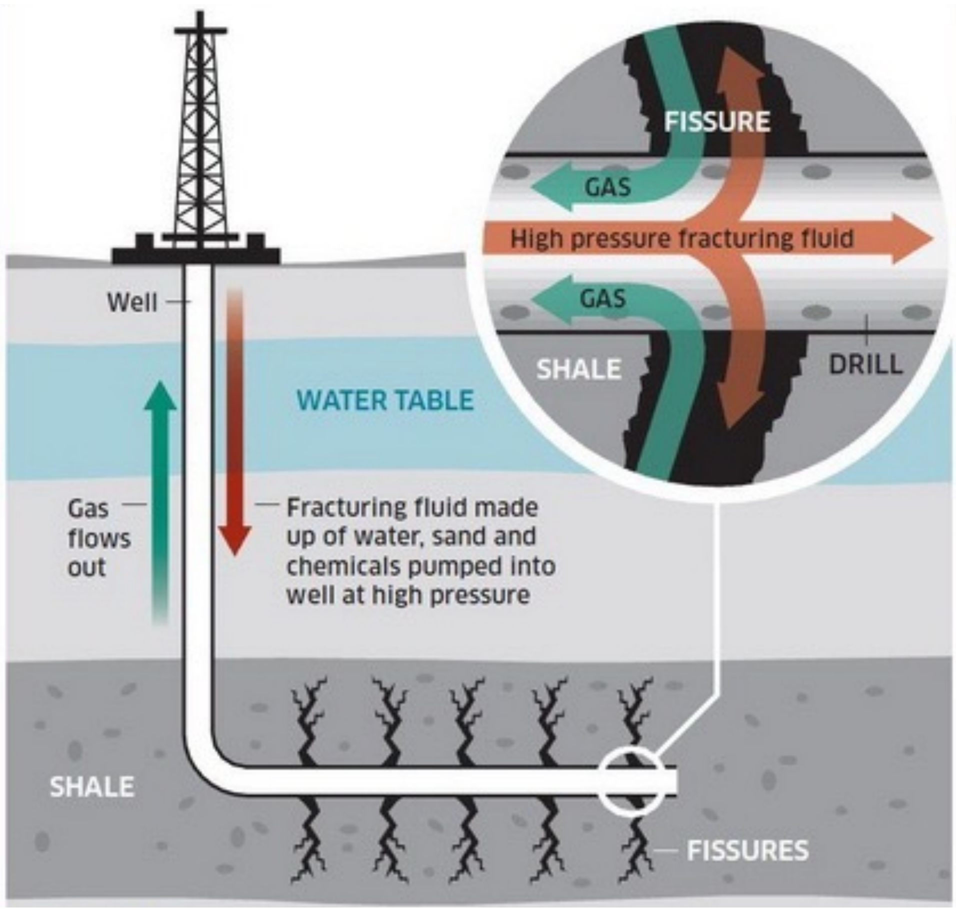


Fatemeh Pourahmadian
Bojan Guzina
Houssem Haddar

Mapping & characterization of fractures

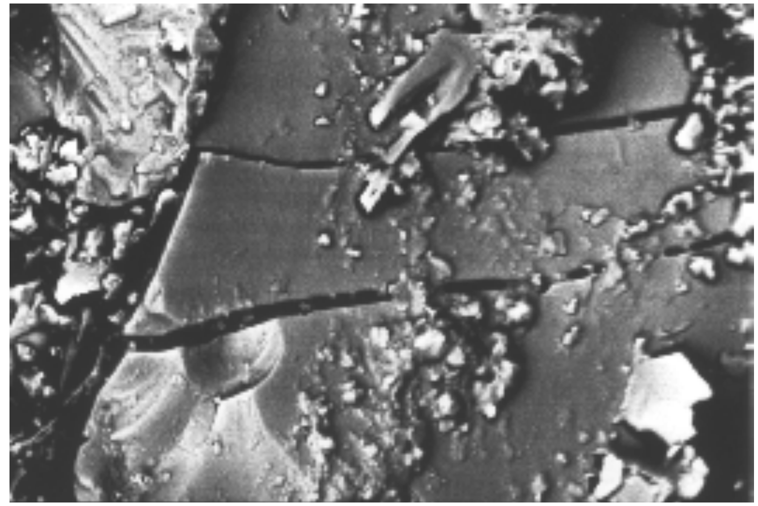
Gas/Geo-thermal energy
enhanced/optimal production

R Snieder
CM Sayers
P Geiser
B Lecampion

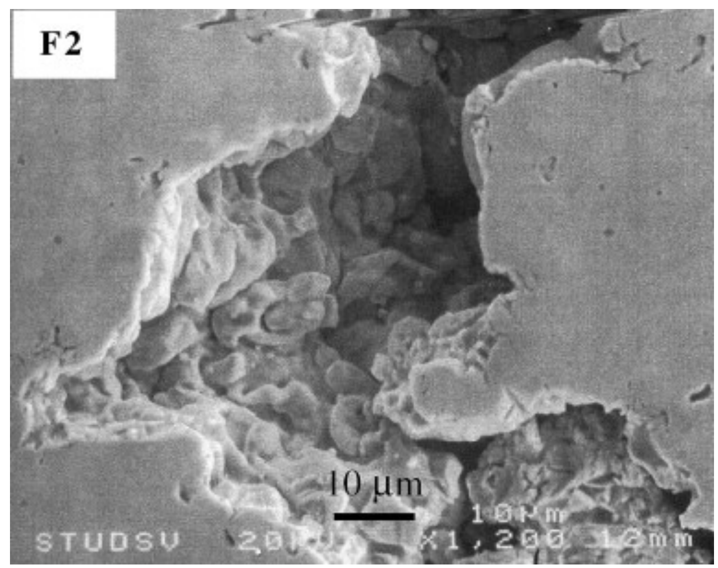


United States Geological Survey Bulletin 426

Non-destructive evaluation
early-stage fracture tomography



Brittle fracturing in sandstone, D. T. Nicholson

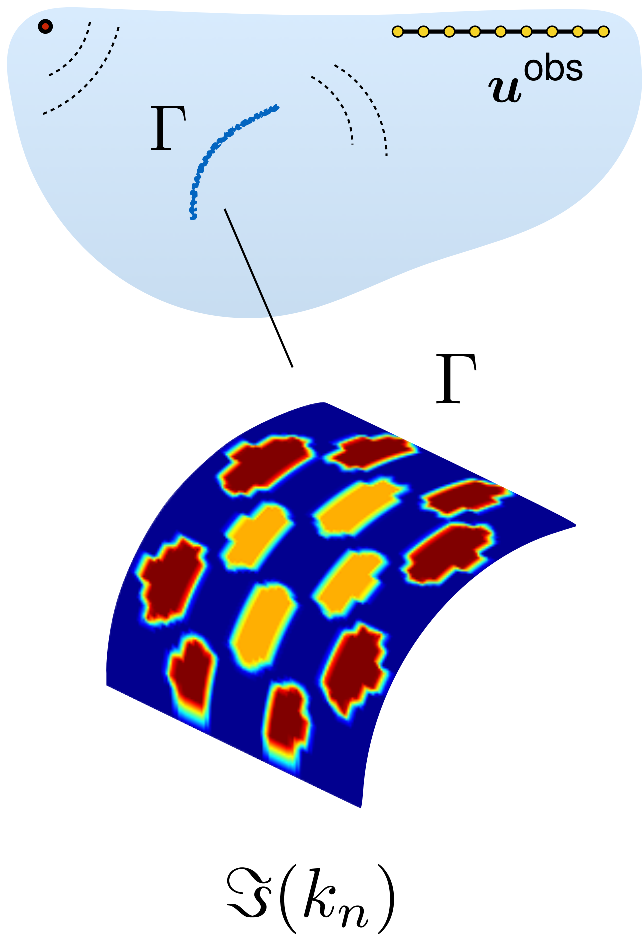


Interface of a heterogeneous fracture, C. Pecorari

G Papanicolaou
Liliana Borcea
H Haddar
L Audibert

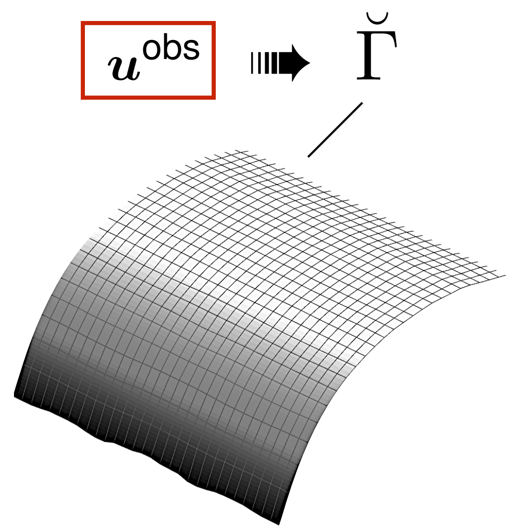
Geometric Reconstruction

Seismic experiment



Geometric reconstruction

Non-iterative waveform tomography

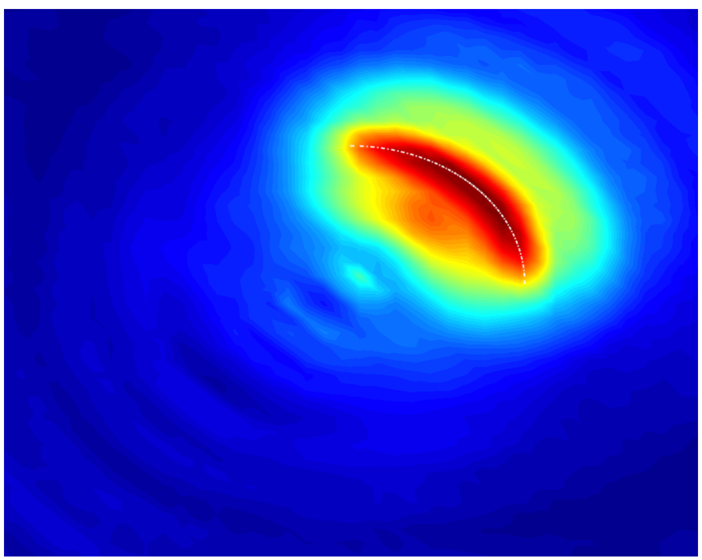


3D imaging of fractures

- non-iterative
- shape reconstruction
- agnostic w.r.t. interfacial condition
- flexible sensing configuration
- arbitrary geometries

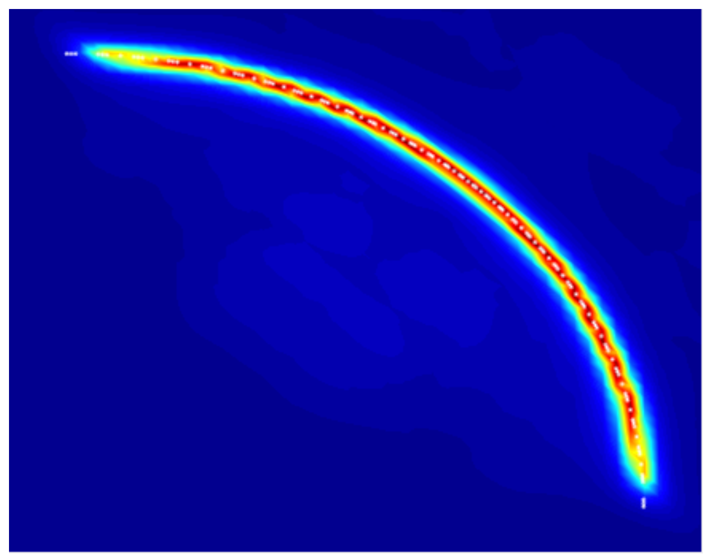
Topological Sensitivity

- heuristic
- robust against noise
- shape reconstruction requires high-frequency data

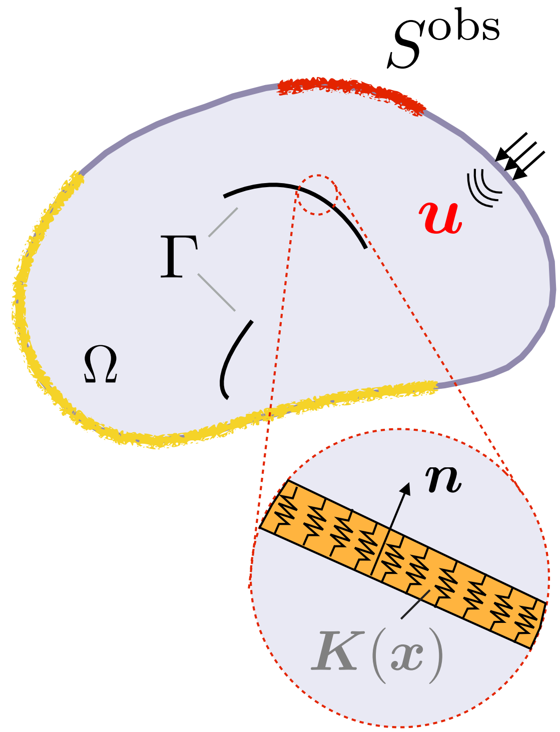


Generalized Linear Sampling Method

- rigorous
- superior localization properties
- obstacles in EM



TS for heterogeneous fractures



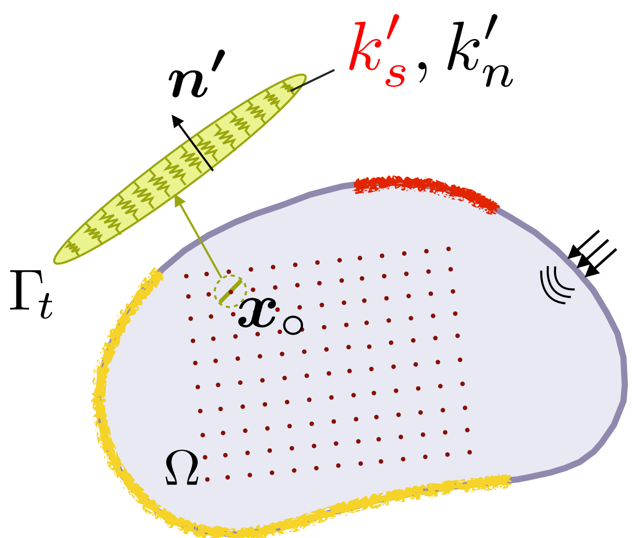
Field equations

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{u}) + \rho\omega^2\boldsymbol{u} = \boldsymbol{0}, \qquad \Omega\backslash\overline{\Gamma}$$

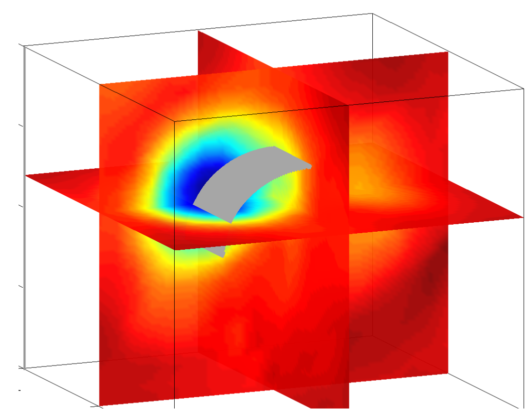
$$\boldsymbol{n}\cdot\boldsymbol{C}:\nabla\boldsymbol{u} = \boldsymbol{K}(x)[\![\boldsymbol{u}]\!], \qquad \Gamma$$

Least-squares cost functional

$$J(\Gamma_t) = J(\emptyset) + \epsilon^{\mathrm{N}}\,\mathsf{T}(\boldsymbol{x}_\circ) + o(\epsilon^{\mathrm{N}})$$



FP & BG, *Int J Solids Struct* (2015)



TS formula

$$\mathsf{T}(\boldsymbol{x}_\circ, k'_n, k'_s) = \nabla\boldsymbol{u}_{\mathrm{i}}\cdot\mathbf{A}\cdot\nabla\boldsymbol{u}_{\mathrm{a}}$$

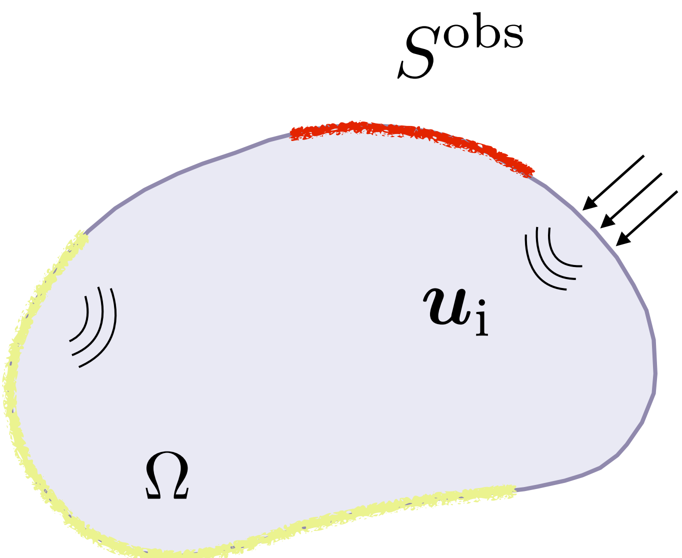
TS for heterogeneous fractures

TS formula

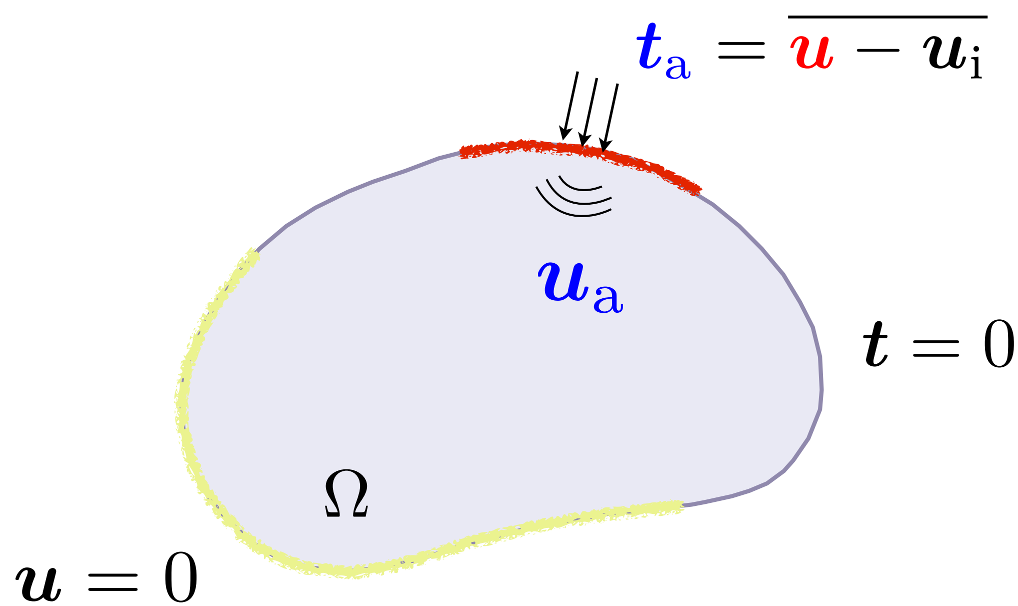
4th order polarization tensor

$$\mathsf{T}(\boldsymbol{x}_\mathrm{o}, k'_n, k'_s) = \nabla \boldsymbol{u}_\mathrm{i} \cdot \mathbf{A} \cdot \nabla \boldsymbol{u}_\mathrm{a}$$

Free field

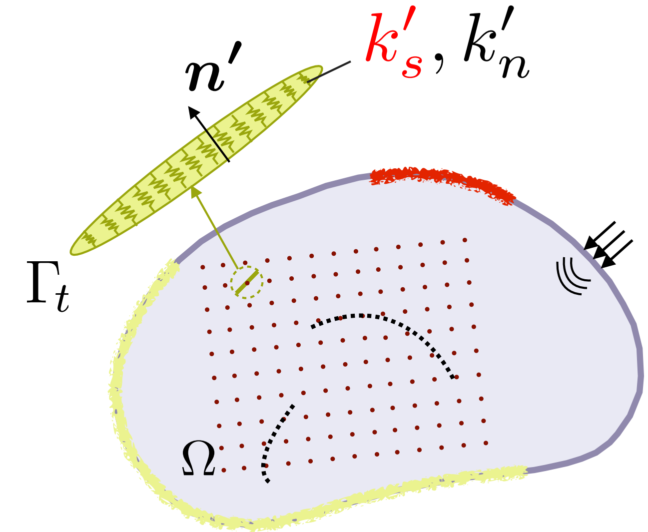


Adjoint field



Polarization tensor

$$\mathsf{T}(\boldsymbol{x}_{\rm o}, k'_n, k'_s) = \nabla \boldsymbol{u}_{\rm i} \cdot \mathbf{A} \cdot \nabla \boldsymbol{u}_{\rm a}$$



