High-frequency inverse scattering by Topological Sensitivity

This work investigates the performance of topological sensitivity (TS) as a tool for dealing with the inverse scattering of scalar/acoustic waves in the high-frequency regime, when the wave length of the incident field is small relative to the remaining length scales in the problem. To provide a focus in the study, it is assumed that the obstacle is convex and impenetrable (of Dirichlet or Neumann type), and that the full-waveform measurements of the scattered field are taken over a sphere whose radius is finite, yet large relative to the size of the sampling region. In this setting, the TS formula is expressed a pair of nested surface integrals – one taken the measurement sphere, and the other over the surface of a hidden obstacle. By way of multipole expansion, the inner integral (over the measurement surface) is reduced to a set of antilinear forms in terms of the Greens function and its gradient. The remaining expression is distilled by evaluating the scattered field on the surface of the obstacle via Kirchhoff approximation, and pursuing an asymptotic expansion of the resulting Fourier integral. In this way, the TS is found to survive upon three asymptotic lynchpins, namely (i) the near-boundary approximation for sampling points close to the 'exposed' surface of an obstacle; (ii) uniform expansions synthesizing the diffraction catastrophes for sampling points near caustic surfaces, lines and points; and (iii) stationary phase approximation. Within the framework of catastrophe theory, it is shown that, in the case of the full source aperture, the TS is asymptotically dominated by the (explicit) near-boundary term—which explains the previously reported reconstruction capabilities of this class of indicator functionals. The analysis further shows that, when the (Dirichlet or Neumann) character of an anomaly is unknown beforehand, the latter can be effectively exposed by assuming point-like Dirichlet perturbation and considering the sign of the leading term inside the reconstruction. The analysis is illustrated by numerical simulations and an application to laboratory observations of the scattered field.