In the Name of God

Identification of contact parameters in Nonlinear systems with internal resonance

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✓ *Purpose*

- ✓ To perform a dual-mode identification procedure in nonlinear systems
  - ✓ to study the nonlinear behavior of the system when two modes considerably contribute in the response.
  - ✓ to investigate the contact behavior in presence of micro-slap & micro-macro slip using a non-parametric identification procedure
  - ✓ to identify the parameters of a proposed friction model simulating the effects of micro-slap on friction force

✓Internal Resonance

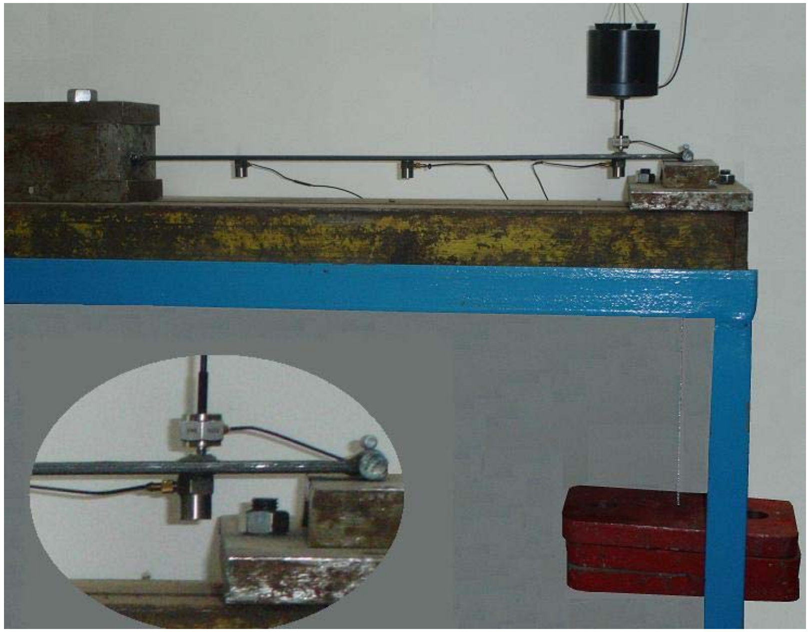
- Suppose that n real and non-zero natural frequencies of an n degree of freedom system are denoted by $\omega_1,\omega_2,\ldots,\omega_n$.

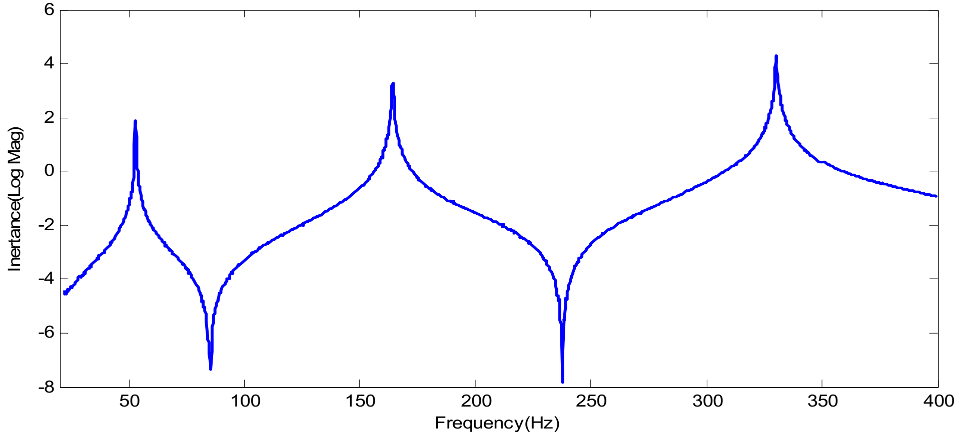
- An important case occurs whenever two or more are commensurable or nearly commensurable *i.e.* $\omega_2 \approx 3\omega_1$.

- Depending on the order of nonlinearity in the system, the commensurable relationship of frequencies can cause the corresponding modes to be strongly coupled, and an internal resonance is said to exist.

- In these cases, The nonlinearity may lead to qualitatively new effects such as instability of cooperative modes, so stability analysis is needed.

✓Overview





Natural frequencies of the base linear system

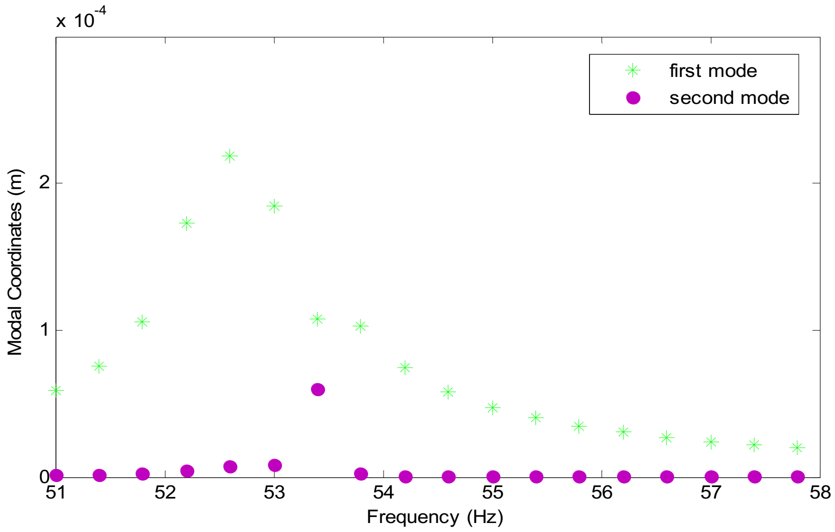
ω_1	ω_2	ω_3
52.85	164.00	330.25

✓Overview

- The arrangement of transducers are changed in order to satisfy the necessary condition for internal resonance to occur.

- Using the identified contact model, we try to solve the problem analytically in order to see if internal resonance takes place and to investigate the stability of the obtained branch of solution.

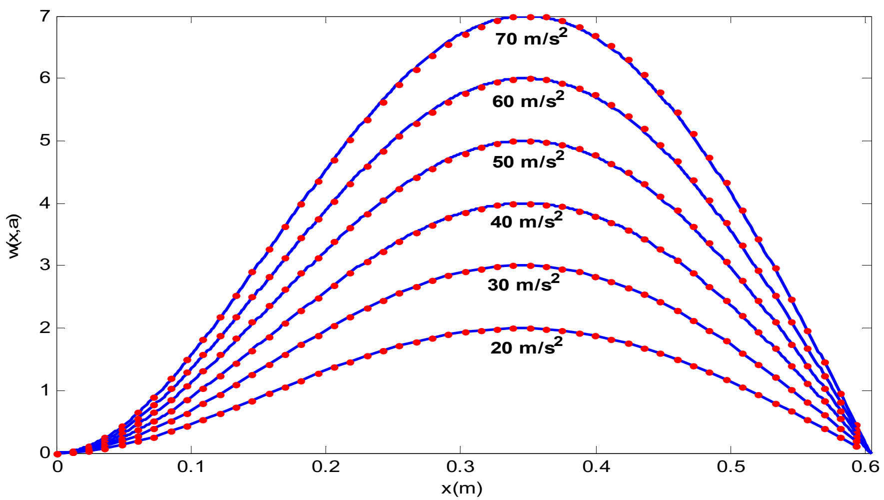
The Nonlinear FRF using IHB method along with continuation procedure



✓ Overview

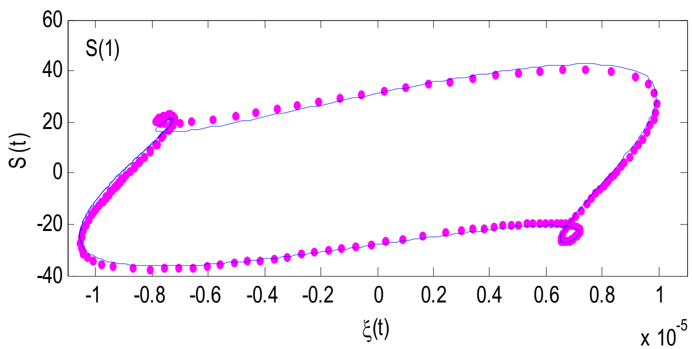
- A method is presented for identification of the nonlinear normal modes. It is based on the assumption that a nonlinear normal mode can be expanded using the normal modes of the base linear system.
- By using the measured responses, the contribution of each linear mode shape in nonlinear normal mode is calculated.