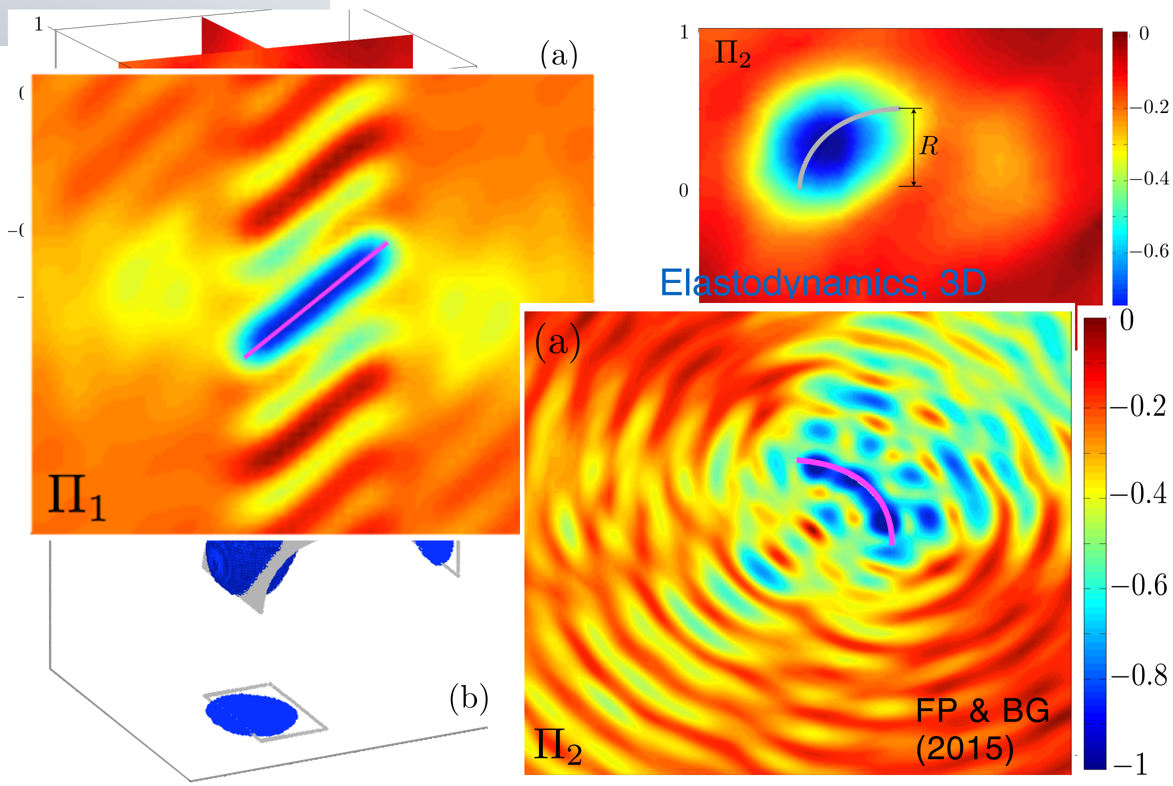
Polarization tensor

$$\mathbf{A} = \alpha_n (\boldsymbol{n}' \otimes \boldsymbol{n}' \otimes \boldsymbol{n}' \otimes \boldsymbol{n}') + \alpha_s \sum_{i=1}^{2} (\boldsymbol{n}' \otimes \boldsymbol{e}'_i + \boldsymbol{e}'_i \otimes \boldsymbol{n}') \otimes (\boldsymbol{n}' \otimes \boldsymbol{e}'_i + \boldsymbol{e}'_i \otimes \boldsymbol{n}')$$

$$\alpha_n(k'_n) = \frac{8(1-\nu)(2\nu-1)}{3\mu(k'_n\Lambda + 2\nu + 1)}$$

$$\alpha_s(k'_s) = \frac{4(1-\nu^2)}{3\mu(2-\nu)(k'_s\Lambda + \nu + 1)}$$

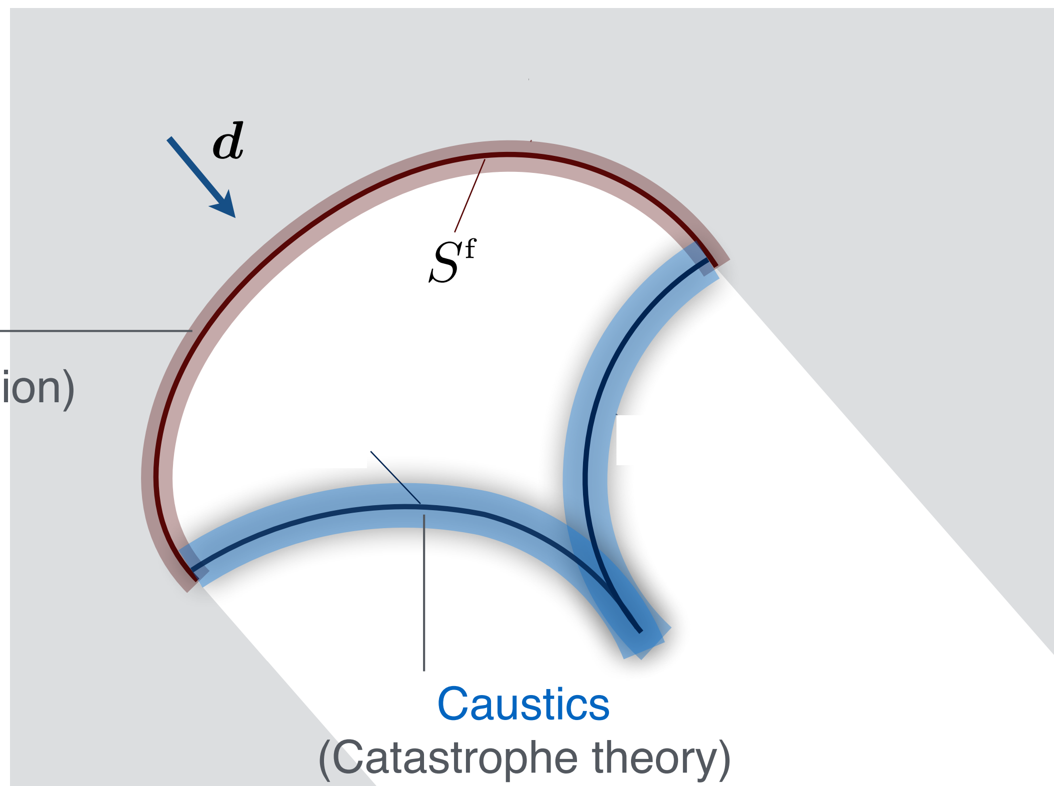
FP & BG, Int J Solids Struct (2015)



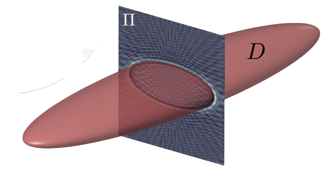
High-frequency TS

$$\mathsf{T}(\boldsymbol{x}^{\circ}) = \int_{S^{\mathrm{f}}} f(\boldsymbol{\zeta})\, e^{\mathrm{i}k\varphi(\boldsymbol{\zeta})}\, \mathrm{d}\boldsymbol{\zeta}$$

Single plane-wave illumination



High-frequency TS

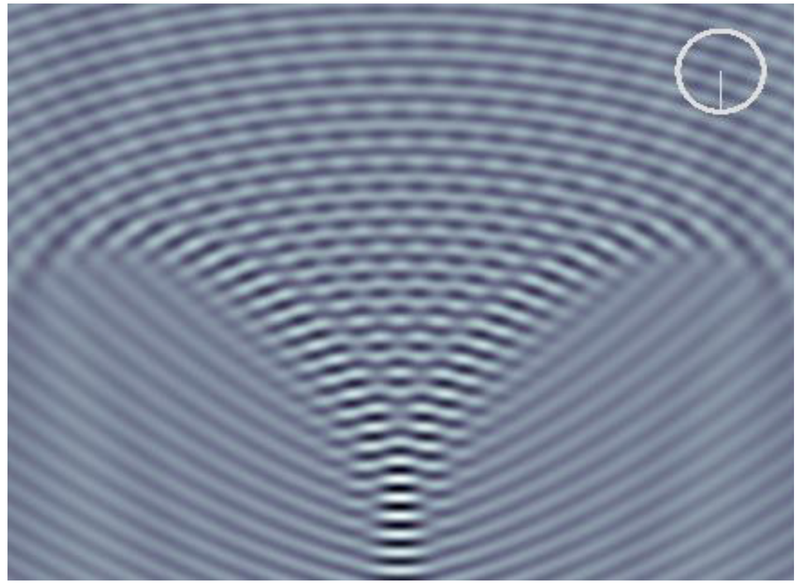


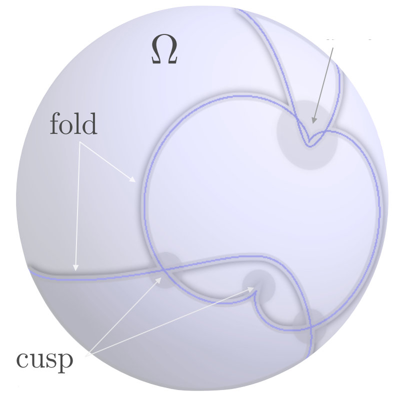
Full aperture illumination

Stationary phase (self-cancellation) $O(k^{\frac{1}{3}})$

Caustics (diffraction scaling) $O(k^{\frac{2}{3}})$

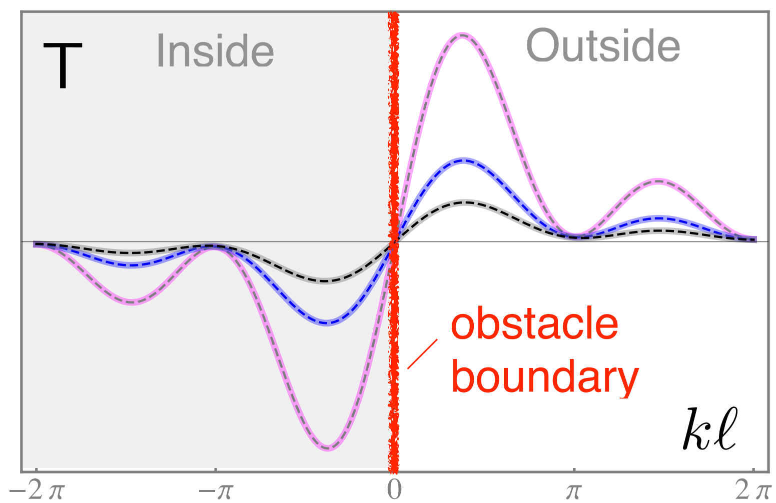
near boundary $O(k)$

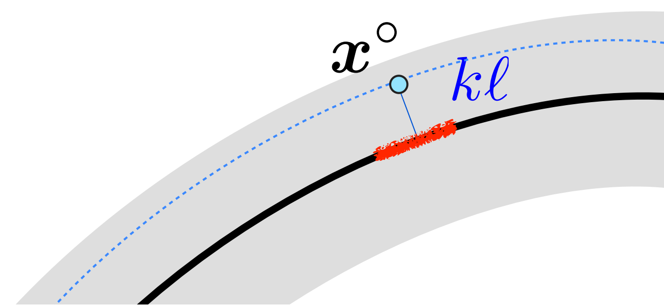




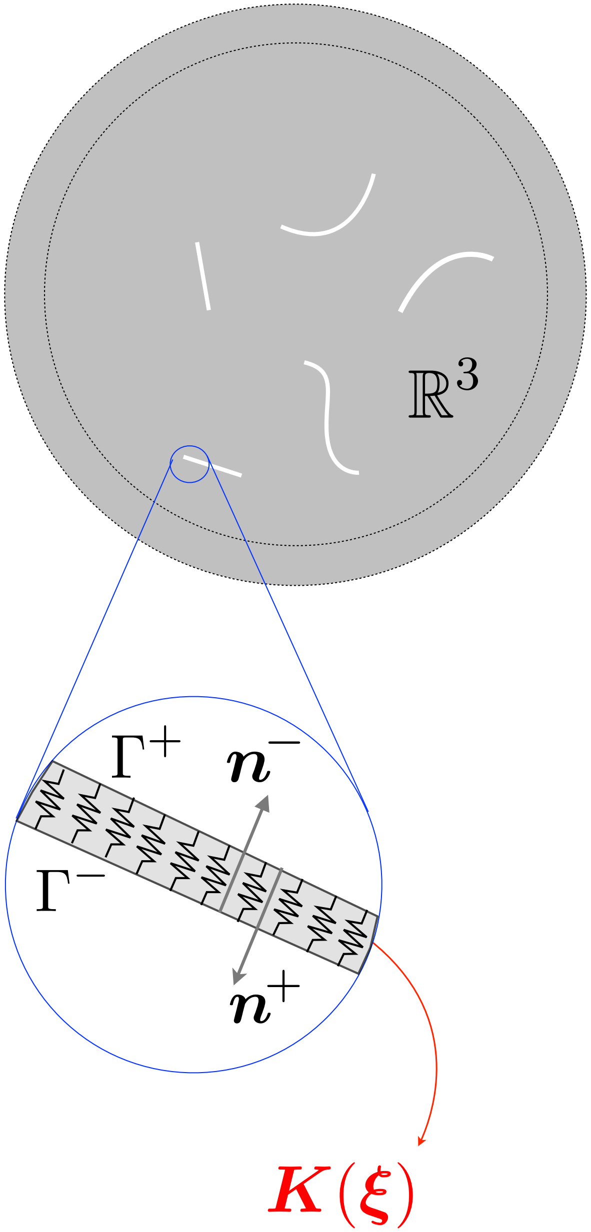
$$\mathrm{T} \doteq \frac{\pi k}{(k\ell)^3}\left\{\frac{3(1-\beta)}{2+\beta}\left(k\ell\cos(k\ell)-\sin(k\ell)\right)^2-(1-\beta\gamma^2)\,(k\ell)^2\sin(k\ell)^2\right\}$$

BG & FP, *Proc R Soc A* (2015)





Generalized Linear Sampling Method



1998-
Colton, Kirsch, Kress, Monk, Arens, Guzina, Nintcheu Fata, Madyarov , Bellis, Bonnet, ...

Obstacles

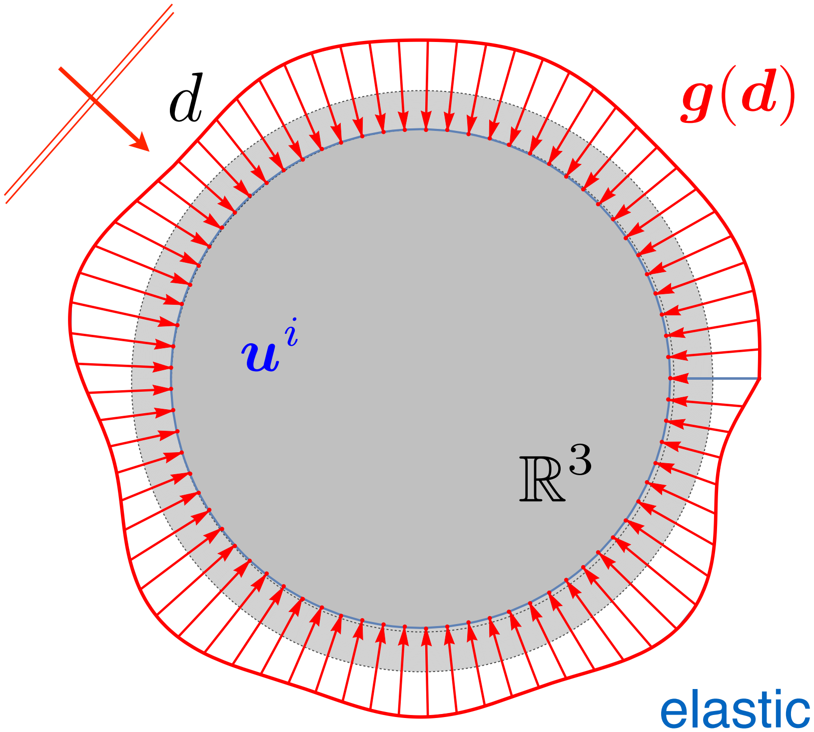
2003-
Cakoni, Haddar, Boukari, Kress, Ritter, Potthast, Monch, Park, Bourgeois, ...

EM Cracks

2014-
Haddar, Audibert, Cakoni, ...

GLSM

Generalized Linear Sampling Method



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EM Cracks

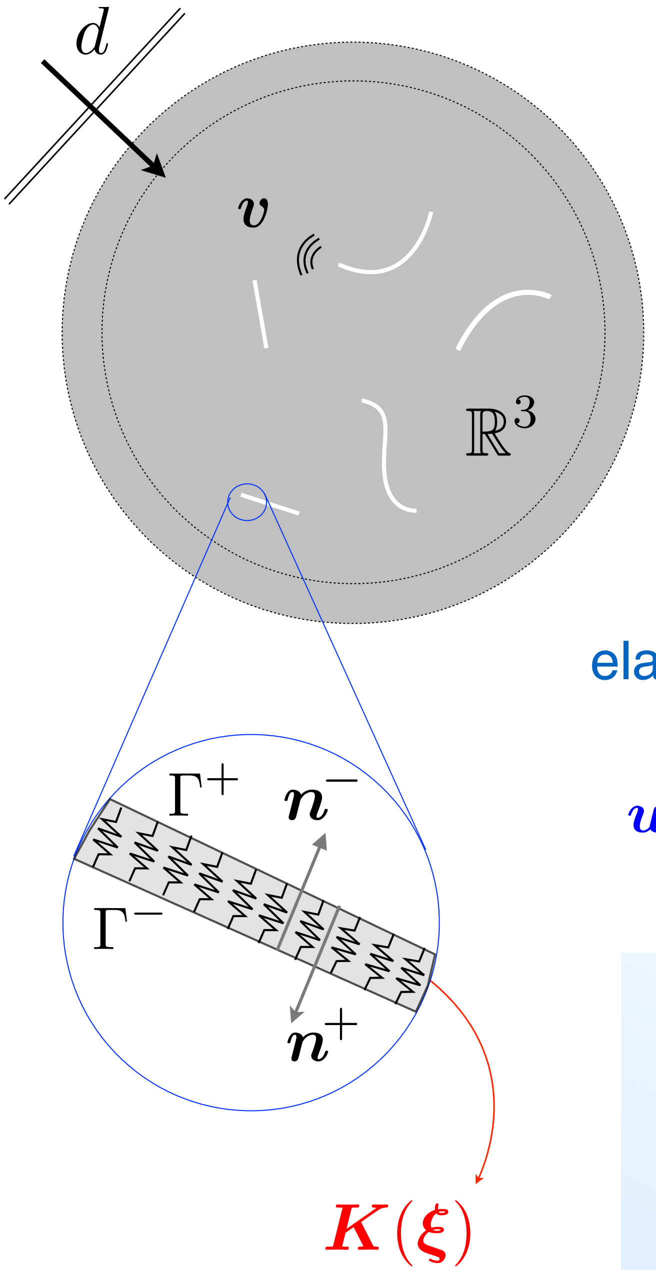
2014-
Haddar, Audibert, Cakoni, ...