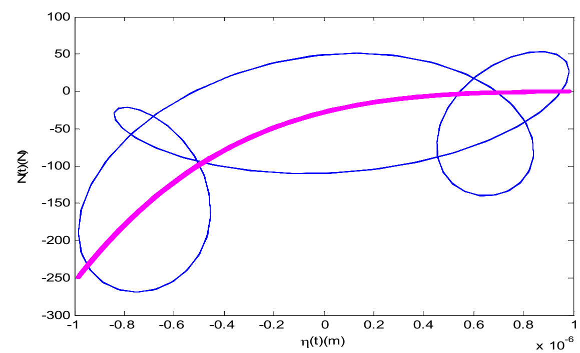
First NNMs: the equivalent linearized systems (blue), the proposed method (red)

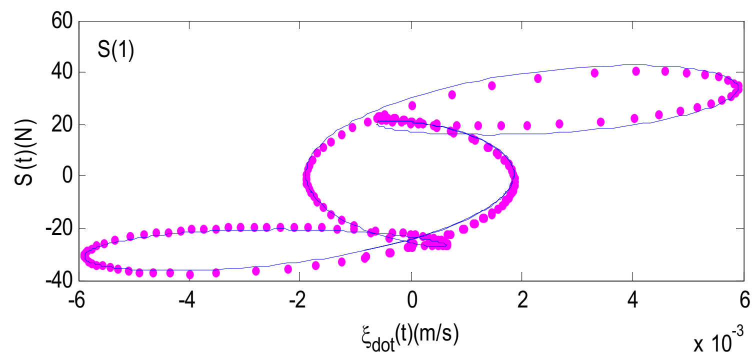


Overview

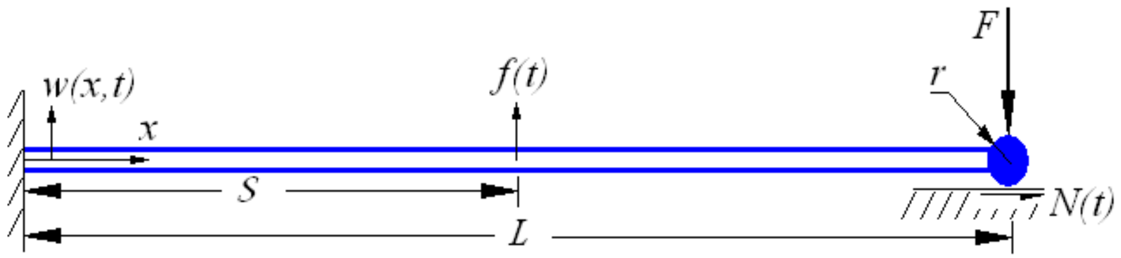
- ✓ we consider the nonlinear behavior of a beam with frictional contact support in the situations in which micro slap develops in normal direction. A friction law based on Valanis model is introduced which accounts for the effects of micro impact on friction force.







✓Nonlinear Model Identification of a Frictional Contact Support



A slender beam with frictional contact boundary condition

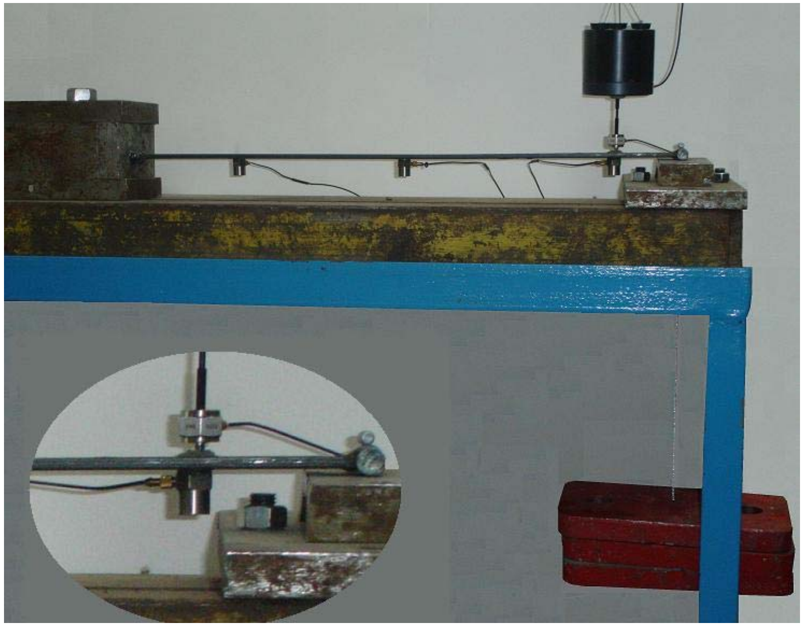
$$EI\frac{\partial^4 w}{\partial x^4}-N(t)\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}=f(t)\delta(x-S)-rN(t)\delta'(x-L)$$

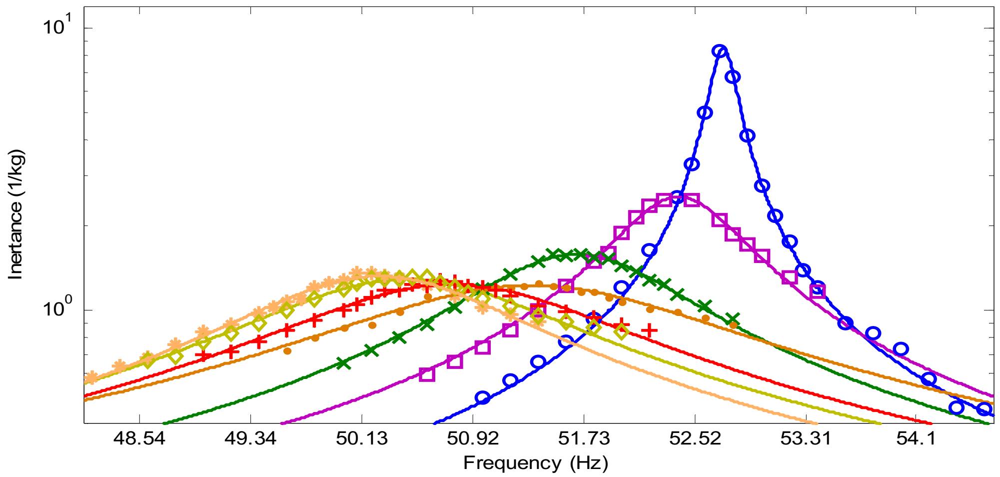
$$w(x,t)=\sum_{i=1}^{n}\tilde{\phi}_i(x,a)q_i(t)$$

$$\ddot{q}_i+\omega_i^2 q_i-f(t)\tilde{\varphi}_i(S,a)-k_\theta(a)\frac{\partial\tilde{\varphi}_i(L,a)}{\partial x}\sum_{r=1}^{n}\frac{\partial\tilde{\varphi}_r(L,a)}{\partial x}q_r(t)$$
$$=\left(r\frac{\partial\tilde{\varphi}_i(L,a)}{\partial x}-\sum_{r=1}^{n}q_r(t)\int_0^L\frac{\partial\tilde{\varphi}_i(x,a)}{\partial x}\frac{\partial\tilde{\varphi}_r(x,a)}{\partial x}dx\right)N(t),\ \ i=1,2,\ldots,n.$$

$$u(t)=-\frac{1}{2}\int_0^L\left(\frac{\partial w(x,t)}{\partial x}\right)^2dx+r\frac{\partial w(L,t)}{\partial x}+N(t)L/AE.$$

✓Nonlinear Model Identification of a Frictional Contact Support





Measured and fitted frequency responses at response amplitudes of 10 m/s²(circles), 20 m/s²(squares), 30 m/s²(×),40 m/s²(points), 50 m/s²(+),60 m/s²(diamonds), and 70 m/s²(*).

a, m/s²	$\omega_1(a)$, Hz	$k_\theta(a)$, kN.m/rad
Linear	52.85	197.4
10	52.79	192.8
20	52.30	167.1
30	51.62	125.4
40	50.63	73.6
50	50.55	71.4
60	50.30	65.7
70	50.11	58.7

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=0$$

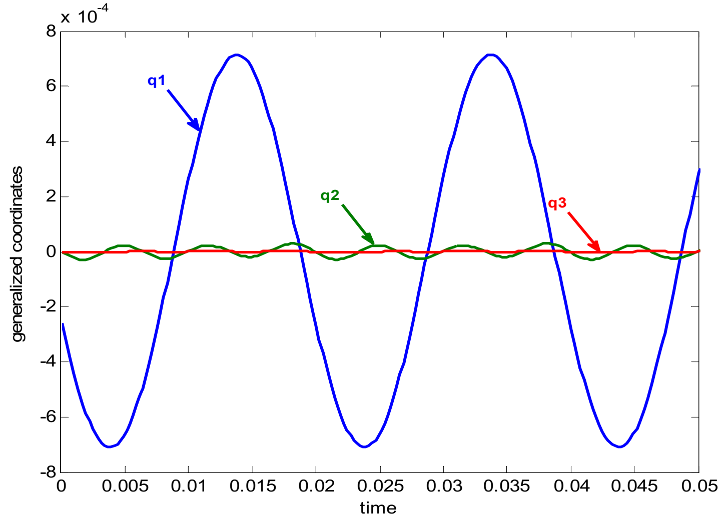
$$w(0,t)=0,\ \ \frac{\partial w(0,t)}{\partial x}=0,\ \ w(L,t)=0,\ \ EI\frac{\partial^2 w(L,t)}{\partial x^2}=k_\theta(a)\frac{\partial w(L,t)}{\partial x}$$

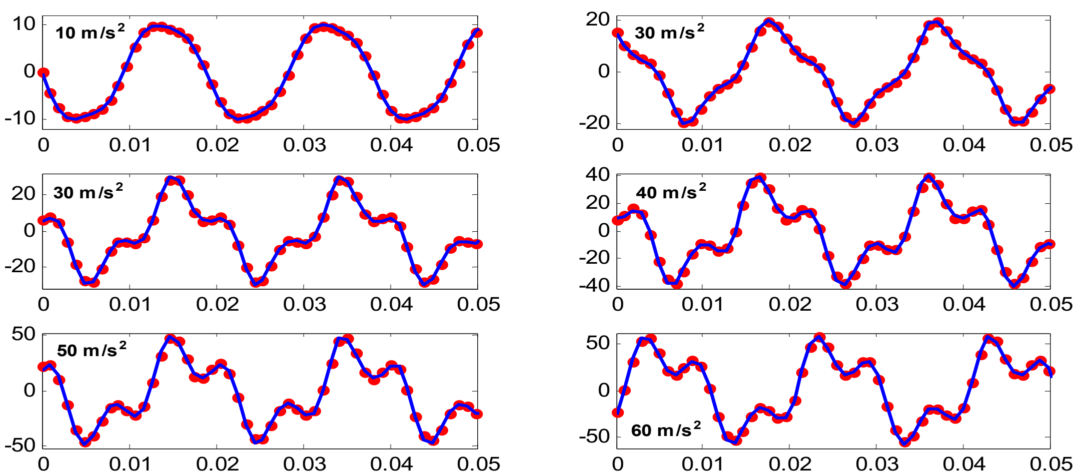
✓ Nonlinear Model Identification of a Frictional Contact Support

$$\ddot{\mathbf{q}}(\mathbf{t}) = \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \\ \ddot{q}_n(t) \end{Bmatrix} = \begin{bmatrix} \tilde{\phi}_1(a,x_1) & \tilde{\phi}_2(a,x_1) & \cdots & \tilde{\phi}_n(a,x_1) \\ \tilde{\phi}_1(a,x_2) & \tilde{\phi}_2(a,x_2) & \cdots & \tilde{\phi}_n(a,x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\phi}_1(a,x_j) & \tilde{\phi}_2(a,x_j) & \cdots & \tilde{\phi}_n(a,x_j) \end{bmatrix}^+ \begin{Bmatrix} \ddot{w}(x_1,t) \\ \ddot{w}(x_2,t) \\ \vdots \\ \ddot{w}(x_j,t) \end{Bmatrix} = \mathbf{\Phi}^+ \ddot{\mathbf{w}}$$

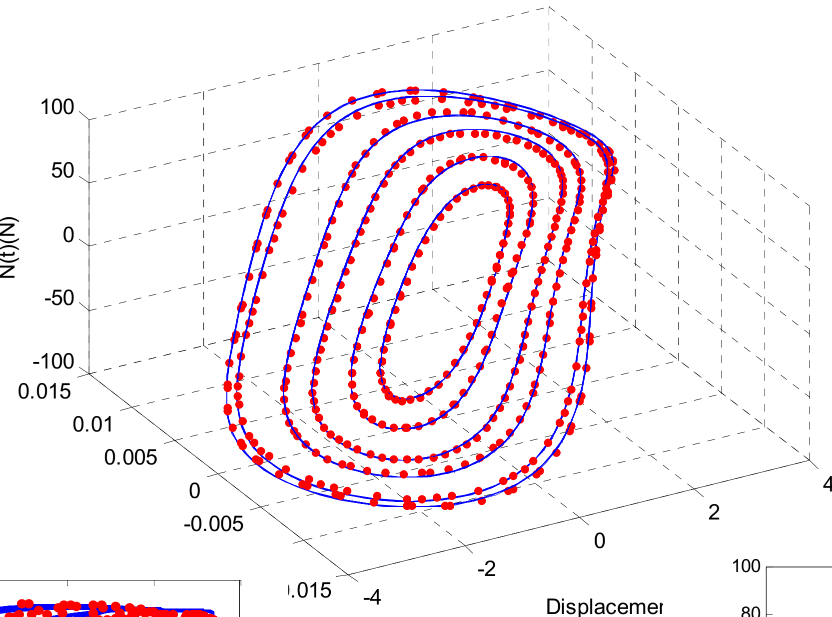
$$\ddot{q}_i(t) = \sum_{m=1}^{M} \left(A_m \cos(m\omega t) + B_m \sin(m\omega t) \right), \;\; i = 1,2,3.$$

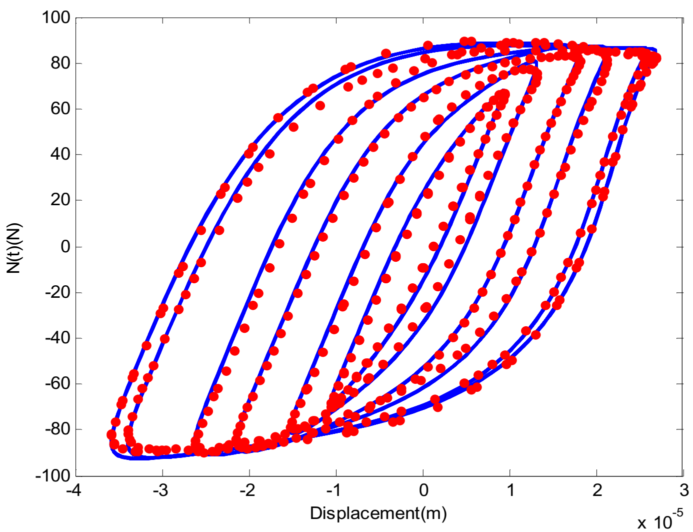
$$q_i(t) = -\sum_{m=1}^{M} \left(\frac{A_m}{(m\omega)^2} \cos(m\omega t) + \frac{B_m}{(m\omega)^2} \sin(m\omega t) \right), \;\; i = 1,2,3.$$