GLSM

elastic Herglotz wave function

$$\boldsymbol{u}^i(\boldsymbol{\xi}) = \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_p(\boldsymbol{d})\, e^{\mathrm{i}k_p \boldsymbol{d}\cdot\boldsymbol{\xi}}\, \mathrm{d}S_{\boldsymbol{d}} + \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_s(\boldsymbol{d})\, e^{\mathrm{i}k_s \boldsymbol{d}\cdot\boldsymbol{\xi}}\, \mathrm{d}S_{\boldsymbol{d}}$$

Generalized Linear Sampling Method



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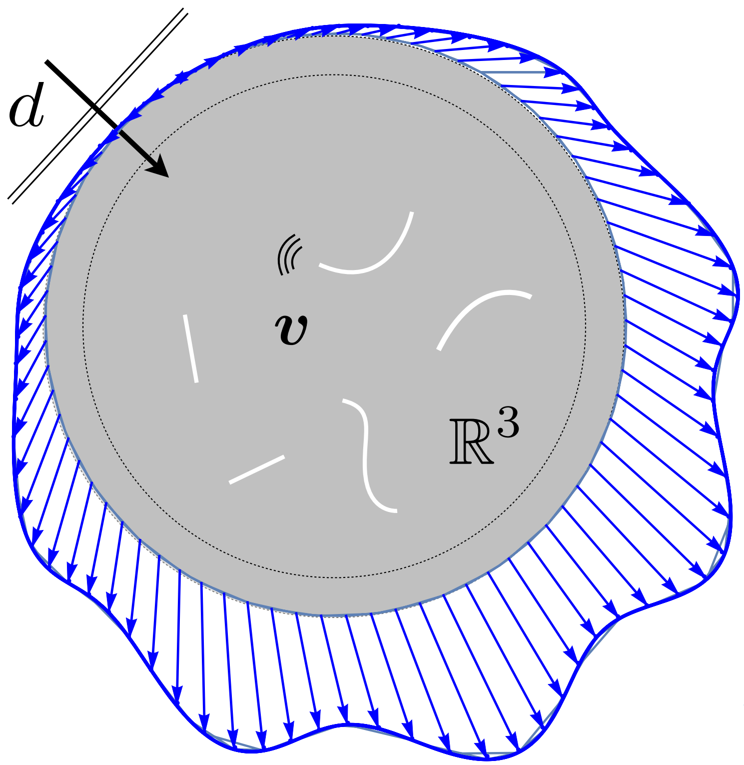
scattered field

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) + \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) = 0,$$

FOD

$$\boldsymbol{n}\cdot\boldsymbol{C}:\nabla\boldsymbol{v} = \boldsymbol{K}(\boldsymbol{\xi})[\![\boldsymbol{v}]\!] - \boldsymbol{t}^f,$$

Generalized Linear Sampling Method



scattered field

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) + \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) = 0,$$

FOD

$$\boldsymbol{n}\cdot\boldsymbol{C}:\nabla\boldsymbol{v} = \boldsymbol{K}(\boldsymbol{\xi})[\![\boldsymbol{v}]\!] - \boldsymbol{t}^f,$$

Scattered far field patterns

unit direction of observation

$$\boldsymbol{v}(\boldsymbol{\xi}) = \frac{e^{ik_p|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|(\lambda+2\mu)}\boldsymbol{v}_p^\infty(\hat{\boldsymbol{\xi}}) + \frac{e^{ik_s|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|\mu}\boldsymbol{v}_s^\infty(\hat{\boldsymbol{\xi}})$$

Physical experiment

$$\boldsymbol{W}^\infty(\boldsymbol{d},\hat{\boldsymbol{\xi}})$$

Far-field pattern observed along $\hat{\boldsymbol{\xi}}$ due to incident wave in direction $\boldsymbol{d}$

Far field operator

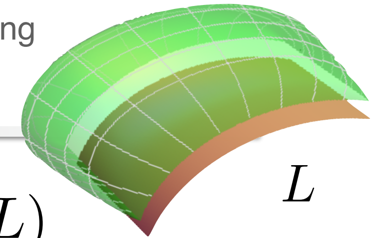
$$\boldsymbol{v}^\infty := F(\boldsymbol{g}) := \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}(\boldsymbol{d})\cdot\boldsymbol{W}^\infty(\boldsymbol{d},\hat{\boldsymbol{\xi}})\,\mathrm{d}S_{\boldsymbol{d}}$$

$(\boldsymbol{v}_p^\infty, \boldsymbol{v}_s^\infty)$ $(\boldsymbol{g}_p, \boldsymbol{g}_s)$

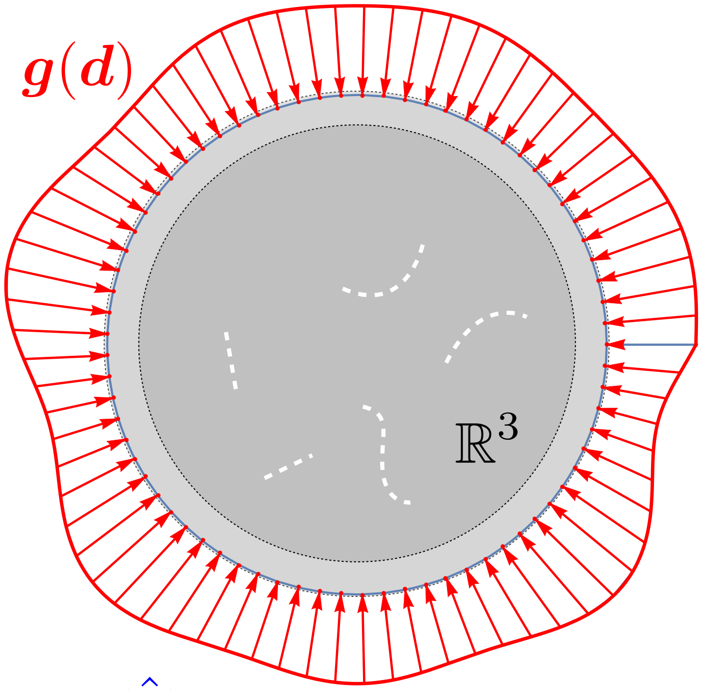
unit sphere

GLSM IDEA

Fracture opening displacement



$\boldsymbol{a} \in \tilde{H}^{\frac{1}{2}}(L)$



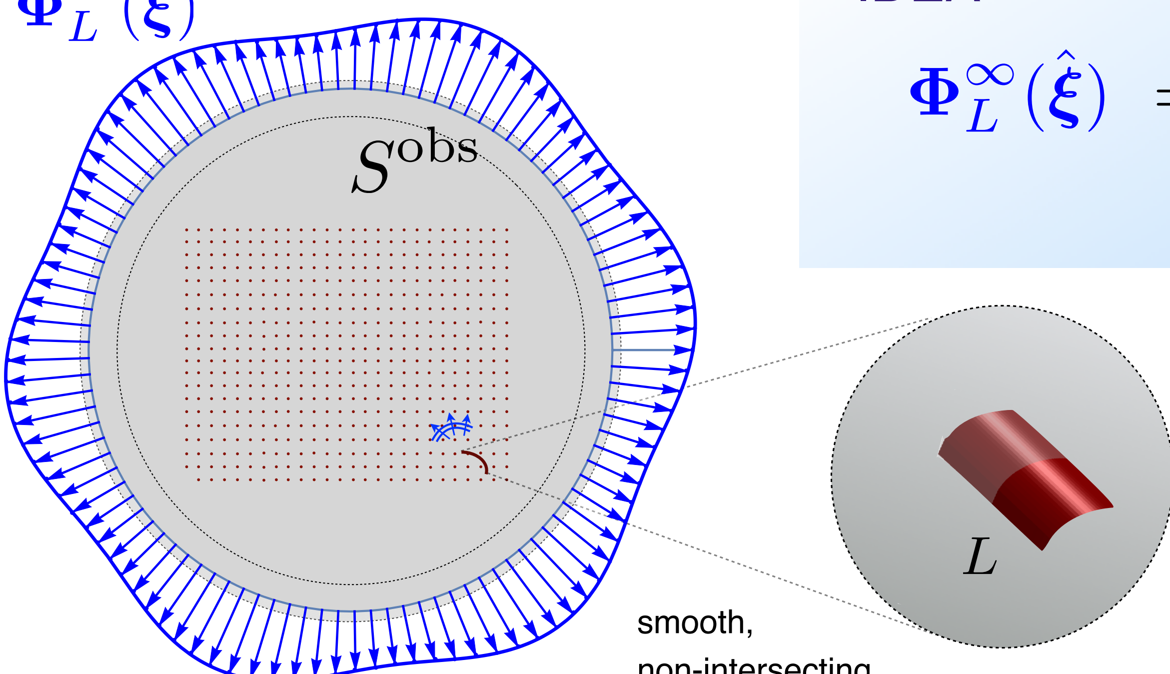
$$\boldsymbol{\Phi}_L^{\infty}[\boldsymbol{a}](\hat{\boldsymbol{\xi}}) \;=$$

$$-\Big(ik_p\,\hat{\boldsymbol{\xi}} \int_L \{\lambda(\boldsymbol{a}\cdot\boldsymbol{n}) + 2\mu(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\}\, e^{-ik_p\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\, \mathrm{d}S_{\boldsymbol{y}},$$

$$ik_s\,\hat{\boldsymbol{\xi}} \times \int_L \{\mu(\boldsymbol{a}\times\hat{\boldsymbol{\xi}})(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}}) + \mu(\boldsymbol{n}\times\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\}\, e^{-ik_s\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\, \mathrm{d}S_{\boldsymbol{y}}\Big)$$

P wave patten

S wave patten



IDEA

$$\boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}}) \;=\; \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}(\boldsymbol{d}) \cdot \boldsymbol{W}^{\infty}(\boldsymbol{d},\hat{\boldsymbol{\xi}})\, \mathrm{d}S_{\boldsymbol{d}}$$

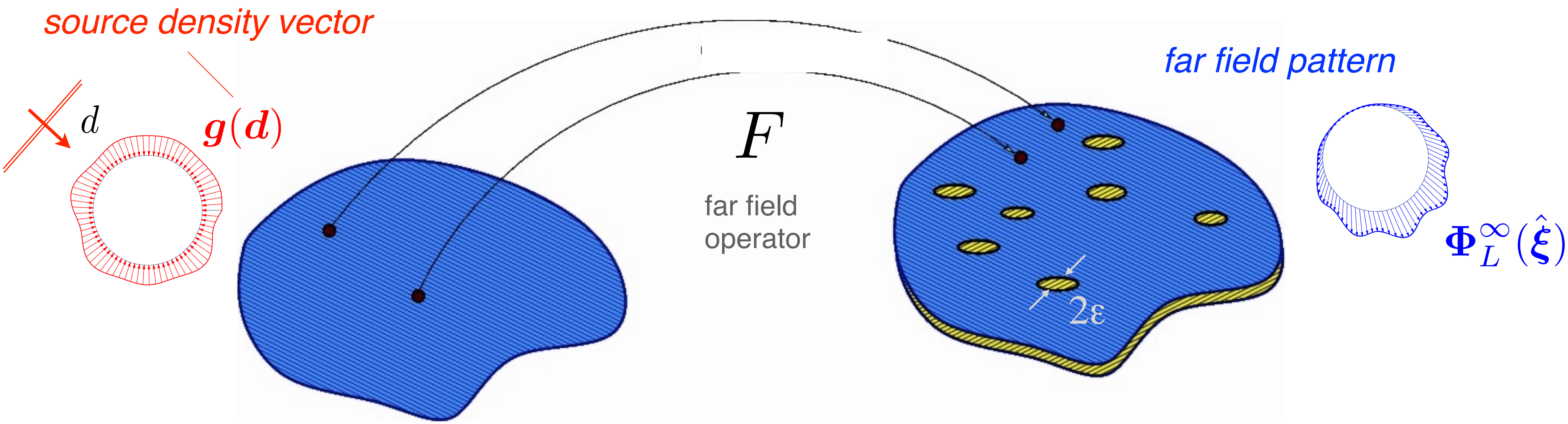
experiment

synthetic rearrangement of sources

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$

Ill-posedness

$$F\boldsymbol{g} = \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$



$$F: L^2(\Omega_{\boldsymbol{d}}) \times L^2(\Omega_{\boldsymbol{d}}) \rightarrow L^2(\Omega_{\hat{\boldsymbol{\xi}}}) \times L^2(\Omega_{\hat{\boldsymbol{\xi}}})$$

Hadamard (1923)

Uniqueness

$$F\boldsymbol{g} = \mathbf{0} \quad \Rightarrow \quad \boldsymbol{g} = \mathbf{0}$$

injective

Existence

$$\forall \varepsilon > 0, \quad \exists \boldsymbol{g}^{\varepsilon}: \; \| F\boldsymbol{g}^{\varepsilon} - \boldsymbol{\Phi}_L^{\infty} \|_{L^2(\Omega_{\hat{\boldsymbol{\xi}}})} < \varepsilon$$

dense range

Stability

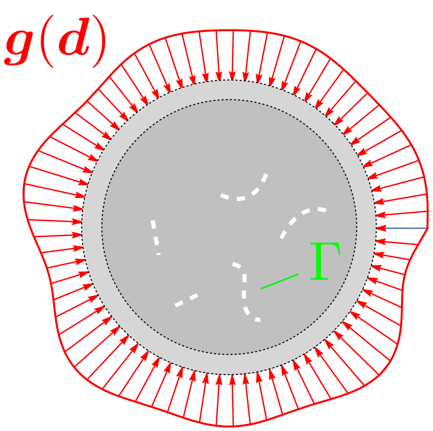
$$\lim_{\alpha \to 0} \| \boldsymbol{g} \| = \infty$$

compact operator of infinite dimension ⟶ no stability

$F = \mathscr{H}^* T \mathscr{H}$

Factorization

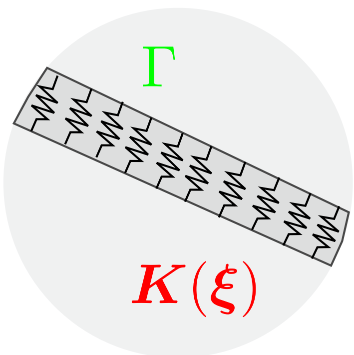
$F\boldsymbol{g} = \boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$



source densities $\boldsymbol{g}$ $\xrightarrow{\mathscr{H}}$ free field traction on Γ

$$\mathscr{H}: L^2(\Omega_{\boldsymbol{d}}) \times L^2(\Omega_{\boldsymbol{d}}) \to H^{-1/2}(\Gamma)$$

$$\mathscr{H}(\boldsymbol{g}_p, \boldsymbol{g}_s) = \boldsymbol{n} \cdot \boldsymbol{C} : \nabla \boldsymbol{u}^i(\boldsymbol{\xi}) \qquad \boldsymbol{t}^f$$



free field traction $\xrightarrow{T}$ FOD

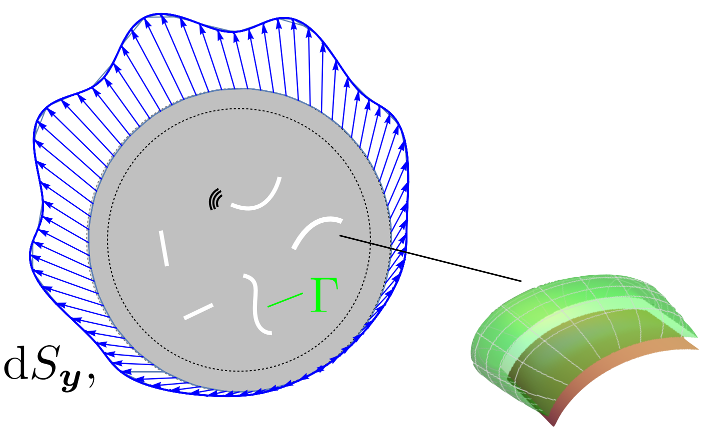
$$T: H^{-1/2}(\Gamma) \to \tilde{H}^{1/2}(\Gamma)$$

fracture
boundary condition

$$T[\boldsymbol{t}^f](\boldsymbol{\xi}) = [\![\boldsymbol{v}]\!], \qquad [\![\boldsymbol{v}]\!](\boldsymbol{\xi}) = (\boldsymbol{t}^f + \boldsymbol{n}\cdot\boldsymbol{C}:\nabla\boldsymbol{v})\cdot\boldsymbol{K}^{-1}(\boldsymbol{\xi})$$

admissible FODs over Γ $\xrightarrow{\mathscr{H}^*}$ far field pattern

$(\boldsymbol{v}_p^\infty, \boldsymbol{v}_s^\infty)$



$$\mathscr{H}^*: \tilde{H}^{1/2}(\Gamma) \to L^2(\Omega_{\hat{\boldsymbol{\xi}}}) \times L^2(\Omega_{\hat{\boldsymbol{\xi}}})$$

$$\mathscr{H}^*[\boldsymbol{a}](\hat{\boldsymbol{\xi}}) = -\Big(ik_p\,\hat{\boldsymbol{\xi}} \int_\Gamma \{\lambda(\boldsymbol{a}\cdot\boldsymbol{n}) + 2\mu(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\}\, e^{-ik_p\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\,\mathrm{d}S_{\boldsymbol{y}},$$

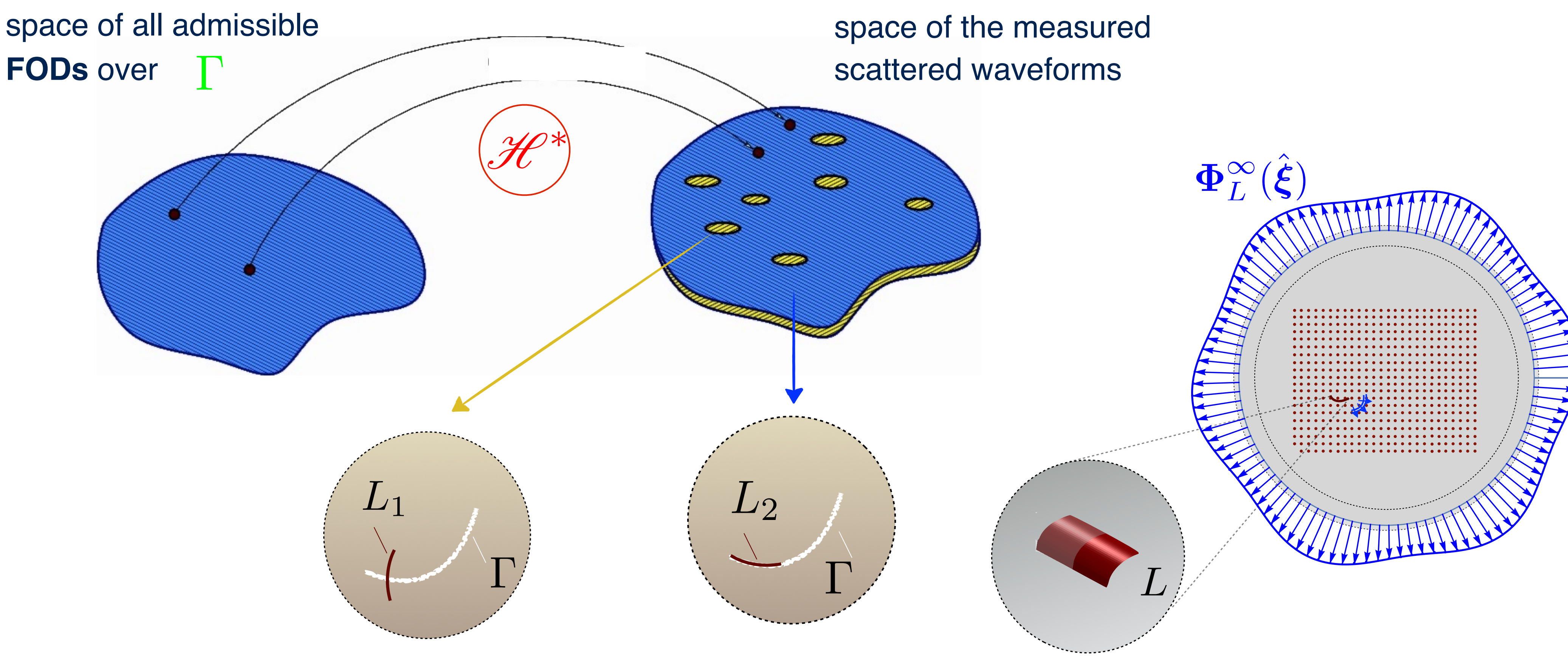
$$ik_s\,\hat{\boldsymbol{\xi}} \times \int_\Gamma \{\mu(\boldsymbol{a}\times\hat{\boldsymbol{\xi}})(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}}) + \mu(\boldsymbol{n}\times\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\}\, e^{-ik_s\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\,\mathrm{d}S_{\boldsymbol{y}}\Big).$$

$\boldsymbol{b} \in \tilde{H}^{\frac{1}{2}}(\Gamma)$

$F = \mathscr{H}^* T \mathscr{H}$

GLSM Theorem

$F\boldsymbol{g} = \boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$



Theorem. Provided $\boldsymbol{K}^{-1} \in L^\infty(\Gamma)$, and that ω is *not* a "Neumann" eigenvalue of the field equation, for *any* trial crack $L \subset \mathscr{B}_1$ – that is smooth and non-intersecting, corresponding to a density function $\boldsymbol{a}(\boldsymbol{\xi}) \in \tilde{H}^{1/2}(L)$,

$$\boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}}) \in \mathit{Range}(\mathscr{H}^*) \quad \text{if and only if} \quad L \subset \Gamma.$$