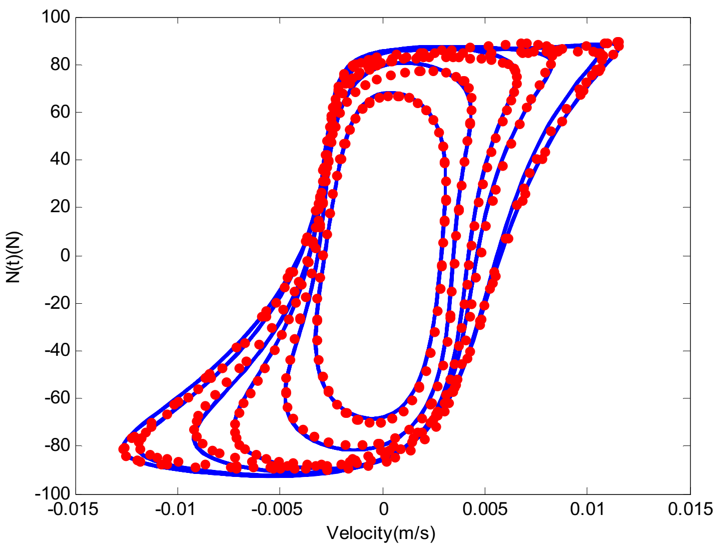




✓Nonlinear Model Identification of a Frictional Contact Support





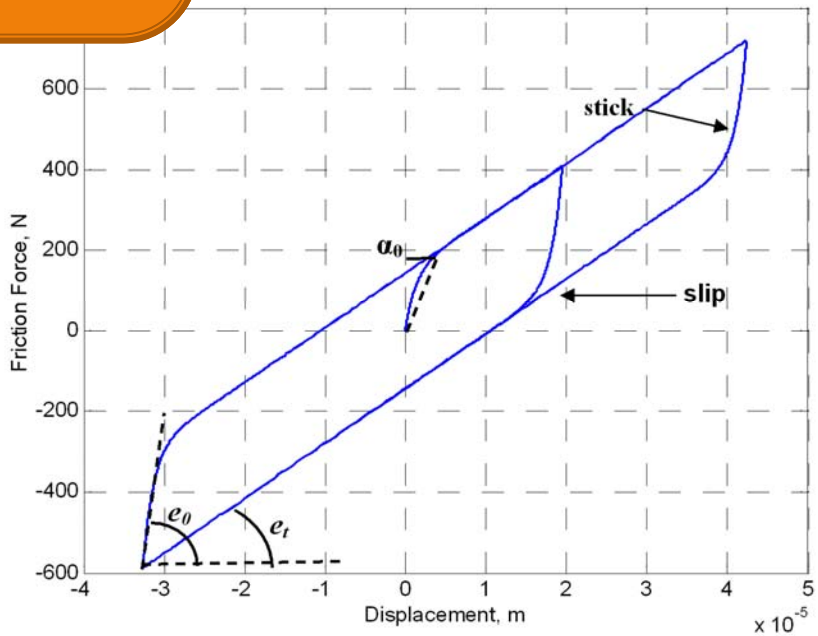


✓Nonlinear Model Identification of a Frictional Contact Support

$$\dot{N}(t)=\frac{e_0\dot{u}\left[1+\frac{\lambda}{e_0}\operatorname{sgn}(\dot{u})\left(e_t u-N(t)\right)\right]}{1+\kappa\frac{\lambda}{e_0}\operatorname{sgn}(\dot{u})\left(e_t u-N(t)\right)},$$

$$\lambda=\frac{e_0}{\alpha_0\left(1-\kappa\frac{e_t}{e_0}\right)}$$

Parameter	Value
e_0 (N/m)	7.9e6
e_t (N/m)	≈ 0.0
κ	0.3
λ (1/m)	8.7e4



✓Analytical and Experimental investigation of the system with near commensurable natural frequencies

$$\ddot{q}_i + \omega_i^2 q_i - f(t)\varphi_i(S) - k_\theta \frac{\partial \varphi_i(L)}{\partial x}\sum_{r=1}^{2}\frac{\partial \varphi_r(L)}{\partial x} q_r(t)$$

$$= \left(r\frac{\partial \varphi_i(L)}{\partial x} - \sum_{r=1}^{2} q_r(t)\int_0^L \frac{\partial \varphi_i(x)}{\partial x}\frac{\partial \varphi_r(x)}{\partial x}dx \right) N(t),\ \ i = 1,2.$$

$$X(t) = -\frac{1}{2}\int_0^L \left(\sum_{r=1}^{2}\frac{\partial \varphi_r(x)}{\partial x} q_r(t)\right)^2 dx + r\sum_{r=1}^{2}\frac{\partial \varphi_r(L)}{\partial x} q_r(t) + N(t)L / AE.$$

$$\dot{N}(t) - e_0\dot{X}(t) + \frac{\lambda}{e_0}\left(e_t X(t) - N(t)\right)\left(\kappa \dot{N}(t) - e_0\dot{X}(t)\right)\mathrm{sgn}\left(\dot{X}(t)\right) = 0.$$

$$X(t) = X\cos\left(\omega t + \theta\right).$$

$$\mathrm{sgn}(\dot{X}(t)) = \mathrm{sgn}\left(-X\omega\sin\left(\omega t + \theta\right)\right)$$

$$= A\cos\left(\omega t\right) + B\sin\left(\omega t\right),$$

$$A = -\frac{4}{\pi}\sin\left(\theta\right), B = -\frac{4}{\pi}\sin\left(\theta\right).$$

$$X(t) = X_1\cos\left(\omega t + \theta_1\right) + X_3\cos\left(3\omega t + \theta_3\right).$$

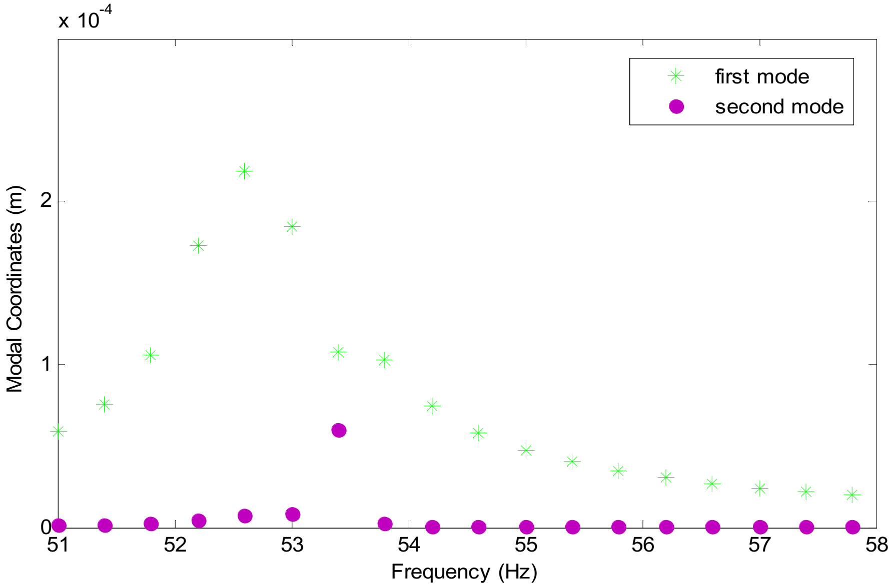
$$\mathrm{sgn}(\dot{X}(t)) = A\cos\left(\omega t\right) + B\sin\left(\omega t\right) + C\cos\left(3\omega t\right) + D\sin\left(3\omega t\right),$$

$$A = \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} \mathrm{sgn}\left(-\omega X_1\sin\left(\omega t + \theta_1\right) - 3\omega X_3\sin\left(3\omega t + \theta_3\right)\right)\cos\omega t\ dt.$$