$F = \mathscr{H}^* T \mathscr{H}$

GLSM Theorem

$F\boldsymbol{g} = \boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$

$\mathscr{H}$

- √ injective
- √ dense range

$\mathscr{H}^*$

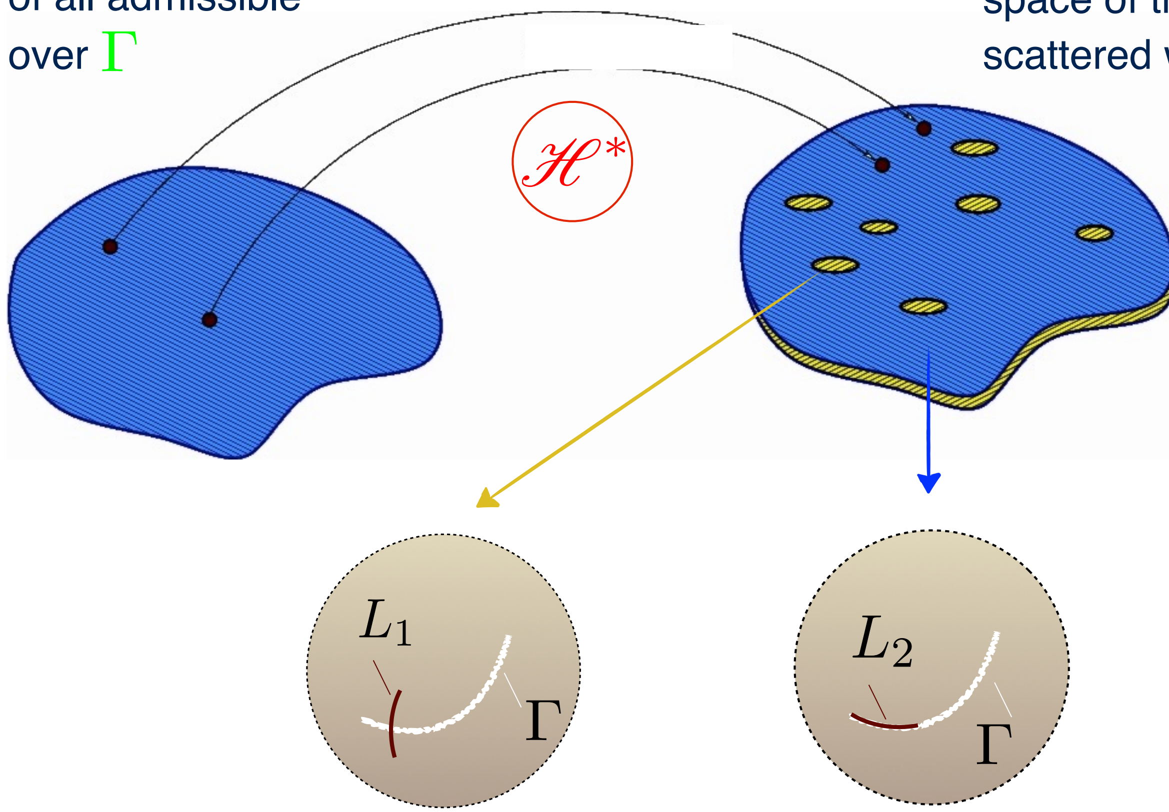
- √ injective
- √ dense range
- √ compact

T

- √ bounded
- √ continuous
- √ continuous inverse

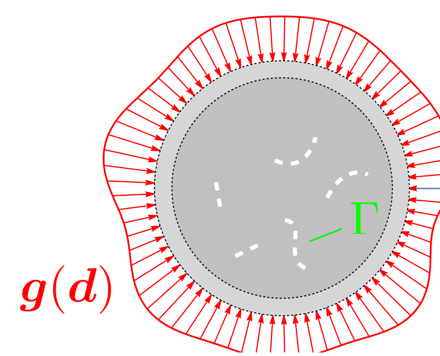
space of all admissible **FODs** over Γ

space of the measured scattered waveforms



$$\lim_{\delta \to 0} \|\mathscr{H} \boldsymbol{g}_\delta^{L_1}\|_{H^{-\frac{1}{2}}(\Gamma)} = \infty \qquad \limsup_{\epsilon \to 0} \|\mathscr{H} \boldsymbol{g}_\epsilon^{L_2}\|_{H^{-\frac{1}{2}}(\Gamma)} < \infty$$

source densities $\boldsymbol{g}$ $\xrightarrow{\mathscr{H}}$ free field traction on Γ



$F = \mathscr{H}^* T \mathscr{H}$

Imaging functional

$F\boldsymbol{g} = \boldsymbol{\Phi}_L^\infty(\hat{\xi})$

LSM cost functional

$$J_\alpha^{\text{LSM}} = \| F^\delta \boldsymbol{g} - \boldsymbol{\Phi}_L^\infty \|^2 + \alpha \|\boldsymbol{g}\|^2$$

noisy operator

LSM indicator

$$\mathscr{I}_{\text{LSM}} = 1/ \|\boldsymbol{g}\|^2$$

GLSM cost functional

$$|(\boldsymbol{g}, F\boldsymbol{g})| = |\mathscr{H}\boldsymbol{g}, T\mathscr{H}\boldsymbol{g}| \geqslant c\|\mathscr{H}\boldsymbol{g}\|^2$$

$$\|\mathscr{H}\boldsymbol{g}\|^2$$

$$J_{\alpha,\delta}^{\text{GLSM}} = \| F^\delta \boldsymbol{g} - \boldsymbol{\Phi}_L^\infty \|^2 + \alpha \left(\delta \|\boldsymbol{g}\|^2 + |(\boldsymbol{g}, \boldsymbol{F}^\delta \boldsymbol{g})|\right)$$

non-convex

$\boldsymbol{F}_\sharp$-factorization

$$\|\mathscr{H}\boldsymbol{g}\|^2 \rightarrow \|F_\sharp^{\frac{1}{2}}\boldsymbol{g}\|^2 = |(\boldsymbol{g}, F_\sharp \boldsymbol{g})|$$

$$F_\sharp = |\Re F| + \Im F$$

Imaging functional

$$F\boldsymbol{g} = \boldsymbol{\Phi}_L^\infty(\hat{\xi})$$

convex GLSM cost functional

$$F_\sharp^\delta = |\Re(F^\delta)| + \Im(F^\delta)$$

noisy kernel

$$J_{\alpha,\delta}^{\mathrm{GLSM}} = \|F^\delta \boldsymbol{g} - \boldsymbol{\Phi}_L^\infty\|^2 + \alpha\left(\delta\|\boldsymbol{g}\|^2 + |(F_\sharp^\delta \boldsymbol{g}, \boldsymbol{g})|\right)$$

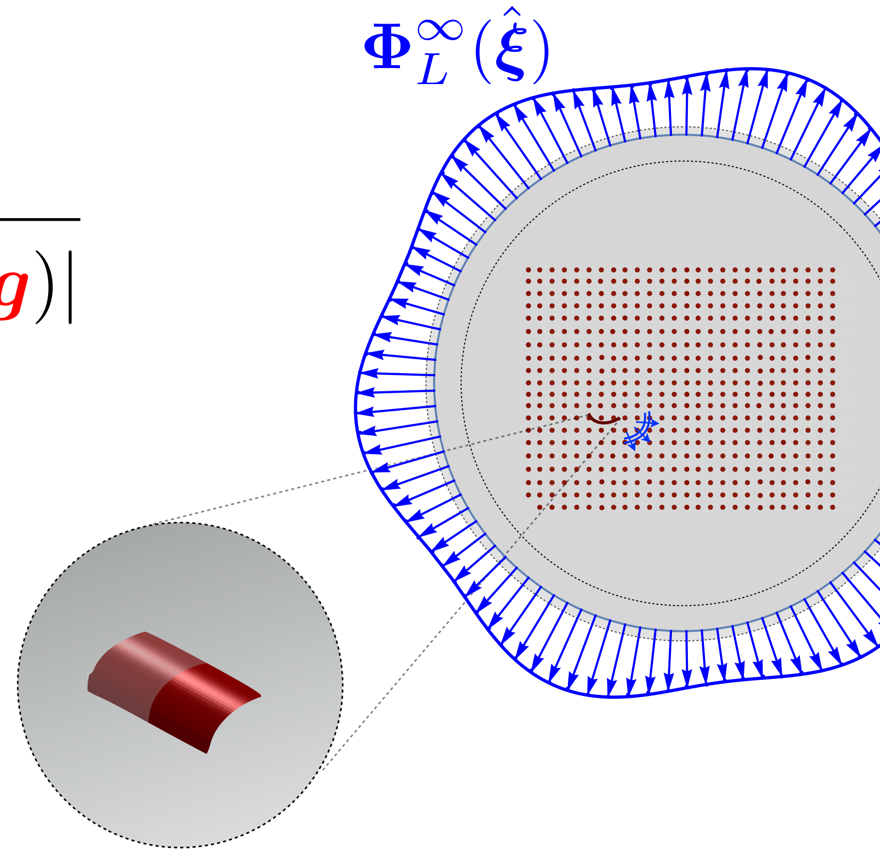
$$J_\alpha = \|F^\delta \boldsymbol{g} - \boldsymbol{\Phi}_L^\infty\|^2 + \alpha\,\|\boldsymbol{g}\|^2$$

GLSM indicator

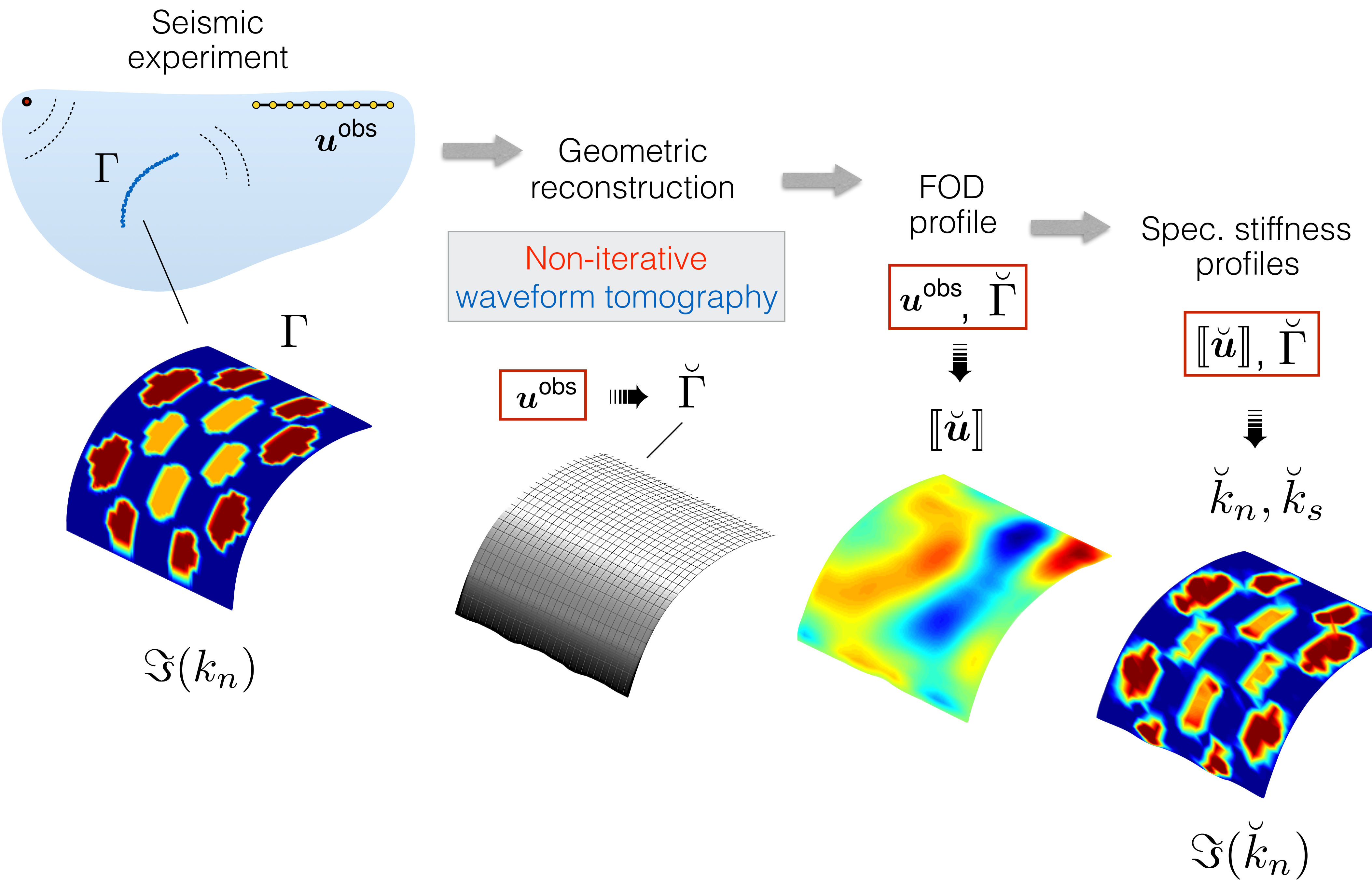
$$\mathscr{I}_{\mathrm{GLSM}} = 1/\sqrt{\delta\,\|\boldsymbol{g}\|^2 + |(F_\sharp^\delta \boldsymbol{g}, \boldsymbol{g})|}$$

LSM indicator

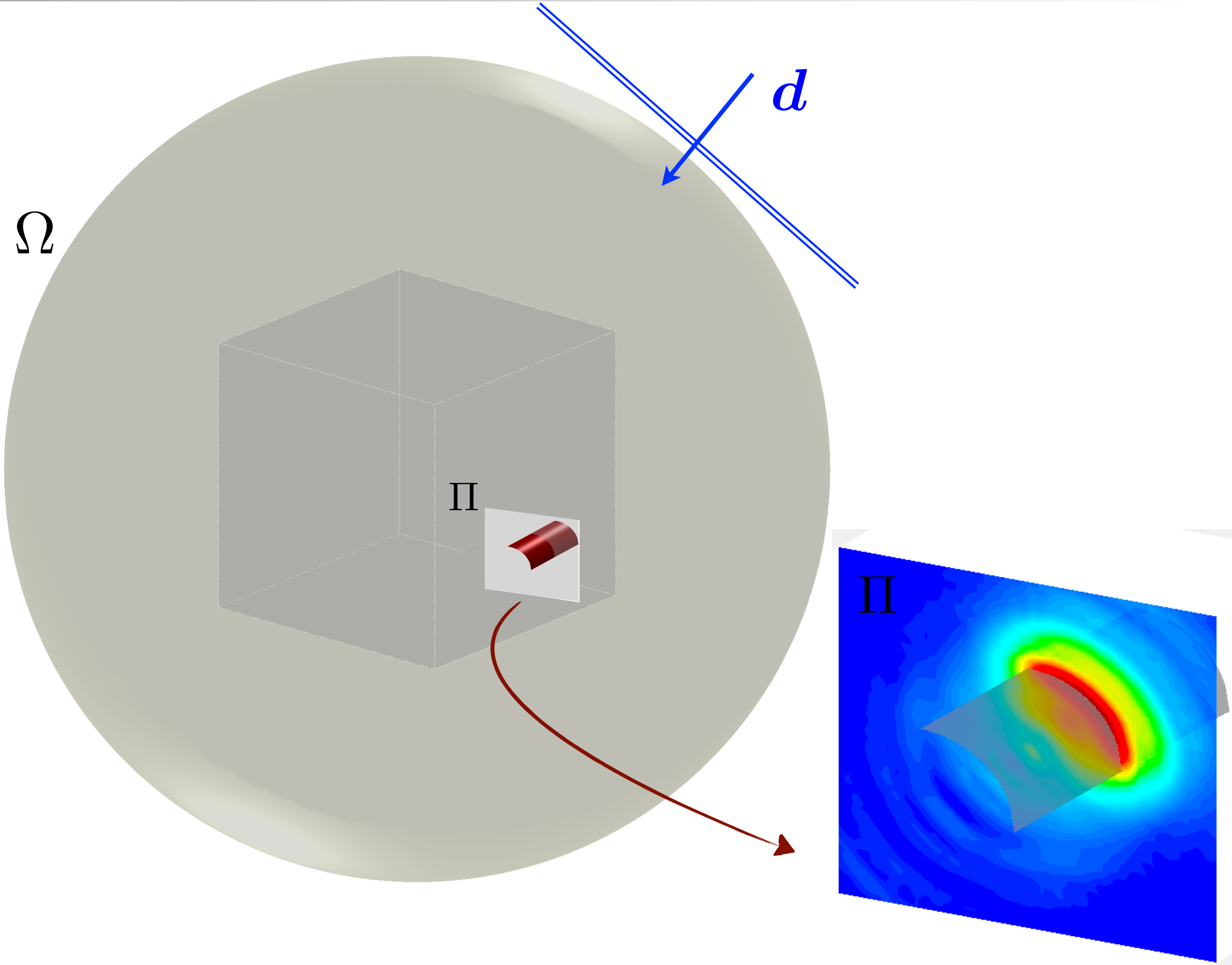
$$\mathscr{I}_{\mathrm{LSM}} = 1/\,\|\boldsymbol{g}\|^2$$