→ ***IHB Method***

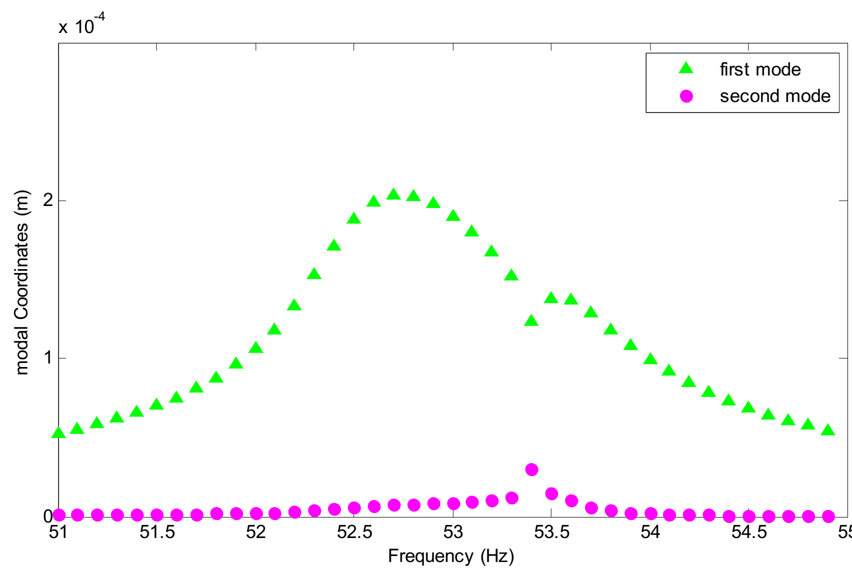
✓ Analytical and Experimental investigation of the system with near commensurable natural frequencies

مکان شتاب سنج	$x_1(m)$	$x_2(m)$	$x_3(m)$	$\omega_2 - 3\omega_1 (Hz)$
الف	۰.۵۴	۰.۱۹	۰.۱۳	۰.۵
ب	۰.۵۴	۰.۲۸	۰.۰۷	۰.۸۵

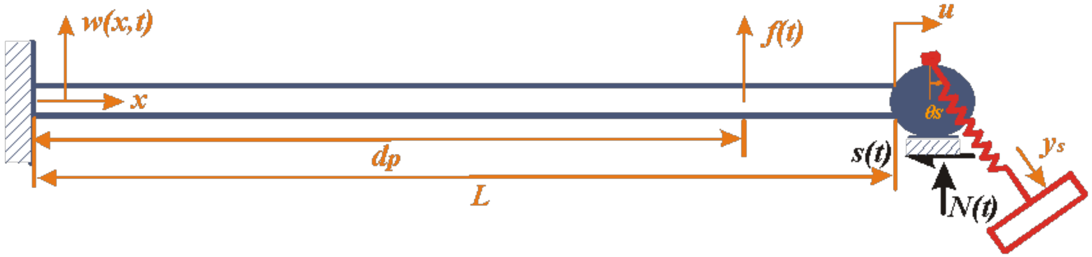


Nonlinear FRF A

Nonlinear FRF B



✓identifying slip-slap forces in the contact interface using dual-mode excitation



A slender beam with frictional contact boundary condition

The discrete part of the system

$$
\begin{cases}
m_s\left(\ddot{u}-r\dfrac{\partial\ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\dfrac{\partial^2}{\partial t^2}\displaystyle\int_0^L\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2dx\right)+k_b u+S(t)=0.\\[2ex]
m_s\left(\ddot{w}(L,t)+r\dfrac{\partial\ddot{w}(L,t)}{\partial x}-\ddot{y}_s\right)-k_s y_s=0.\\[2ex]
J_s\ddot{\theta}_s+m_s l_s\left(\ddot{u}-r\dfrac{\partial\ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\dfrac{\partial^2}{\partial t^2}\displaystyle\int_0^L\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2dx\right)+m_s g l_s\theta_s=0.
\end{cases}
$$

Beam equation of motion

$$
EI\frac{\partial^4 w}{\partial x^4}+\left(S(t)+m_s\left(\ddot{u}-r\frac{\partial\ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2dx\right)\right)\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}=f(t)\delta(x-d_p).
$$

✓identifying slip-slap forces in the contact interface using dual-mode excitation

Boundary Conditions

✓ $$w(0,t)=0,\quad \frac{\partial w(0,t)}{\partial x}=0.$$

✓ $$EI\frac{\partial^3 w(L,t)}{\partial x^3}-m_p\left(\ddot{w}(L,t)+r\frac{\partial \ddot{w}(L,t)}{\partial x}\right)-m_s\left(\ddot{w}(L,t)+r\frac{\partial \ddot{w}(L,t)}{\partial x}-\ddot{y}_s\right)+N(t)$$
$$+\frac{\partial w(L,t)}{\partial x}\left(S(t)+m_s\left(\ddot{u}-r\frac{\partial \ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2dx\right)\right)=0.$$

✓ $$EI\frac{\partial^2 w(L,t)}{\partial x^2}+rEI\frac{\partial^3 w(L,t)}{\partial x^3}+rS(t)\left(1+\frac{\partial w(L,t)}{\partial x}\right)+J_p\frac{\partial \ddot{w}(L,t)}{\partial x}-$$
$$rm_s\left(\ddot{u}-r\frac{\partial \ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2dx\right)\left(1-\frac{\partial w(L,t)}{\partial x}\right)=0.$$

$$\begin{Bmatrix} w(x,t)\\ u(t)\\ y_s(t)\\ \theta_s(t)\end{Bmatrix}=\sum_{i=1}^{n}\begin{Bmatrix}\tilde{\varphi}_i(x)\\ \tilde{u}_i\\ \tilde{y}_{si}\\ \tilde{\theta}_{si}\end{Bmatrix}q_i(t)$$

✓ identifying slip-slap forces in the contact interface using dual-mode excitation

Galerkin Projection

$$\int_0^L \tilde{\varphi}_i(x)\left(EI\frac{\partial^4 w}{\partial x^4}+\left(S(t)+m_s\left(\ddot{u}-r\frac{\partial \ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx\right)\right)\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}-f(t)\delta(x-d_p)\right)dx+$$

$$\tilde{u}_i\left(m_s\left(\ddot{u}-r\frac{\partial \ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx\right)+k_b u+S(t)\right)+$$

$$\tilde{y}_{si}\left(m_s\left(\ddot{w}(L,t)+r\frac{\partial \ddot{w}(L,t)}{\partial x}-\ddot{y}_s\right)-k_s y_s\right)+$$