3-step approach

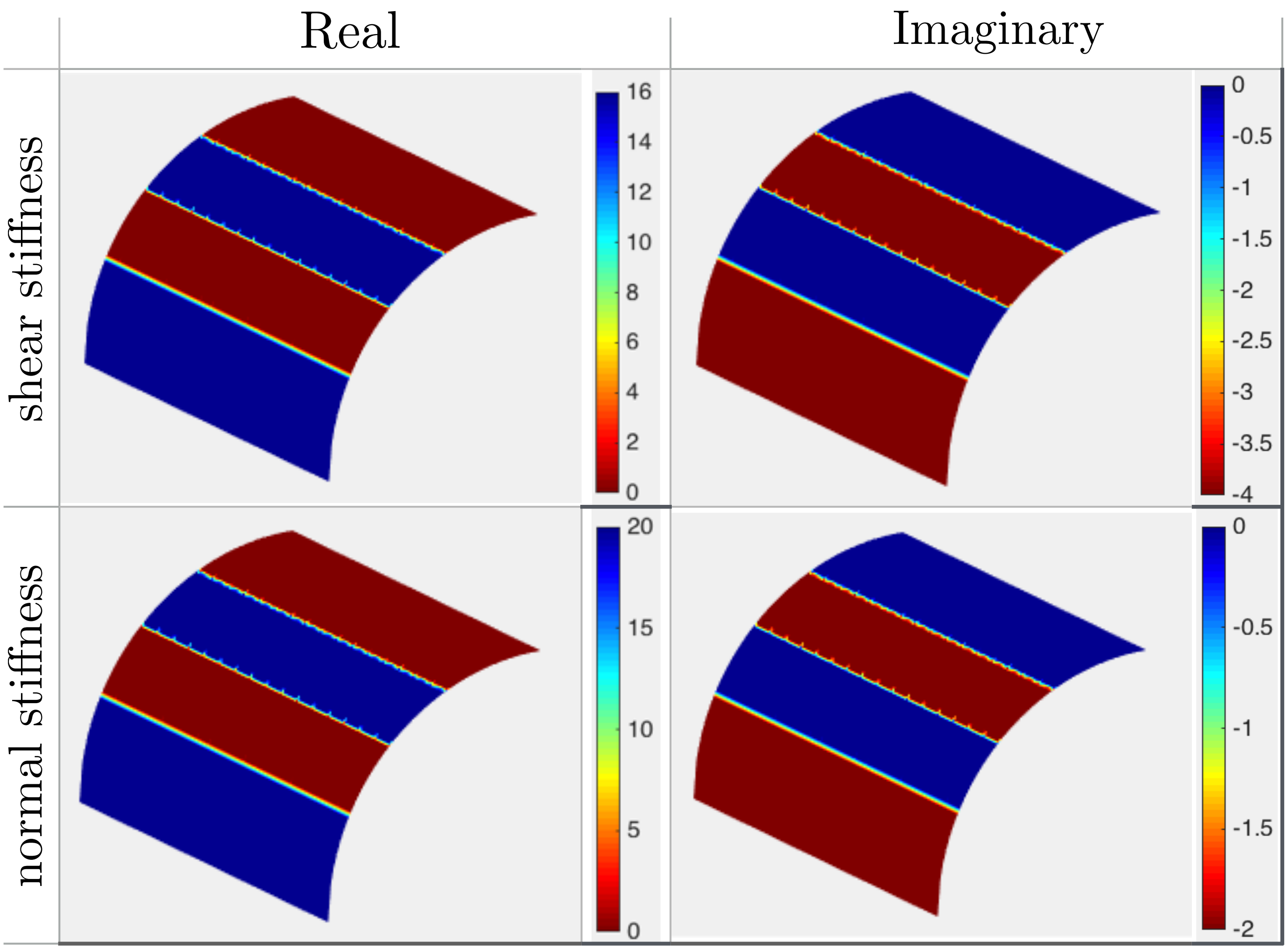


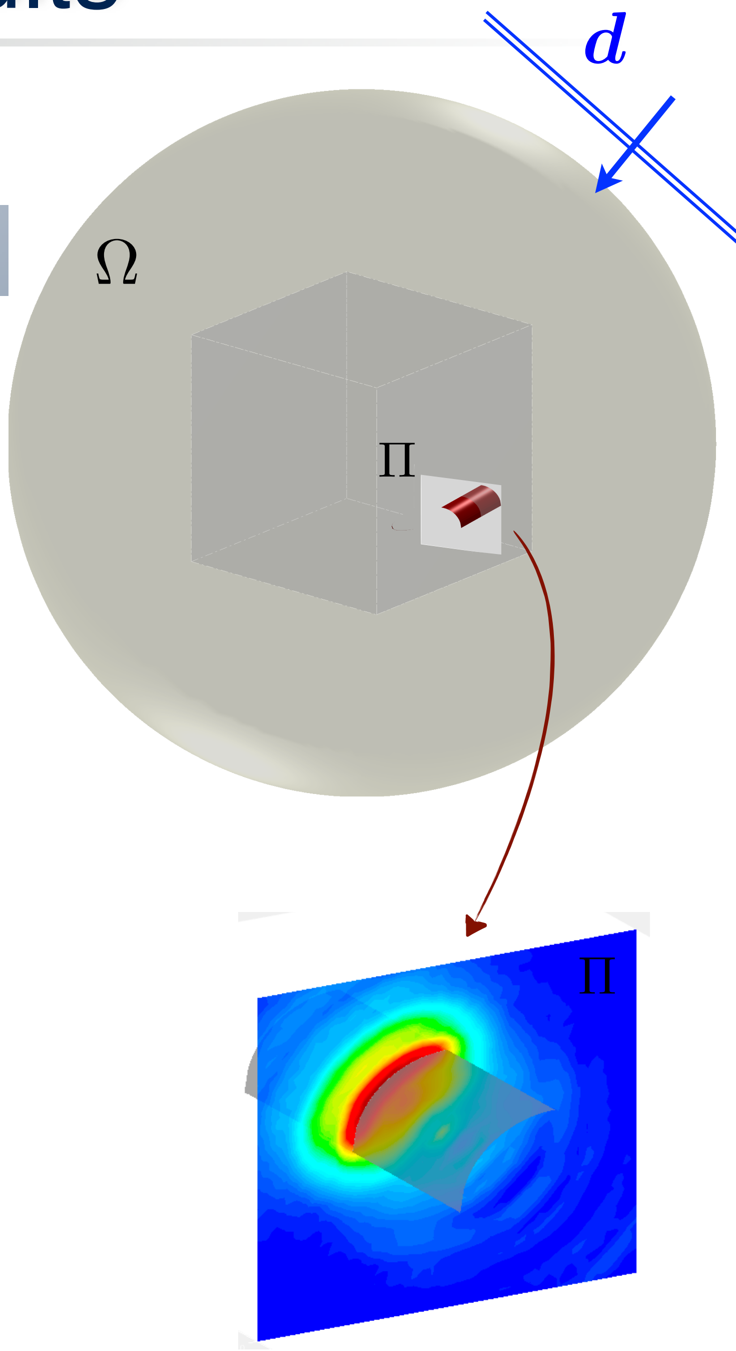
Numerical results



Numerical results

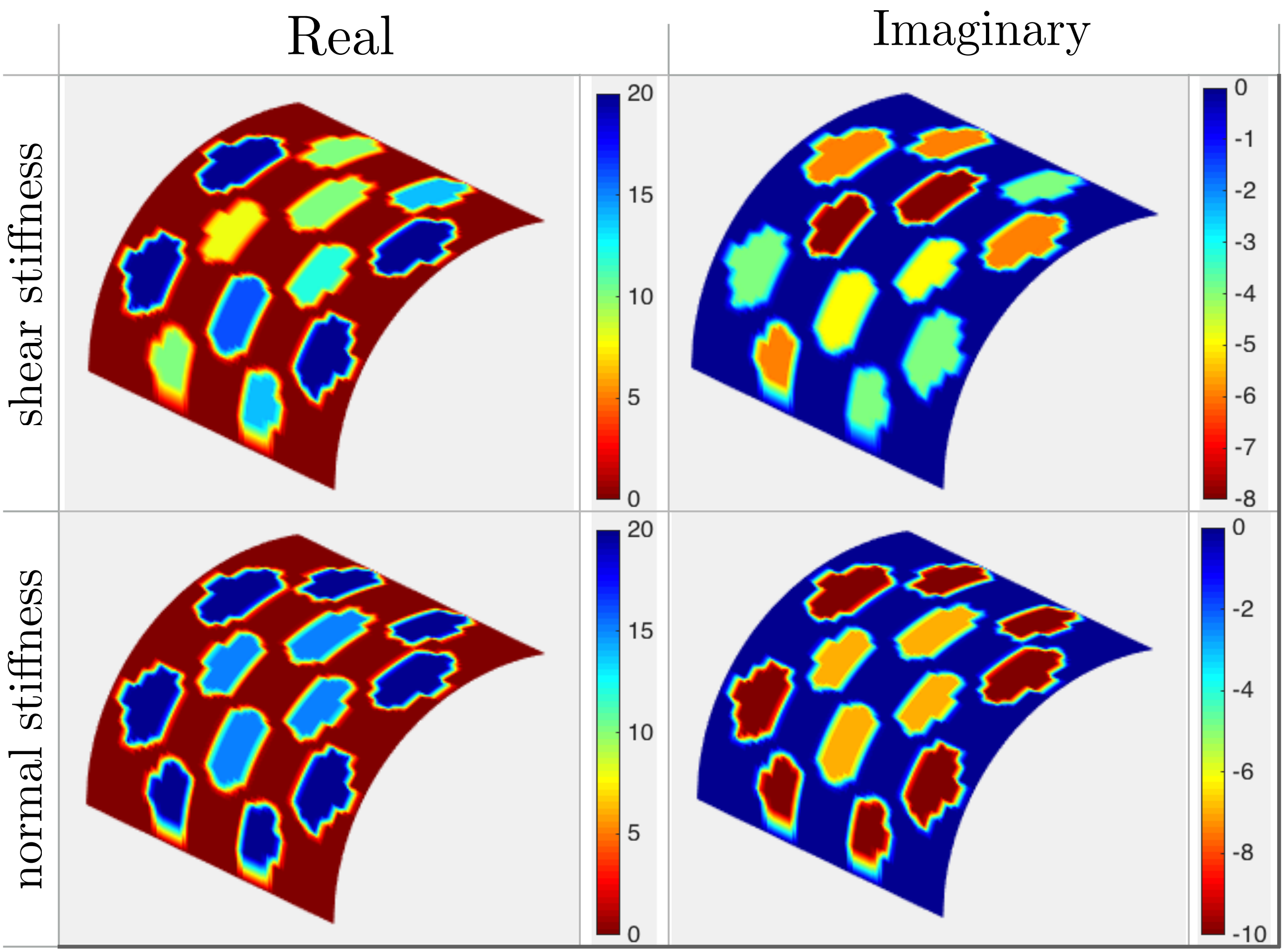
"zebra" interface

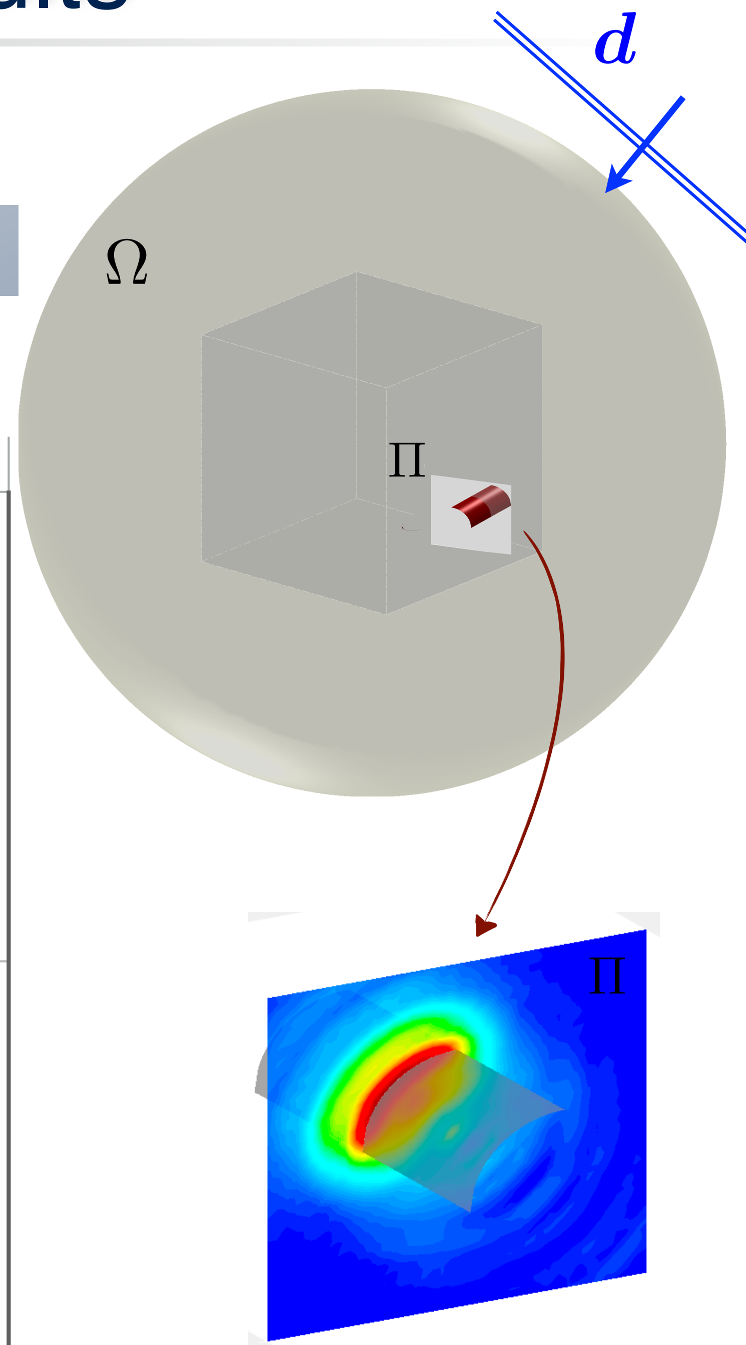




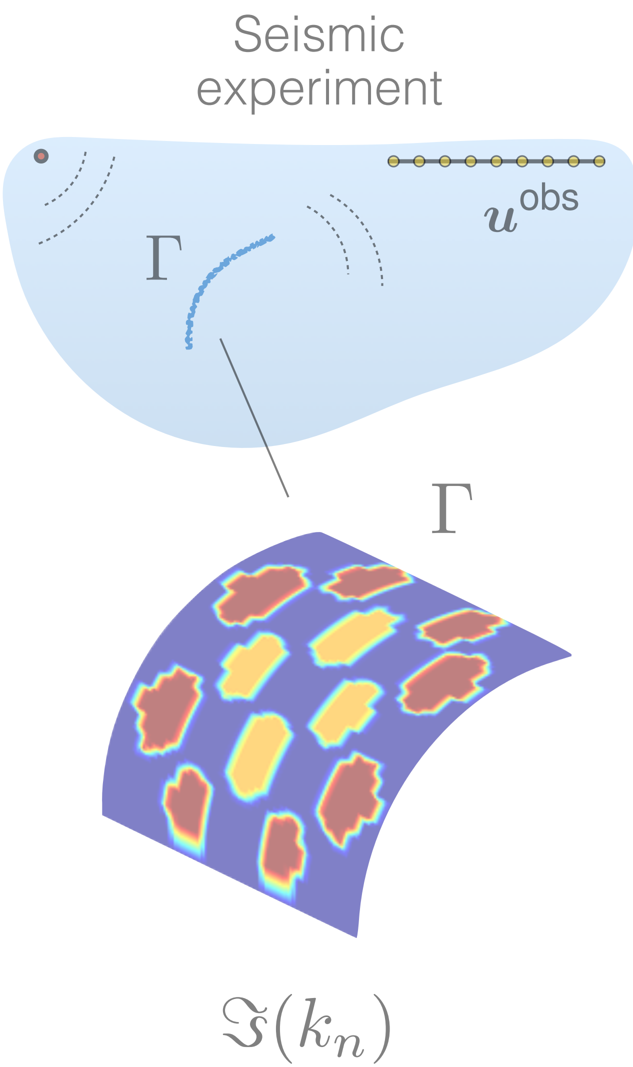
Numerical results

“cheetah” interface



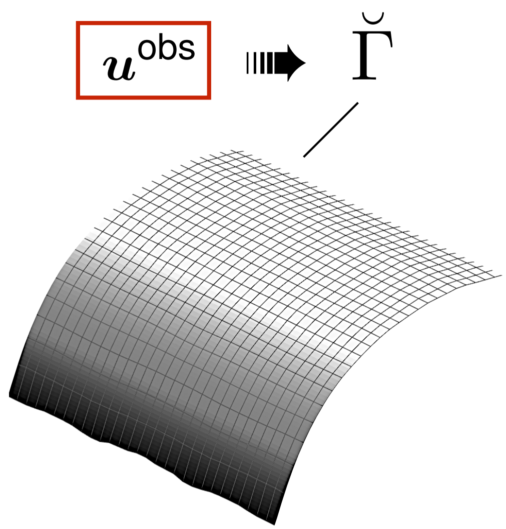


Geometric Reconstruction

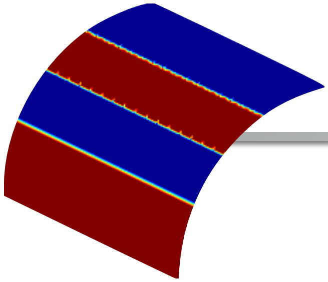


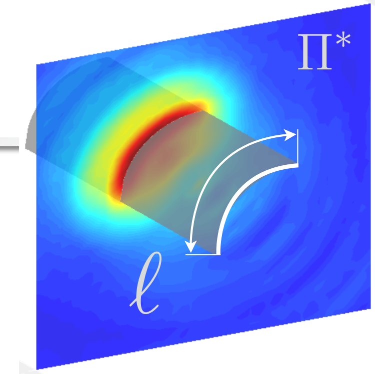
Geometric reconstruction

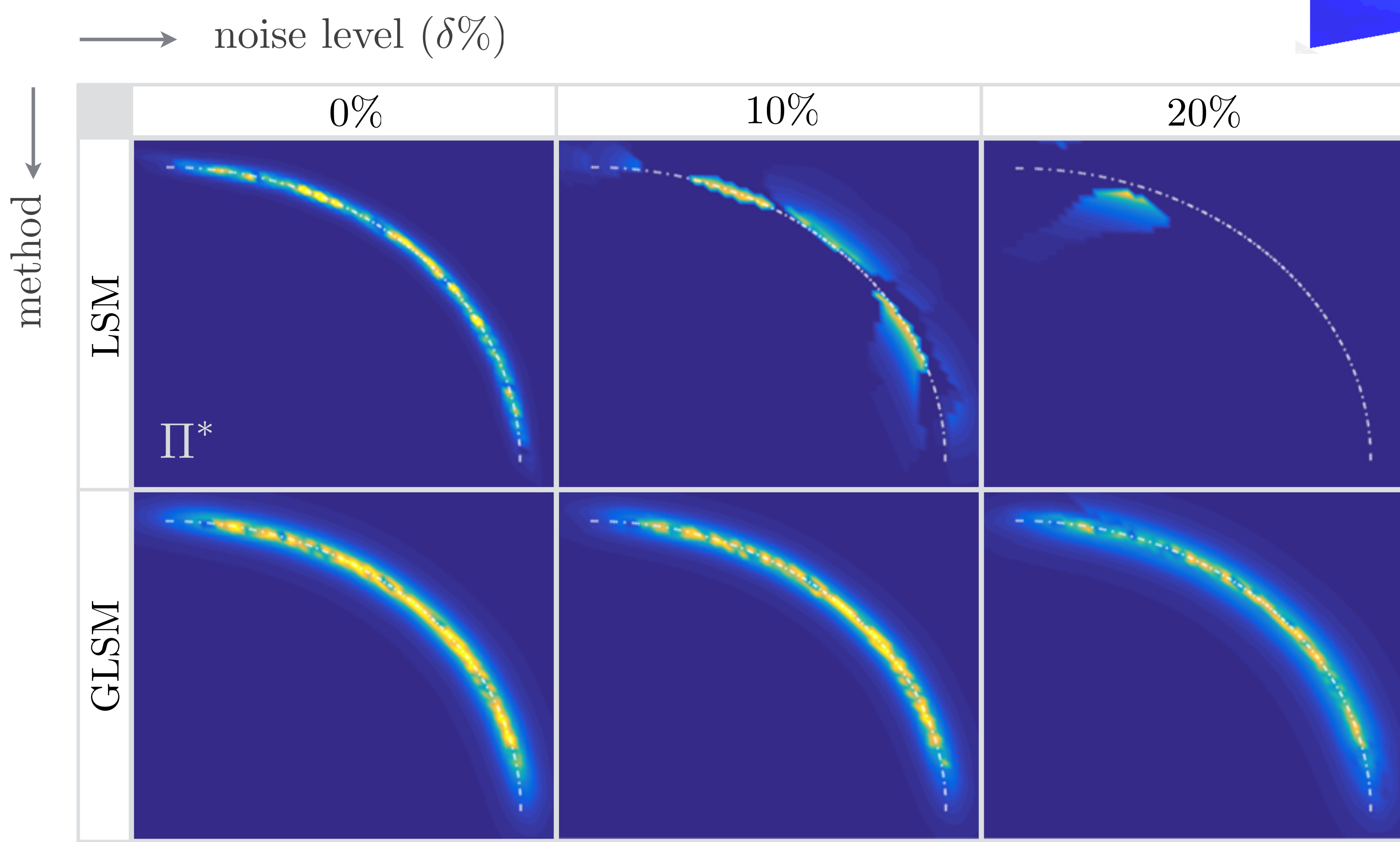
Non-iterative
waveform tomography



Mid-plane reconstruction



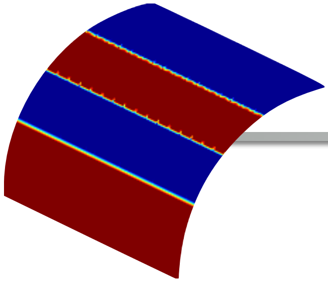


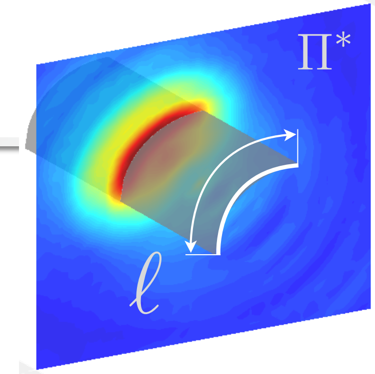


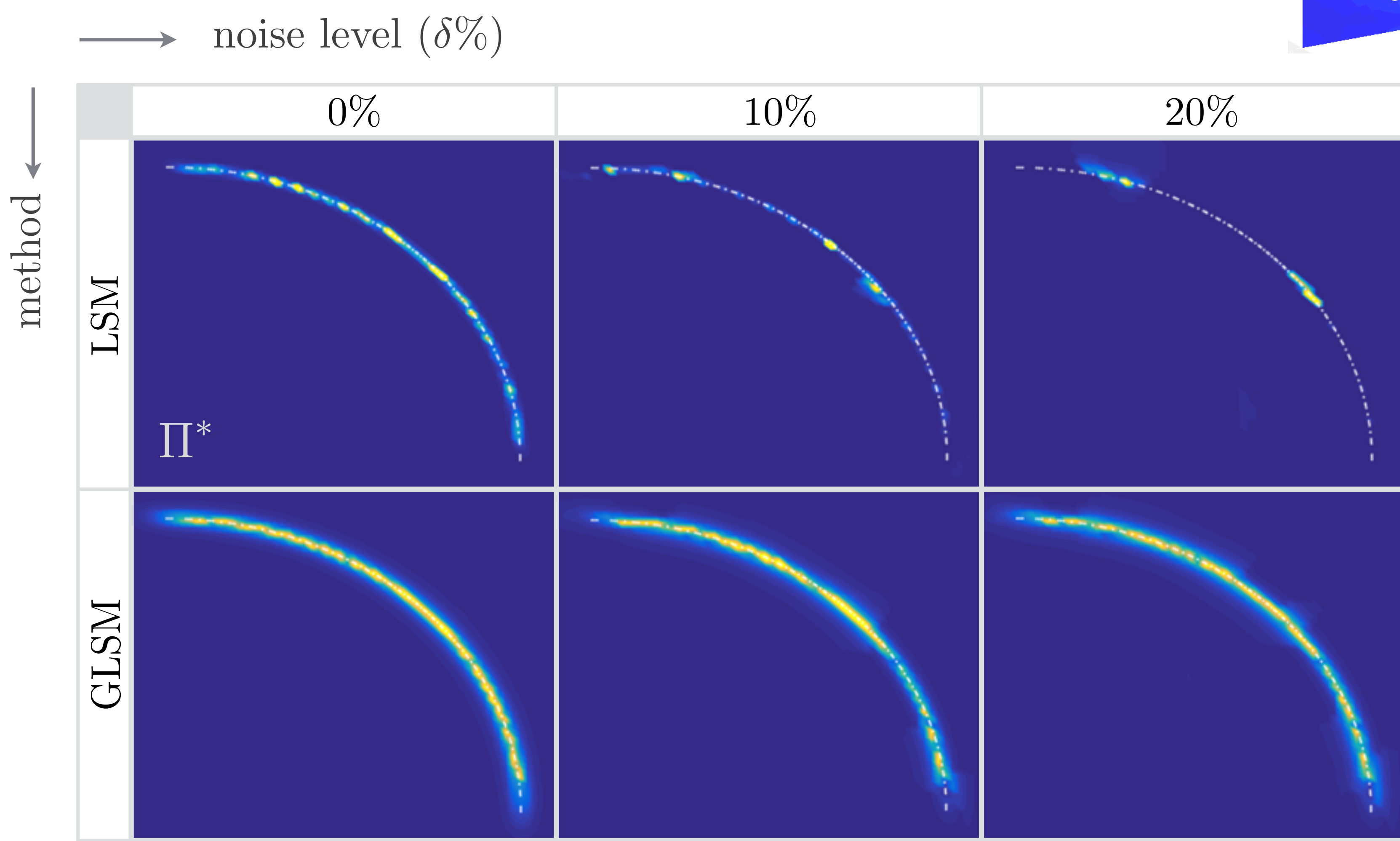
full aperture

$\lambda/\ell = 1$

Mid-plane reconstruction



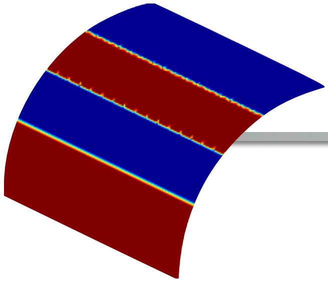


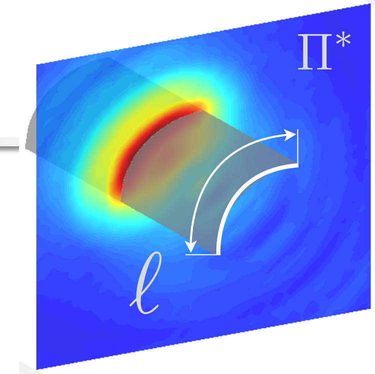


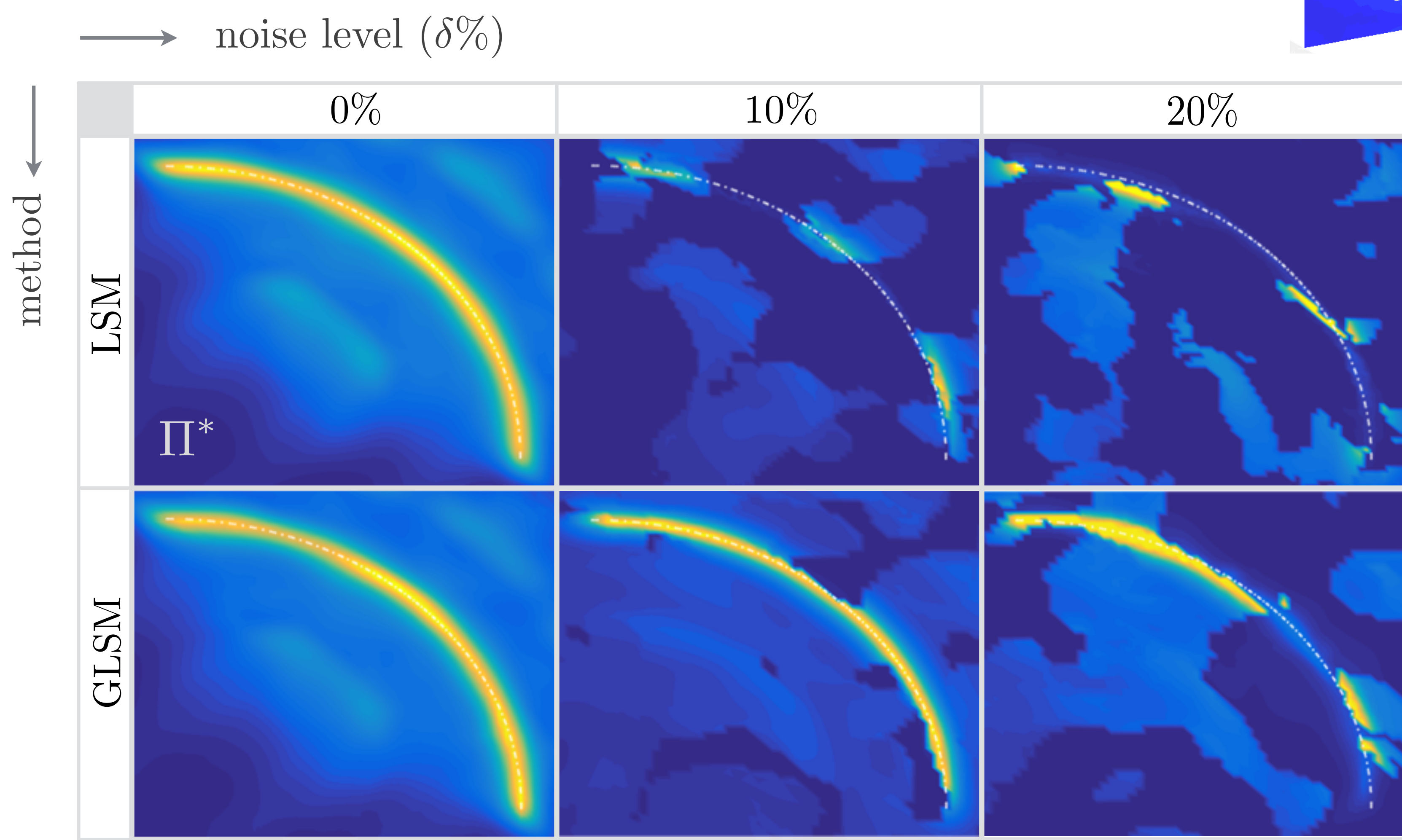
full aperture

$\lambda/\ell = 0.25$

Mid-plane reconstruction

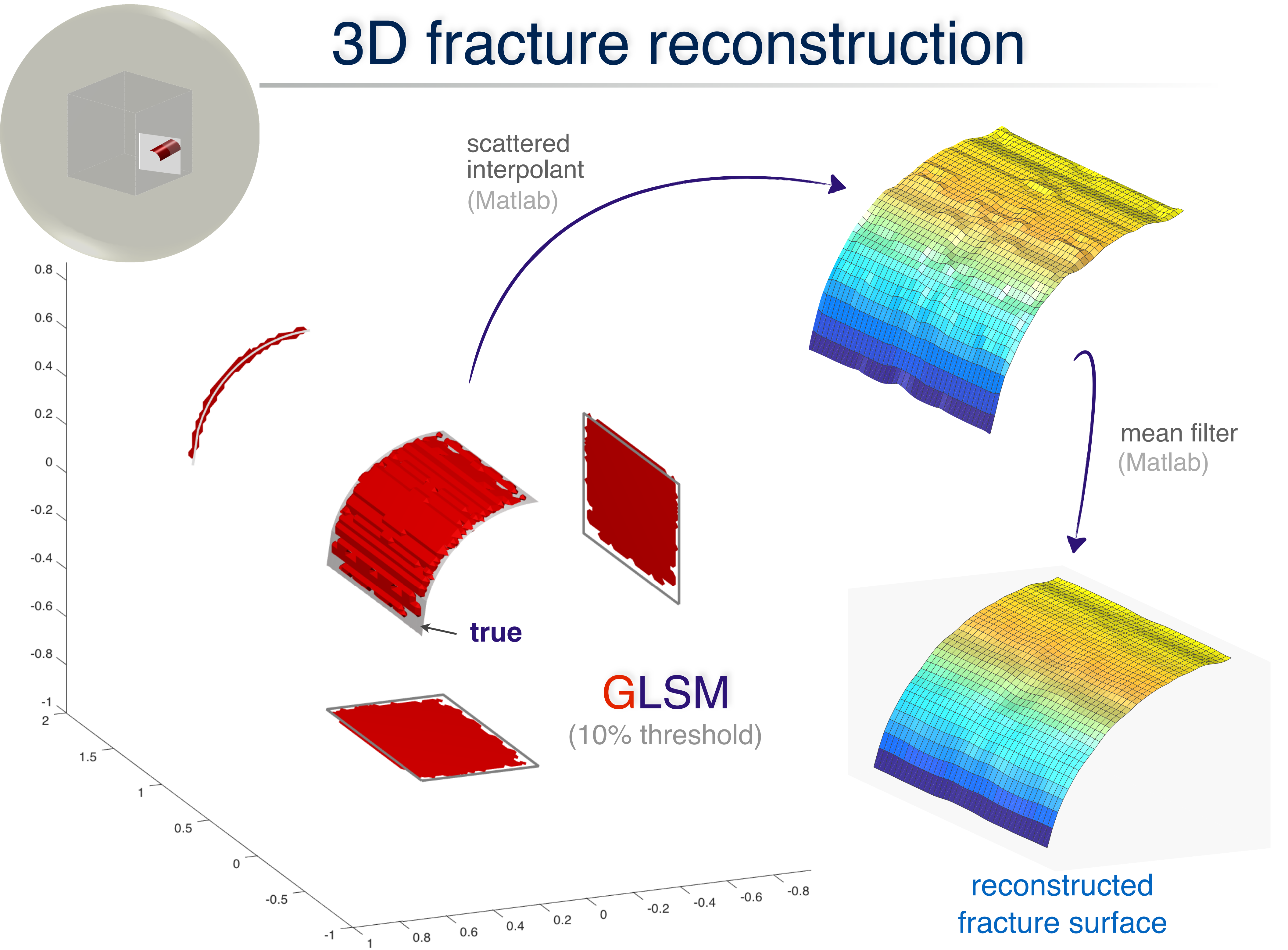


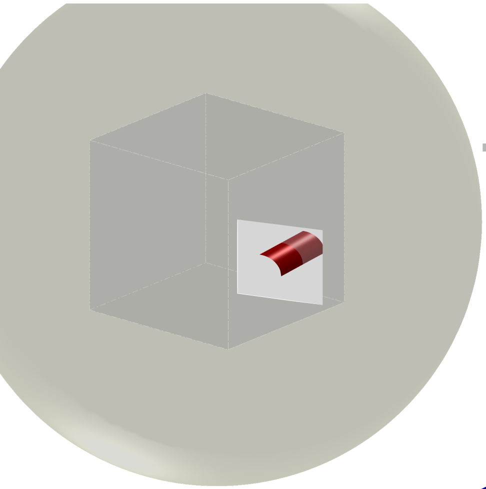




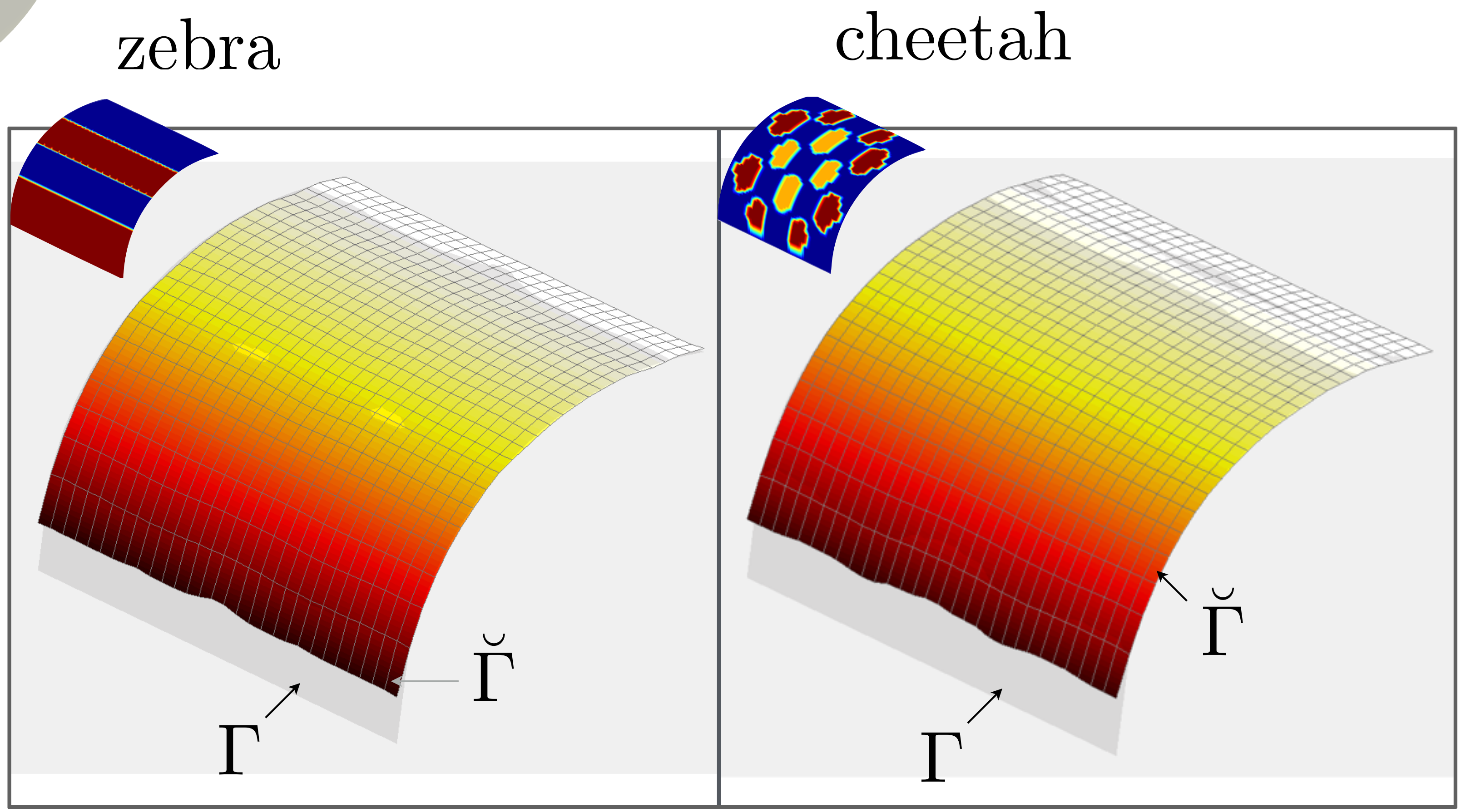
half aperture $\lambda/\ell = 0.25$

3D fracture reconstruction

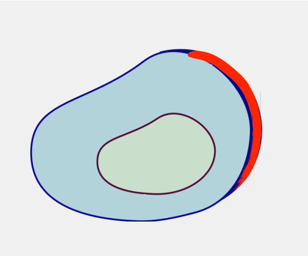


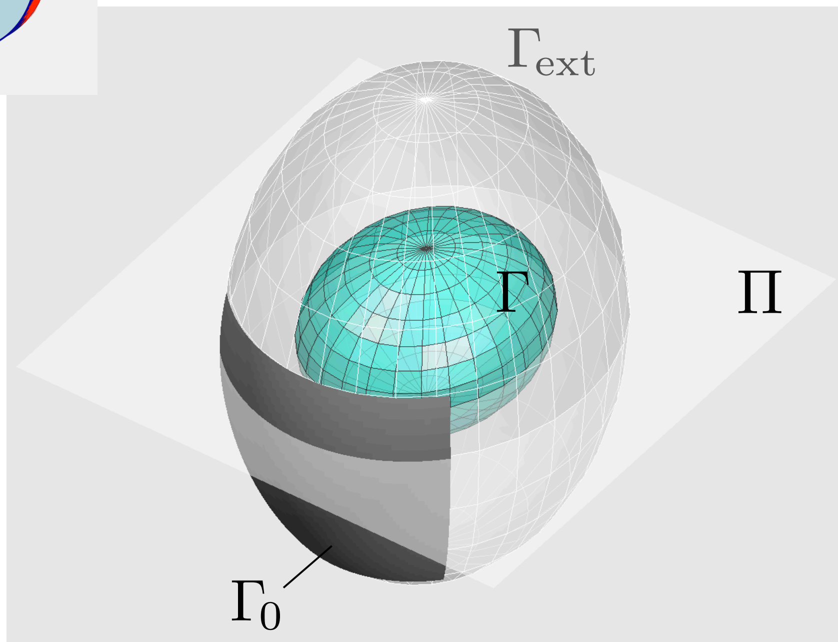


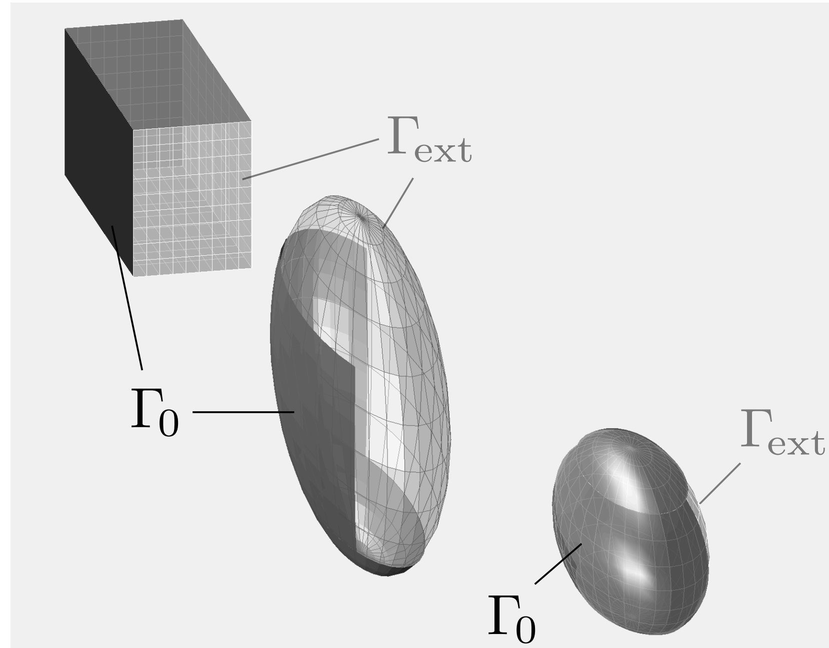
3D fracture reconstruction

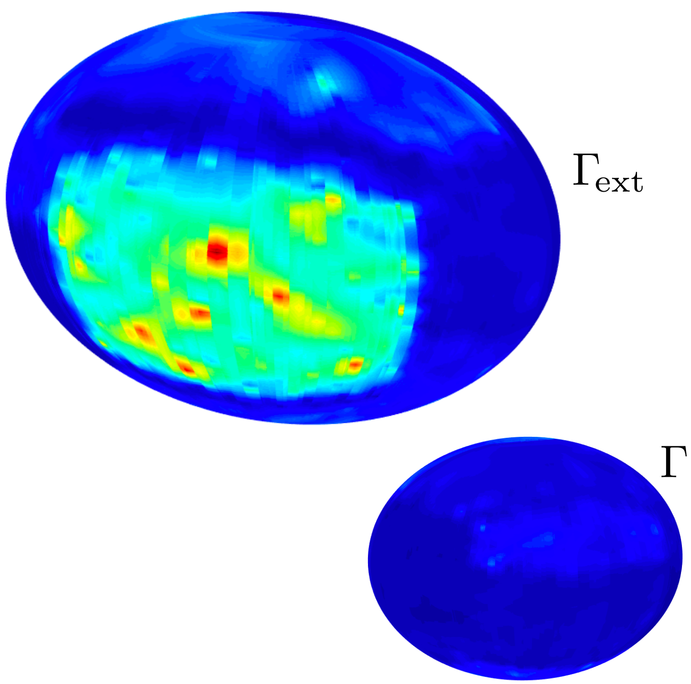


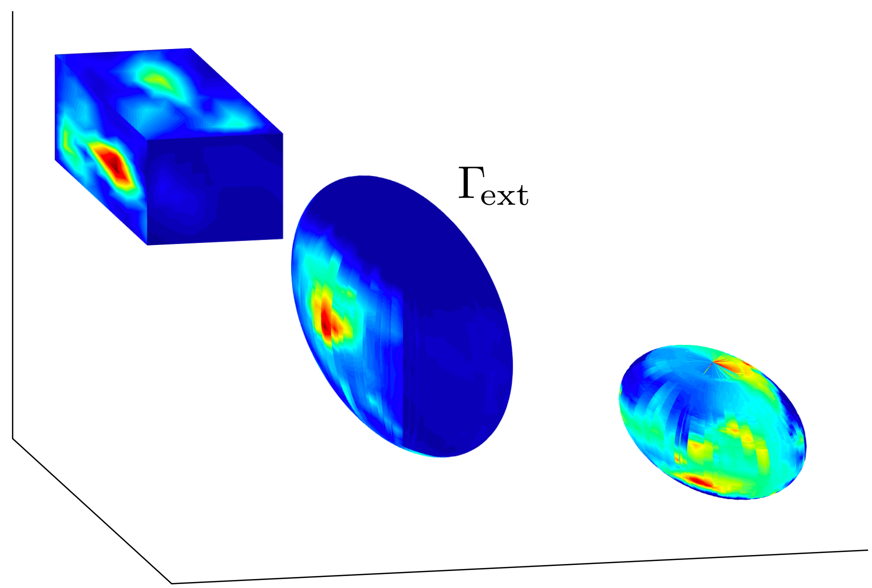