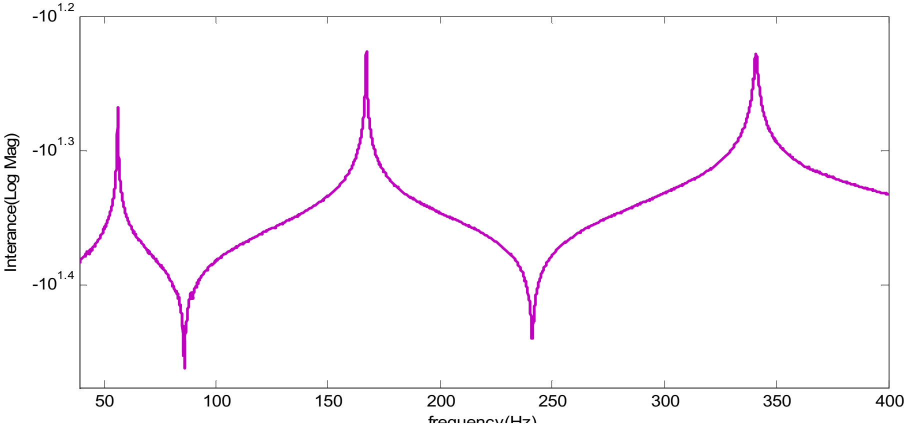
$$\tilde{\theta}_{si}\left(J_s\ddot{\theta}_s+m_s l_s\left(\ddot{u}-r\frac{\partial \ddot{w}(L,t)}{\partial x}+l_s\ddot{\theta}_s-\frac{\partial^2}{\partial t^2}\int_0^L \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx\right)+m_s g l_s\theta_s\right)=0.$$

Relative displacement at the contact interface

✓ $$\xi(t)=-\frac{1}{2}\int_0^L\left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx+r\frac{\partial w(L,t)}{\partial x}+u(t).$$

✓ $$\eta(t)=w(L,t)+r\frac{\partial w(L,t)}{\partial x}$$

✓identifying slip-slap forces in the contact interface using dual-mode excitation



Resonance frequencies at low level random excitations (Hz)

ω_1	ω_2	ω_3
54.85	166.2	339.25

$$S(t)=k_t\left(r\frac{\partial w(L,t)}{\partial x}+u(t)\right),N(t)=k_n\left(w(L,t)+r\frac{\partial w(L,t)}{\partial x}\right)$$

Resonance frequencies at low level random excitations (Hz)

Parameter	k_t (N/m)	k_n (N/m)
Value	3.95e6	2.37e7

✓Identifying Normal Modes of a Nonlinear System

$$\frac{\partial^2 w}{\partial t^2} + L\left[w\left(x,t\right)\right] + N\left[w\left(x,t\right)\right] = f(x,t).$$

$$w(x,t) = \sum_{i=1}^{n} \varphi_i(x) q_i(t).$$

$$\ddot{\mathbf{q}}(\mathbf{t}) = \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \\ \ddot{q}_n(t) \end{Bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1(x_j) & \varphi_2(x_j) & \cdots & \varphi_n(x_j) \end{bmatrix}^{+} \begin{Bmatrix} \ddot{w}(x_1,t) \\ \ddot{w}(x_2,t) \\ \vdots \\ \ddot{w}(x_j,t) \end{Bmatrix} = \mathbf{\Phi}^{+}\ddot{\mathbf{w}}$$

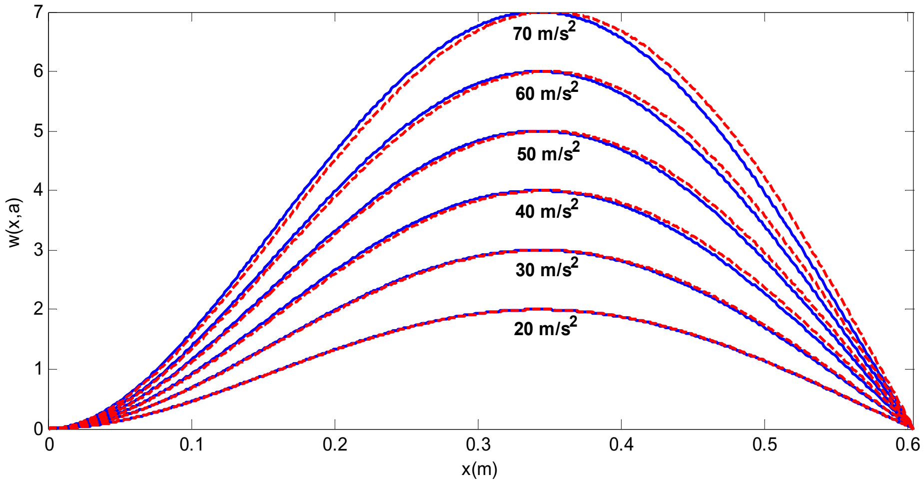
$$\ddot{q}_i(t) = \sum_{m=1}^{M} A_{mi} sin(m\omega t + \psi_{mi}) = A_{1i} sin(\omega t + \psi_{1i}) + \ldots, \;\; i = 1,2,3.$$

$$q_i(t) = A_{0i} - \sum_{m=1}^{M} \frac{A_{mi}}{(m\omega)^2} \sin(m\omega t + \psi_m) = A_{0i} - \frac{A_{1i}}{\omega^2} \sin(\omega t + \psi_1) + \ldots, \;\; i = 1,2,3.$$

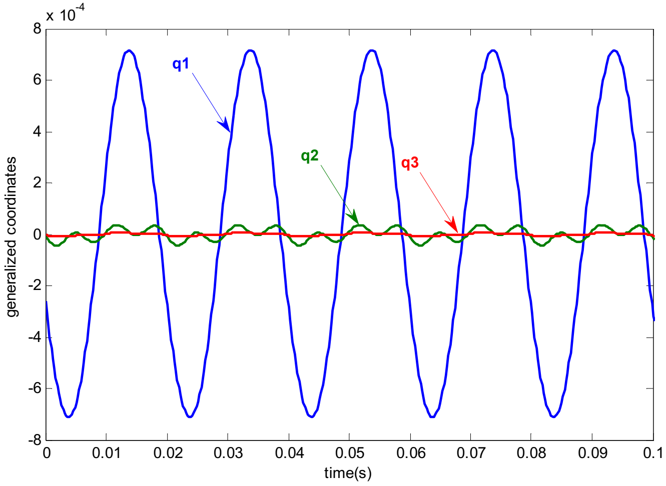
$$\boxed{w(x,t) \simeq -\left(\sum_{i=1}^{n} \frac{A_{1i}\varphi_i(x)}{\omega^2}\right) \sin(\omega t + \psi_1).}$$

$$\boxed{\tilde{\varphi}_i\left(x,a\right) \simeq \frac{1}{\left|A_1\right|} \sum_{i=1}^{n} A_{1i}\varphi_i\left(x\right).}$$

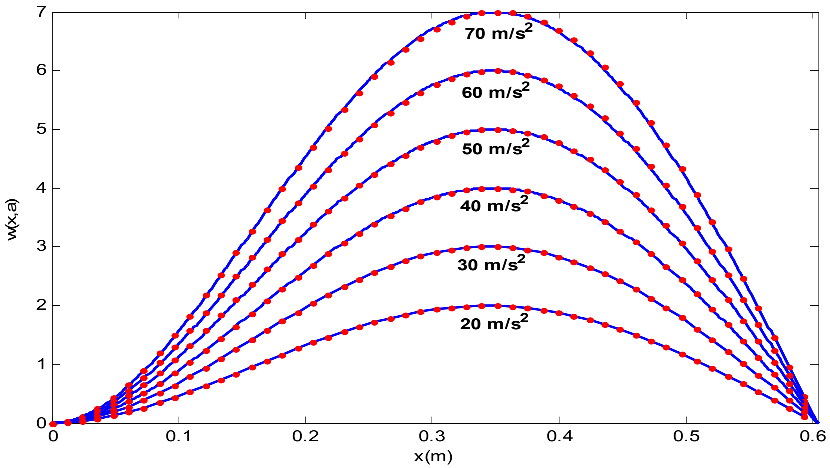
✓Identifying Normal Modes of a Nonlinear System



Comparing NNMs obtained by equivalent linearized system (red) and the first mode of the base linear system (blue)