Layered composites



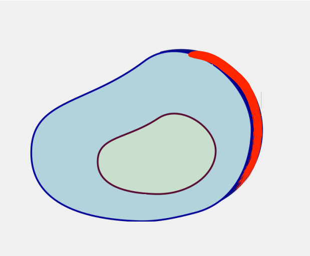


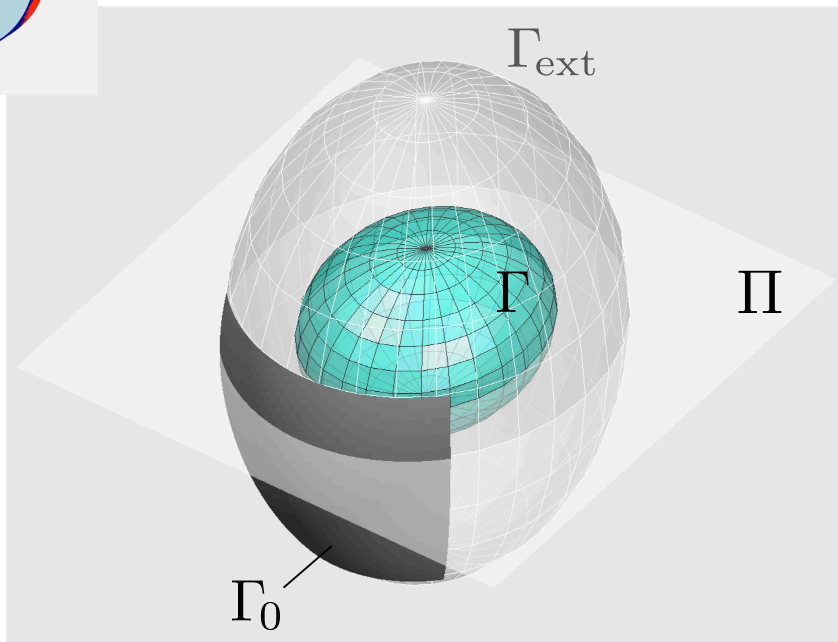


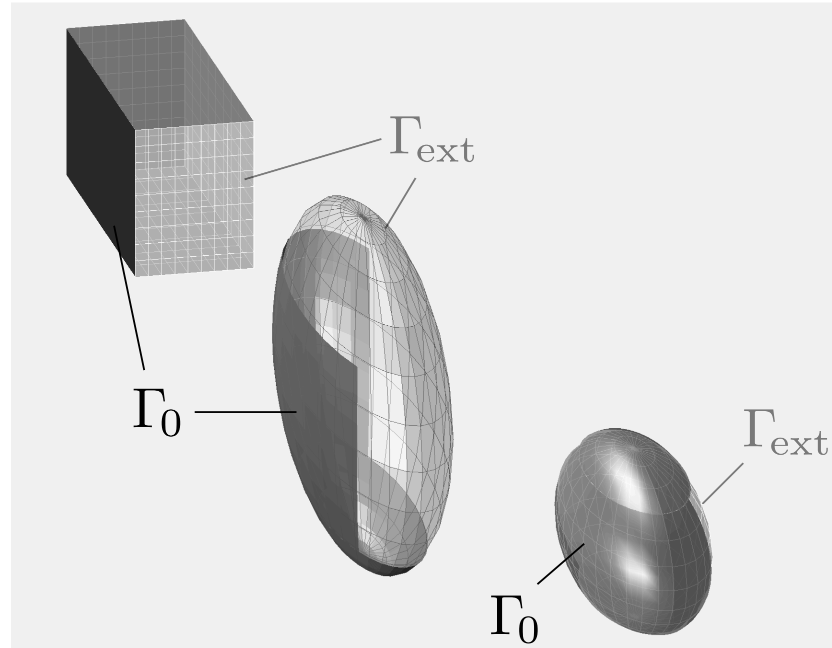


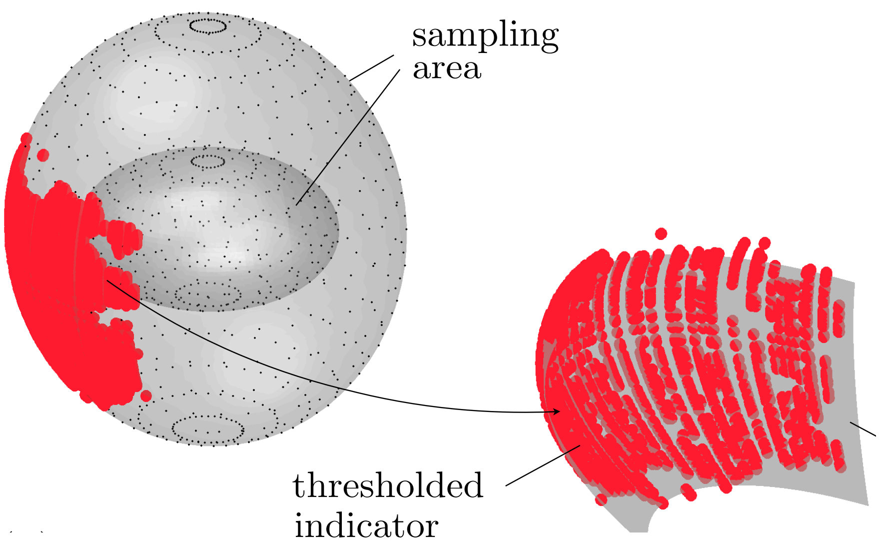


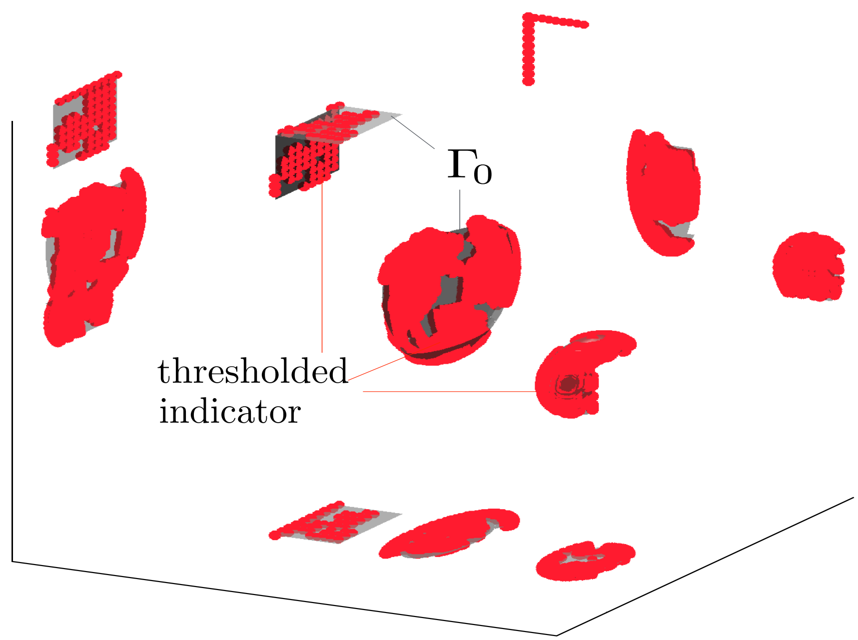
Layered composites



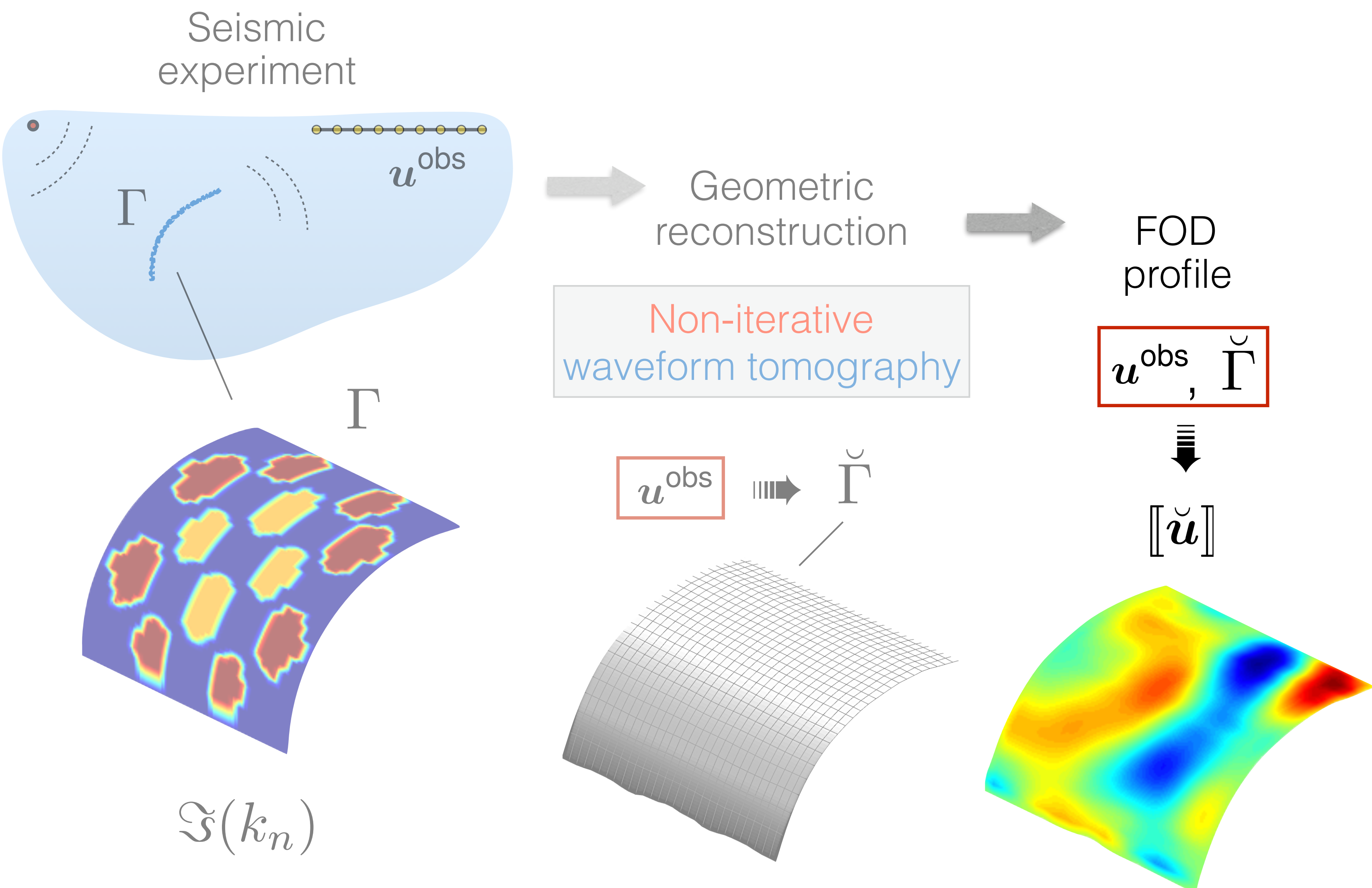




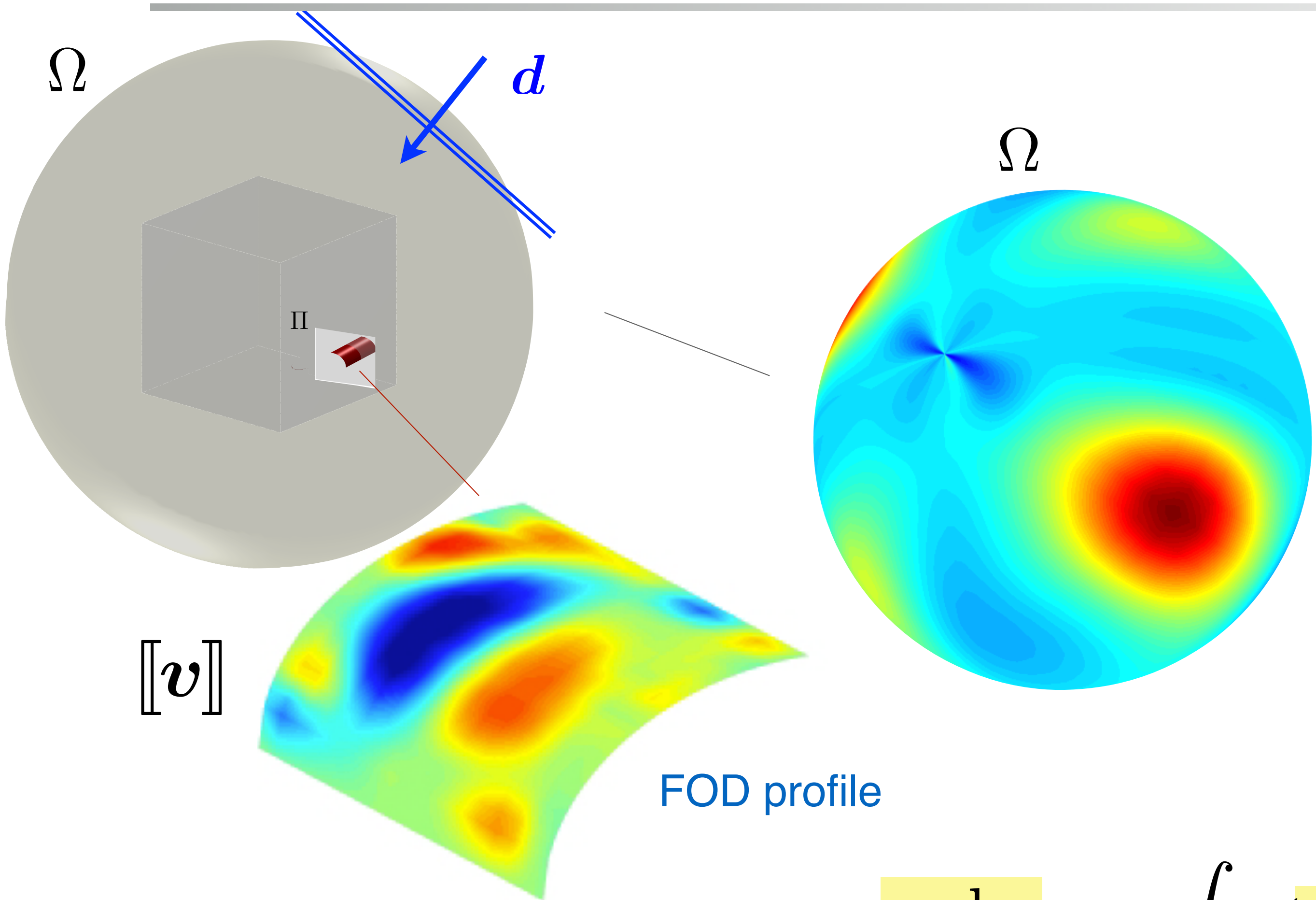




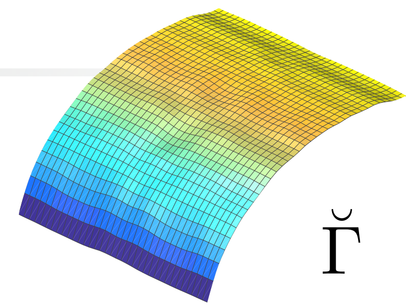
FOD reconstruction



FOD Reconstruction



$\boldsymbol{v}^{\text{obs}}$



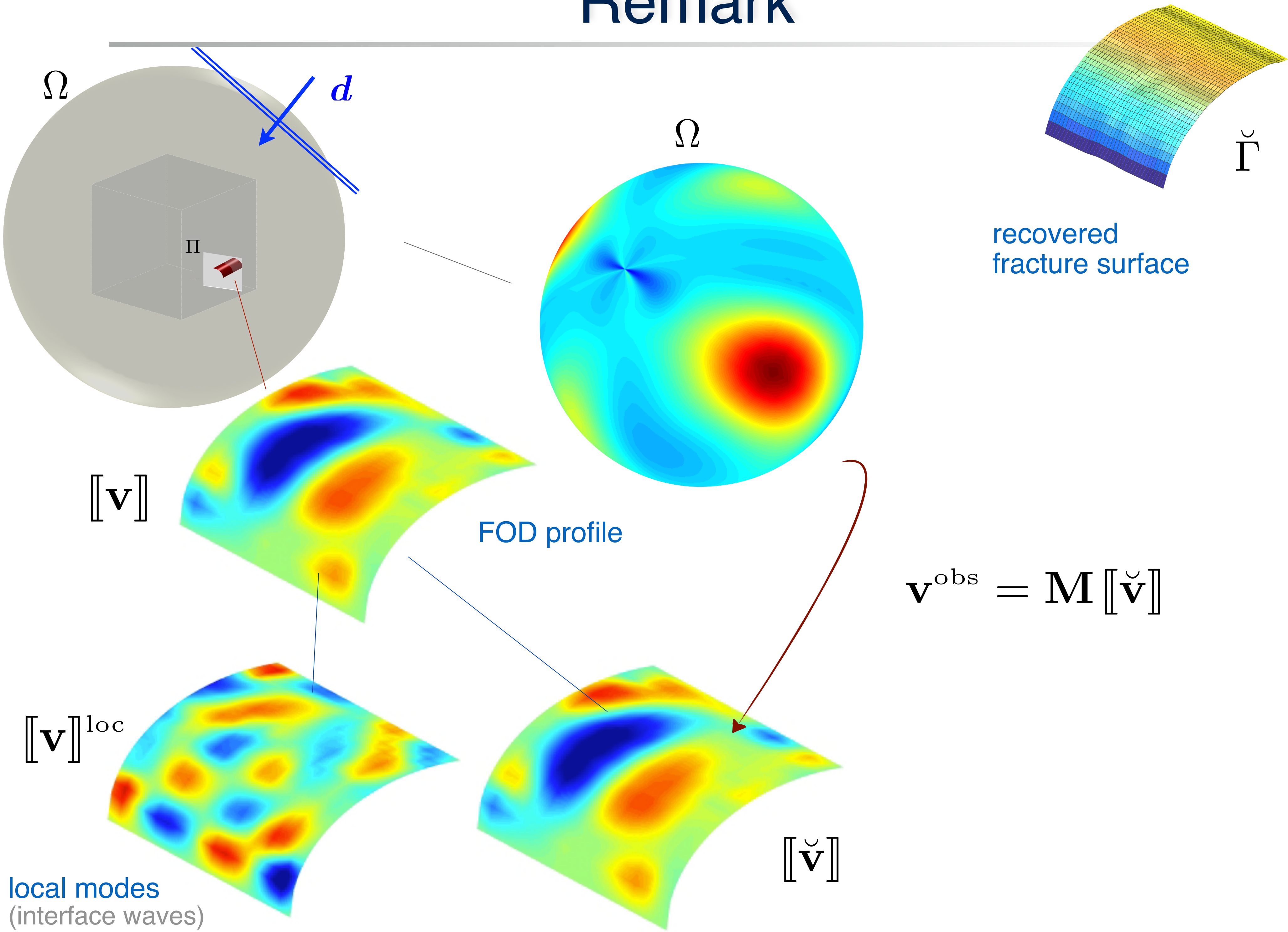
FOD profile

$$\boldsymbol{v}^{\text{obs}} = \int_{\breve{\Gamma}} \left([\![\breve{\boldsymbol{v}}]\!] \otimes \breve{\boldsymbol{n}} \right) : \boldsymbol{\Sigma}^{\infty} \, \mathrm{d}S_{\breve{\Gamma}}$$

discrete

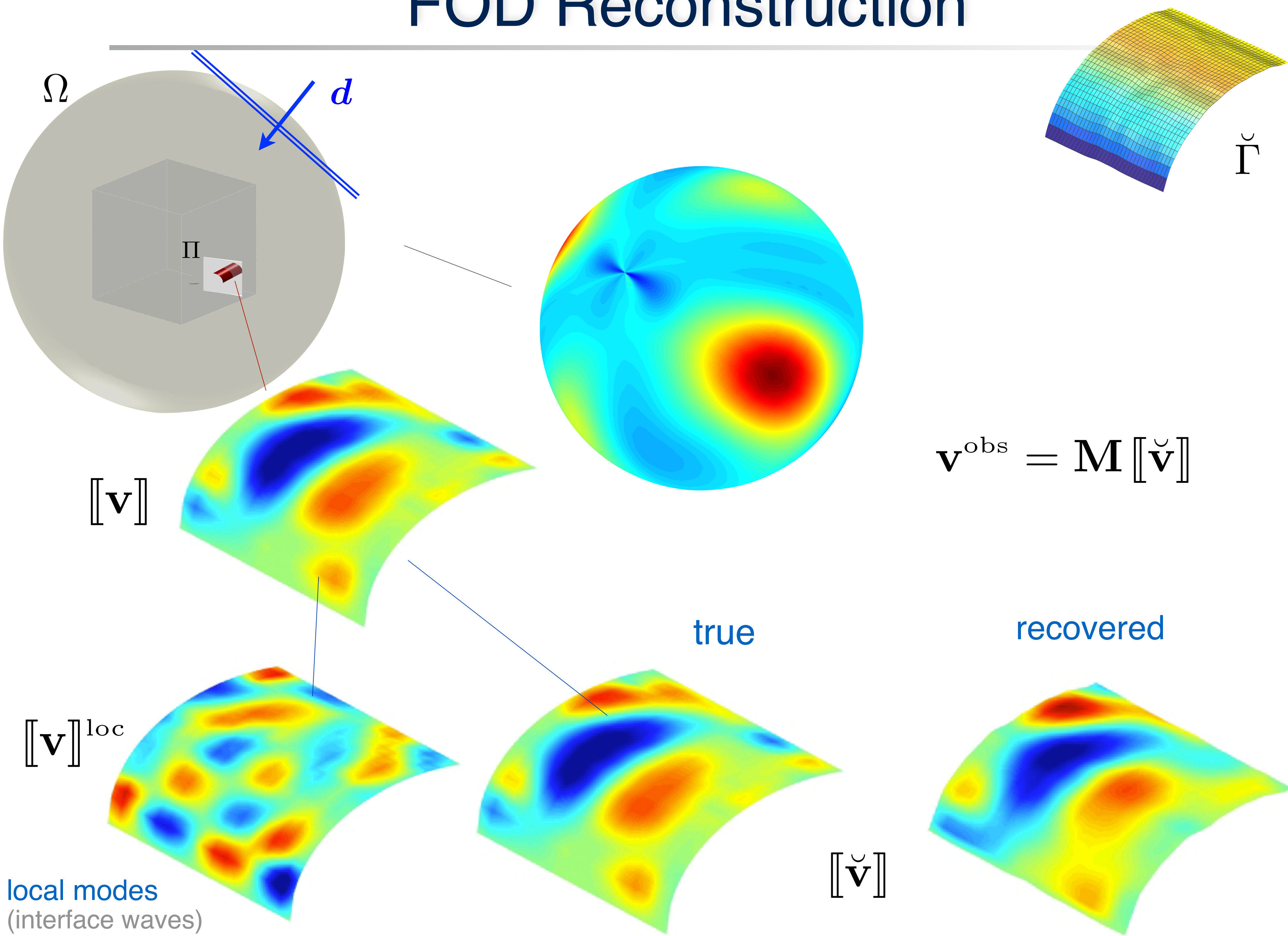
$$\mathbf{v}^{\text{obs}} = \mathbf{M} \, [\![\breve{\mathbf{v}}]\!]$$

Remark

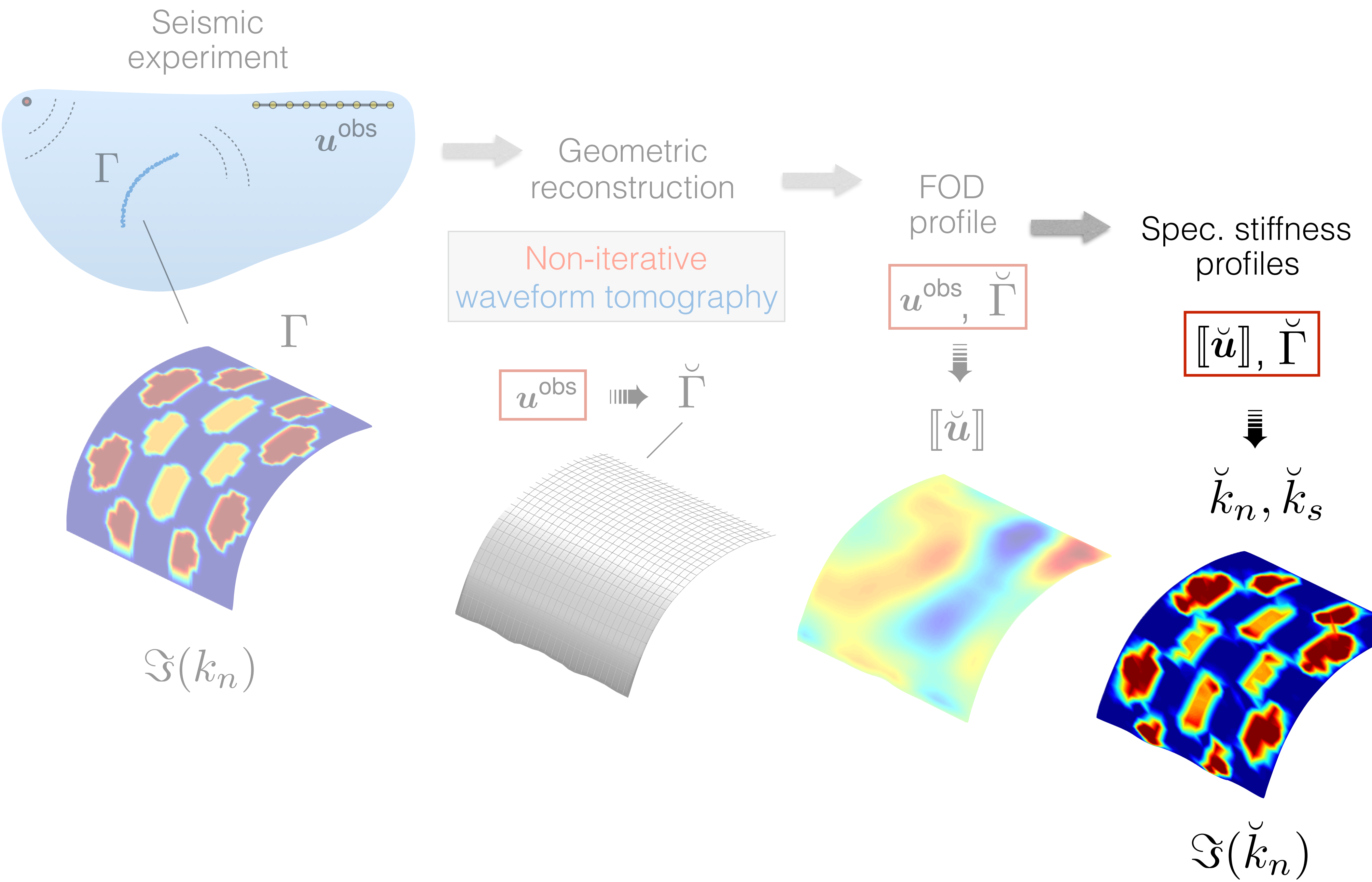


Ω
$\boldsymbol{d}$
Π
Ω
recovered
fracture surface
$\breve{\Gamma}$
$[\![\mathbf{v}]\!]$
FOD profile
$\mathbf{v}^{\rm obs} = \mathbf{M}\,[\![\breve{\mathbf{v}}]\!]$
$[\![\mathbf{v}]\!]^{\rm loc}$
local modes
(interface waves)
$[\![\breve{\mathbf{v}}]\!]$

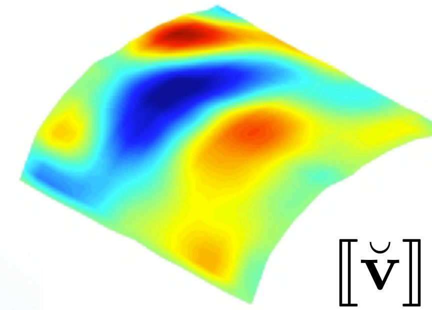
FOD Reconstruction



Identification of interfacial parameters



Reconstruction of *K*



$[\![\breve{\mathbf{v}}]\!]$

TBIE

$\mathbf{T}[\![\breve{\boldsymbol{v}}]\!]$

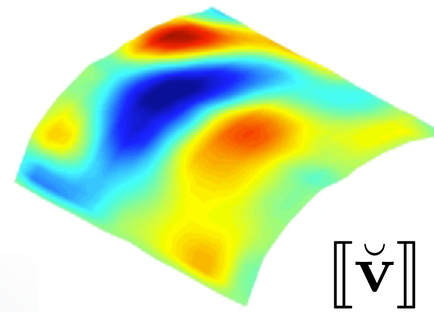
$$\boldsymbol{t}_f(\boldsymbol{\xi}) - \breve{\boldsymbol{K}} \cdot [\![\breve{\boldsymbol{v}}]\!](\boldsymbol{\xi}) = \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \fint_{\breve{\Gamma}} \boldsymbol{\Sigma} : \boldsymbol{D}_{\boldsymbol{x}} [\![\breve{\boldsymbol{v}}]\!](\boldsymbol{x}) \, \mathrm{d}S_x - \rho\omega^2 \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \int_{\breve{\Gamma}} \boldsymbol{U} \cdot \big([\![\breve{\boldsymbol{v}}]\!] \otimes \breve{\boldsymbol{n}}\big)(\boldsymbol{x}) \, \mathrm{d}S_x$$

free field traction

discretization

$$\breve{\mathbf{K}}\,[\![\breve{\mathbf{v}}]\!] = \breve{\mathbf{T}}[\![\breve{\mathbf{v}}]\!] + \breve{\mathbf{t}}_f$$

Reconstruction of K



TBIE

$$\mathbf{T}[\![\breve{v}]\!]$$

$$\boldsymbol{t}_f(\boldsymbol{\xi}) - \breve{\boldsymbol{K}} \cdot [\![\breve{\boldsymbol{v}}]\!](\boldsymbol{\xi}) = \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \int_{\breve{\Gamma}} \boldsymbol{\Sigma} : \boldsymbol{D}_{\boldsymbol{x}} [\![\breve{\boldsymbol{v}}]\!](\boldsymbol{x})\, \mathrm{d}S_x - \rho\omega^2 \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \int_{\breve{\Gamma}} \boldsymbol{U} \cdot ([\![\breve{\boldsymbol{v}}]\!] \otimes \breve{\boldsymbol{n}})(\boldsymbol{x})\, \mathrm{d}S_x$$

free field traction

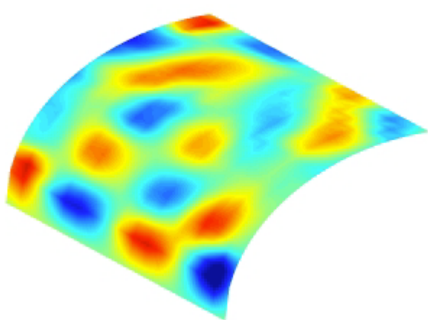
discretization

$$\breve{\mathbf{K}}\,[\![\breve{\mathbf{v}}]\!] = \breve{\mathbf{T}}[\![\breve{\mathbf{v}}]\!] + \breve{\mathbf{t}}_f$$

Regularization

interface-wave suppression → smoothing the differential operator → Tikhonov

$[\![\mathbf{v}]\!]^{\mathrm{loc}}$



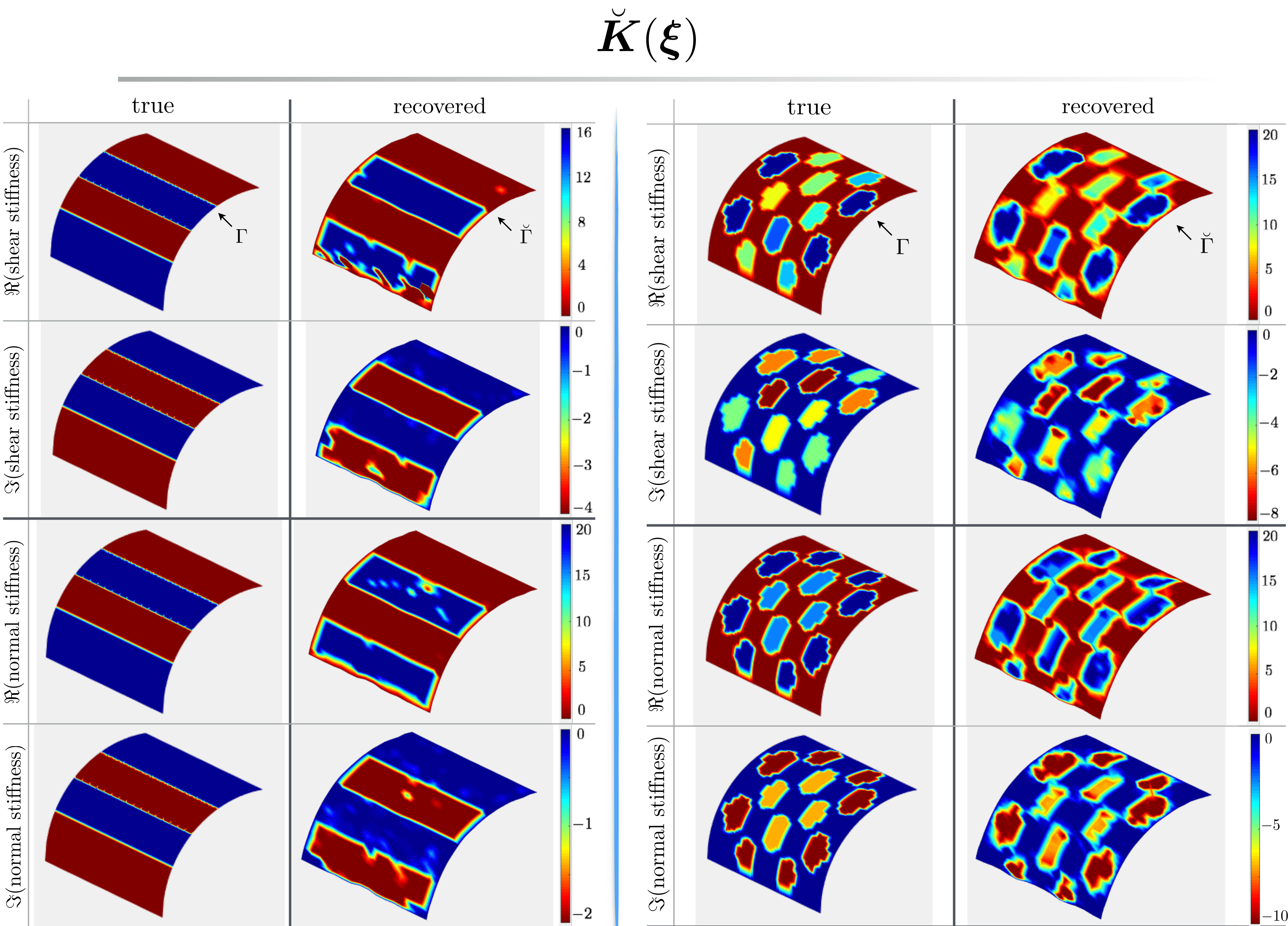
source density rearrangement
$[\![\mathbf{v}]\!]^{\mathrm{loc}} \to \mathbf{0}$

$\mathbf{M}, \mathbf{F}$ ⇒ $\boldsymbol{g}$

SVD of $\mathbf{T}$

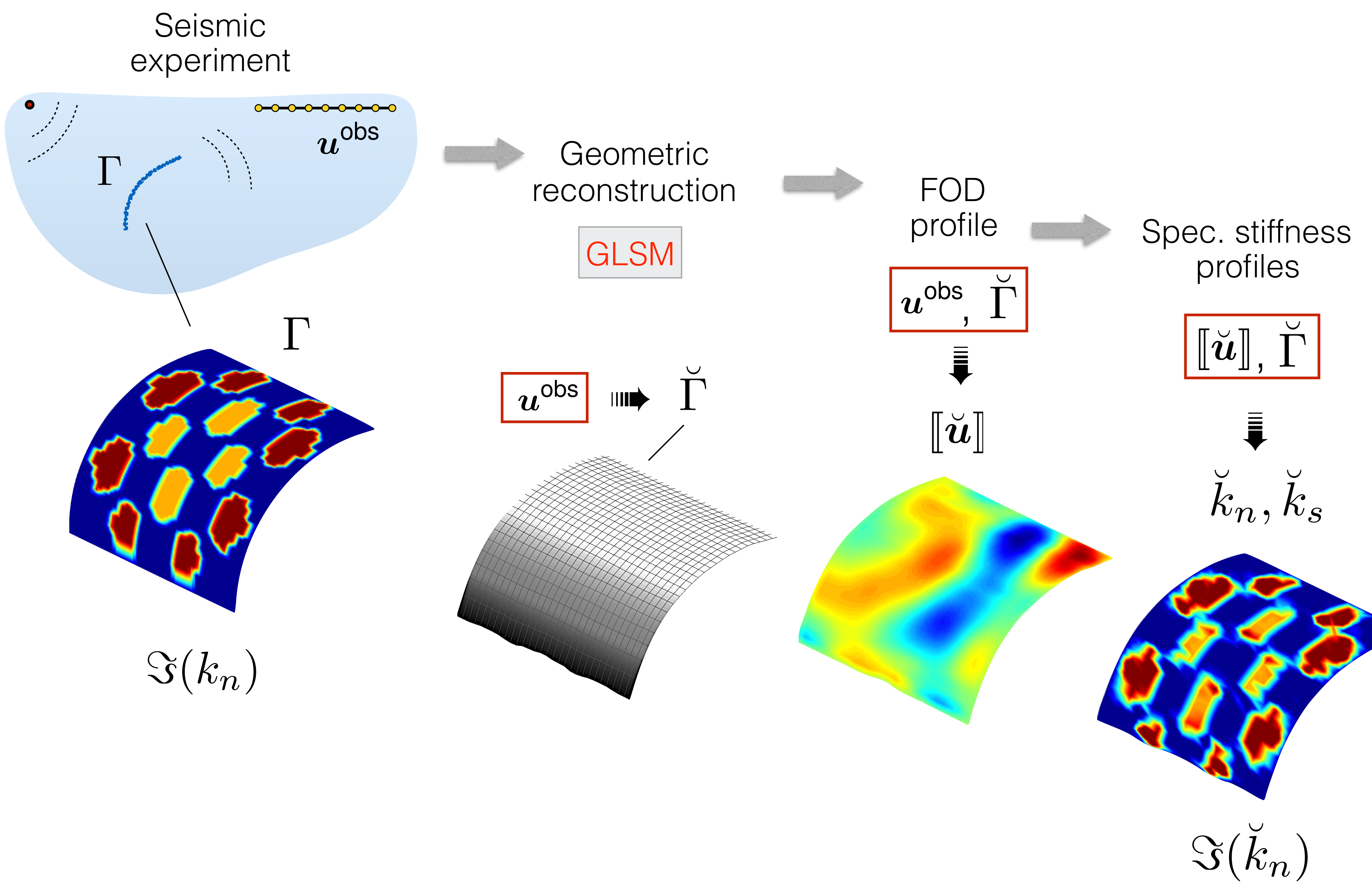
$[\![\breve{\mathbf{v}}]\!], \mathbf{T}$ ⇓ $\breve{\mathbf{T}}$

⇓ $\breve{\mathbf{K}}$



$\breve{\boldsymbol{K}}(\boldsymbol{\xi})$
true
recovered
true
recovered
$\Re$(shear stiffness)
$\Im$(shear stiffness)
$\Re$(normal stiffness)
$\Im$(normal stiffness)
Γ
$\breve{\Gamma}$
16
12
8
4
0
−1
−2
−3
−4
20
15
10
5
−5
−6
−8
−10

Summary



Seismic experiment
u^{obs}
Γ
Γ
$\Im(k_n)$
Geometric reconstruction
GLSM
u^{obs}
$\breve{\Gamma}$
FOD profile
u^{obs}, $\breve{\Gamma}$
$[\![\breve{u}]\!]$
Spec. stiffness profiles
$[\![\breve{u}]\!]$, $\breve{\Gamma}$
$\breve{k}_n, \breve{k}_s$
$\Im(\breve{k}_n)$