The generalized coordinates at a=70m/s^2

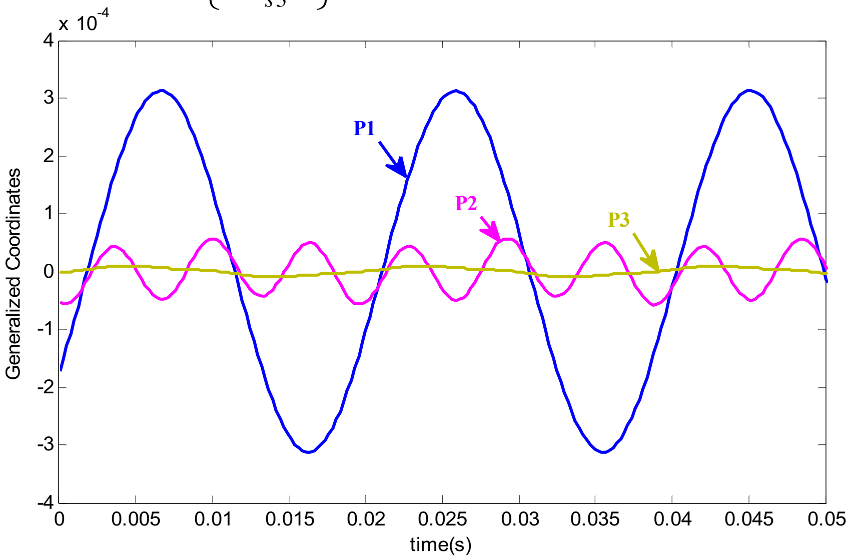


First NNMs: the equivalent linearized systems (blue), the method proposed in this paper (red)

✓identifying slip-slap forces in the contact interface using dual-mode excitation

$$\begin{Bmatrix} w(x,t) \\ u(t) \\ y_s(t) \\ \theta_s(t) \end{Bmatrix} \simeq \begin{Bmatrix} \tilde{\varphi}_1(x) \\ \tilde{u}_1 \\ \tilde{y}_{s1} \\ \tilde{\theta}_{s1} \end{Bmatrix} a_1 \sin(\omega t + \psi_1) + \begin{Bmatrix} \tilde{\varphi}_3(x) \\ \tilde{u}_3 \\ \tilde{y}_{s3} \\ \tilde{\theta}_{s3} \end{Bmatrix} a_3 \sin(3\omega t + \psi_3) + \dots$$

$$\begin{Bmatrix} w(x,t) \\ u(t) \\ y_s(t) \\ \theta_s(t) \end{Bmatrix} = \sum_{i=1}^{m} \begin{Bmatrix} \varphi_i(x) \\ u_i \\ y_{si} \\ \theta_{si} \end{Bmatrix} p_i(t)$$



$$\ddot{p}_i(t) = \sum_{k=1}^{K} A_{ki} sin(k\omega t + \psi_{ki}) \simeq A_{1i} sin(\omega t + \psi_{1i}) + A_{3i} sin(3\omega t + \psi_{3i}).$$

$$p_i(t) = -\sum_{k=1}^{K} \frac{A_{ki}}{(k\omega)^2} \sin(k\omega t + \psi_k) = -\frac{1}{\omega^2}\left(A_{1i} \sin(\omega t + \psi_{1i}) + \frac{A_{3i}}{9} \sin(3\omega t + \psi_{3i}) \right)$$

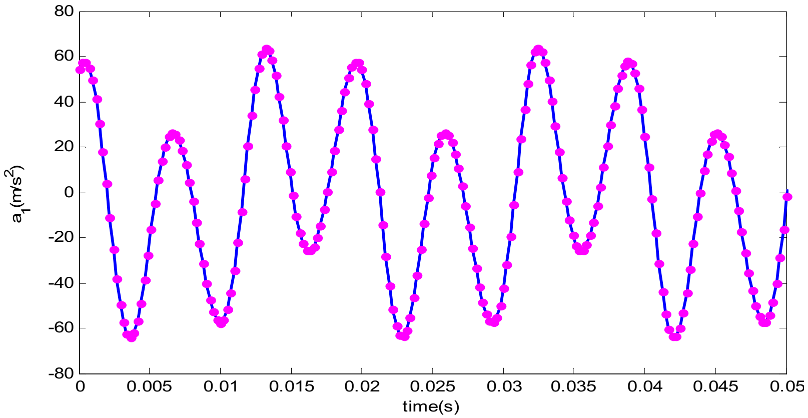
$$i = 1, 2, \dots, m.$$

✓identifying slip-slap forces in the contact interface using dual-mode excitation

$$\begin{Bmatrix} w(x,t) \\ u(t) \\ y_s(t) \\ \theta_s(t) \end{Bmatrix} \simeq -\frac{1}{\omega^2}\left(\sum_{i=1}^{m} A_{1i} \begin{Bmatrix} \varphi_i(x) \\ u_i \\ y_{si} \\ \theta_{si} \end{Bmatrix} \sin(\omega t+\psi_1) + \frac{1}{9}\sum_{i=1}^{m} A_{3i} \begin{Bmatrix} \varphi_i(x) \\ u_i \\ y_{si} \\ \theta_{si} \end{Bmatrix} \sin(3\omega t+\psi_3) \right).$$

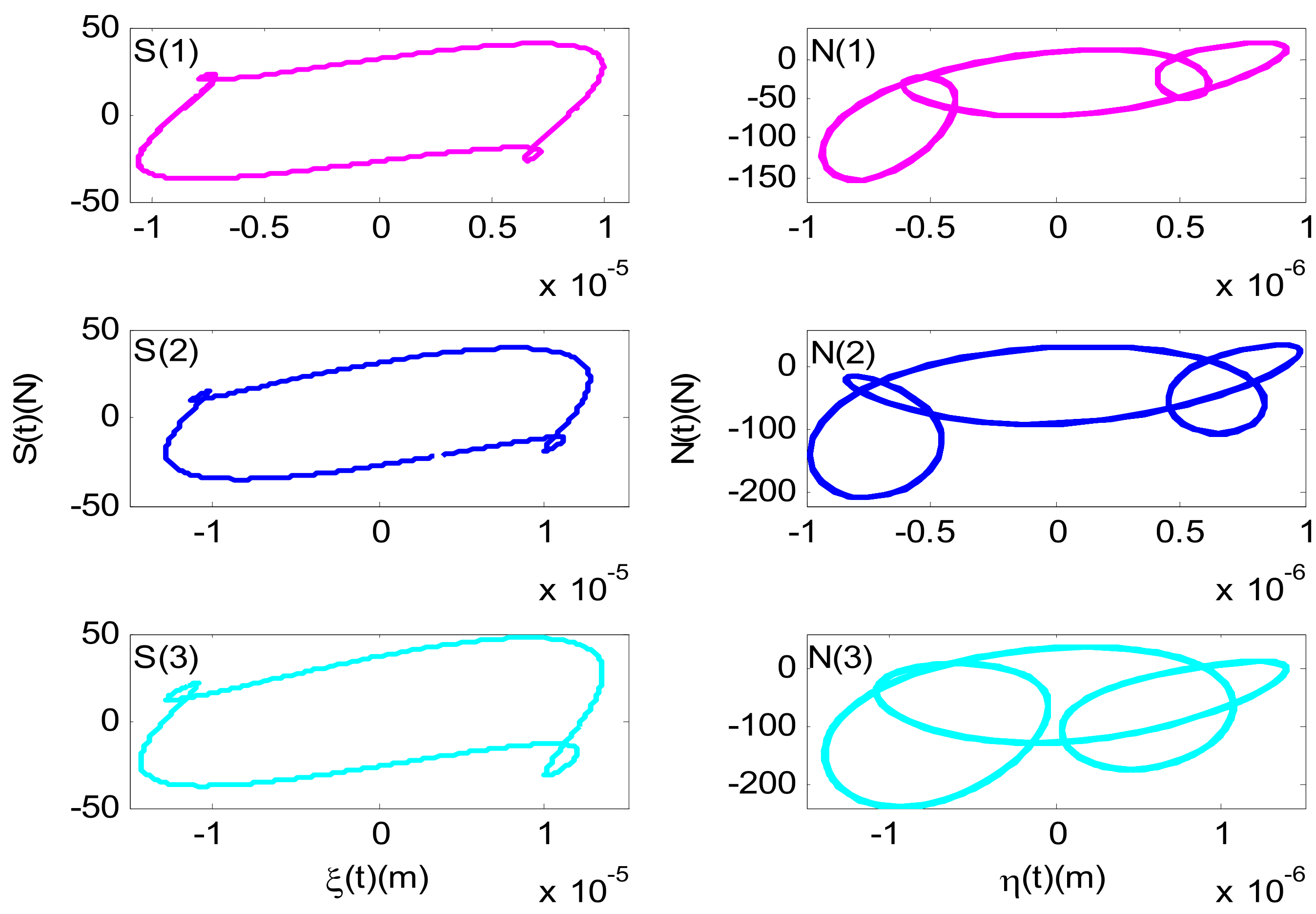
The Nonlinear Modes

$$\begin{Bmatrix} \tilde{\varphi}_j(x) \\ \tilde{u}_j \\ \tilde{y}_{sj} \\ \tilde{\theta}_{sj} \end{Bmatrix} = \frac{1}{\sqrt{A_{j1}^{\ 2} + A_{j2}^{\ 2} + \ldots + A_{jm}^{\ 2}}} \sum_{i=1}^{m} A_{ji} \begin{Bmatrix} \varphi_i(x) \\ u_i \\ y_{si} \\ \theta_{si} \end{Bmatrix}, j = 1,3.$$



Accelerations at the driving point, measured (solid line) and reconstructed (circles)

✓identifying slip-slap forces in the contact interface using dual-mode excitation



S(1)- identified friction force while a_1=14(m/s^2), a_2=34(m/s^2) S(2)- a_1=17.5(m/s^2), a_2=42.5(m/s^2) S(3)- a_1=20(m/s^2), a_2=50(m/s^2) N(1-3)- Corresponding normal force

✓identifying slip-slap forces in the contact interface using dual-mode excitation

$$\dot{S}(t)=\frac{e_0\dot{\xi}\left[1+\frac{\lambda}{e_0}\operatorname{sgn}(\dot{\xi})\left(e_t\xi-S(t)\right)\right]}{1+\kappa\frac{\lambda}{e_0}\operatorname{sgn}(\dot{\xi})\left(e_t\xi-S(t)\right)},\quad \lambda=\frac{e_0}{\alpha_0\left(1-\kappa\frac{e_t}{e_0}\right)}$$

$$\alpha_0=\mu N(t), e_0=e_f\left(1-e^{-\left(\frac{k_c N(t)}{e_f}\right)}\right)$$

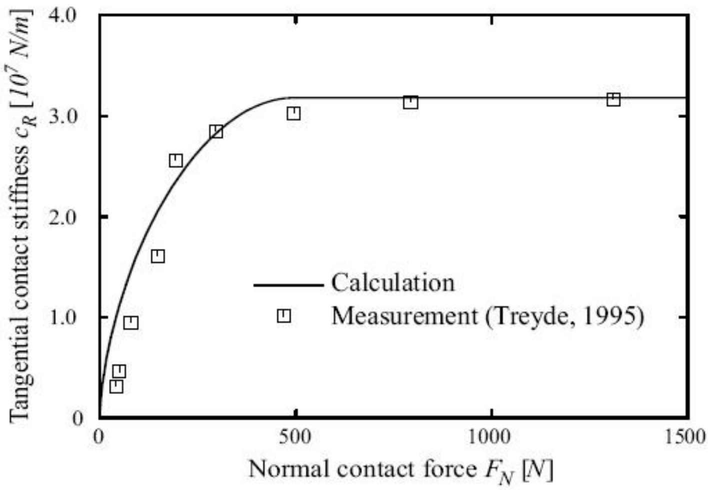
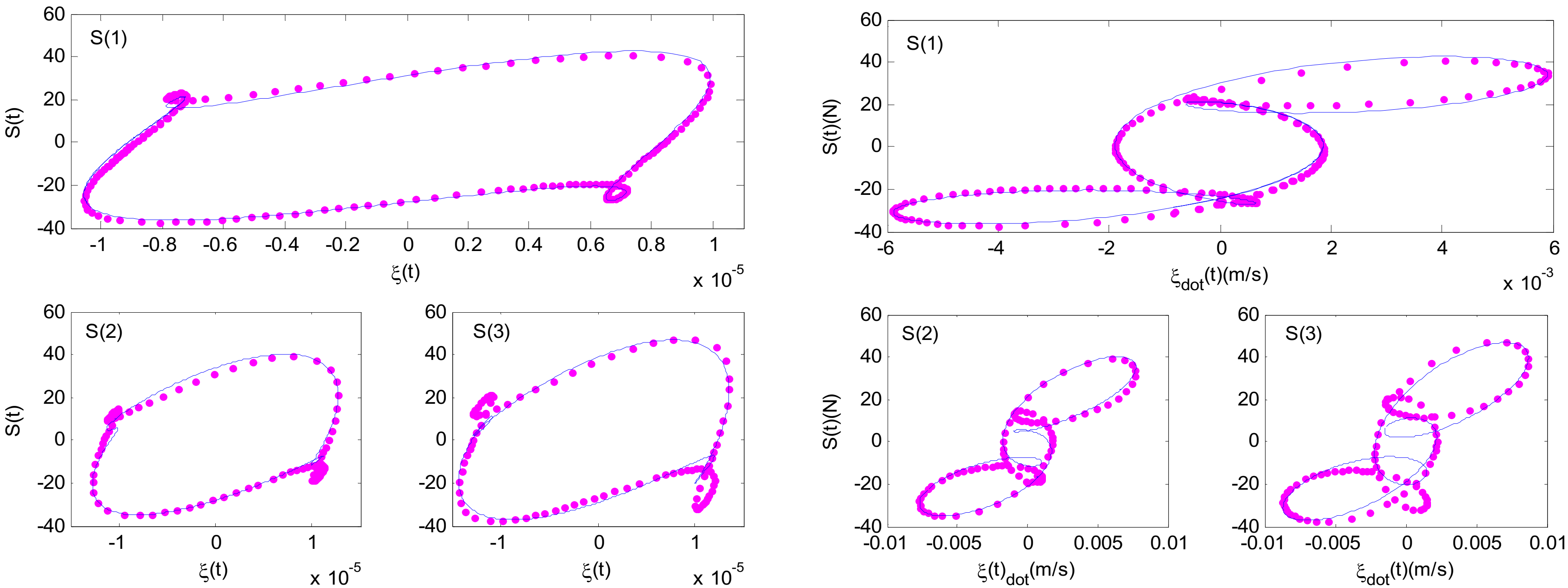


Table 3- The identified parameters for the proposed model

Parameter	Value
e_f (N/m)	3.857e7
k_c (1/m)	2.547e5
e_t (N/m)	≈ 0.0
κ	0.284
μ	0.397

✓identifying slip-slap forces in the contact interface using dual-mode excitation



The hysteresis loops S(t) vs. ξ(t) , $\dot{\xi}(t)$,Measured (circles), Predicted (solid line)

S(1)- a_1=14(m/s^2), a_2=34(m/s^2) S(2)- a_1=17.5(m/s^2), a_2=42.5(m/s^2) S(3)- a_1=20(m/s^2), a_2=50(m/s^2)

THANKS FOR YOUR ATTENTION