Experimental Validation of Differential Evolution Indicators

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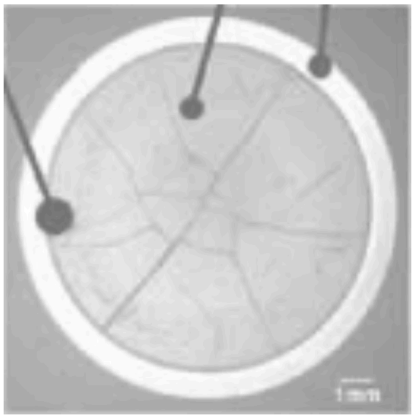
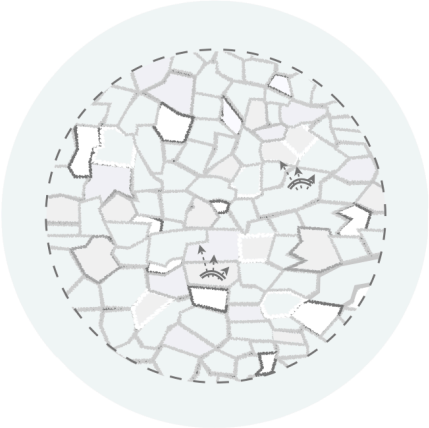
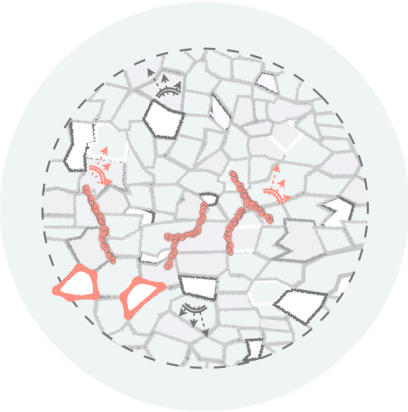
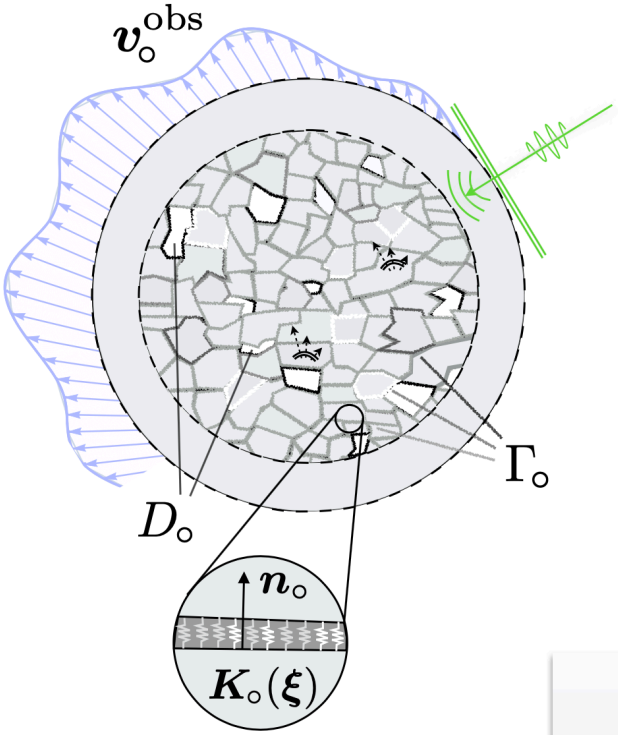




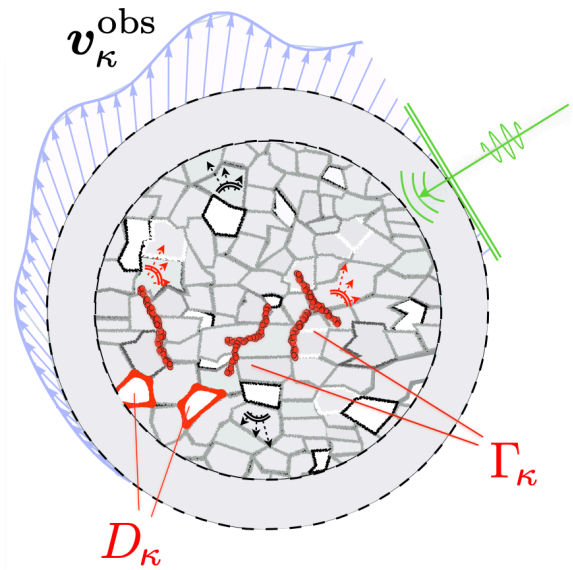


Problem statement

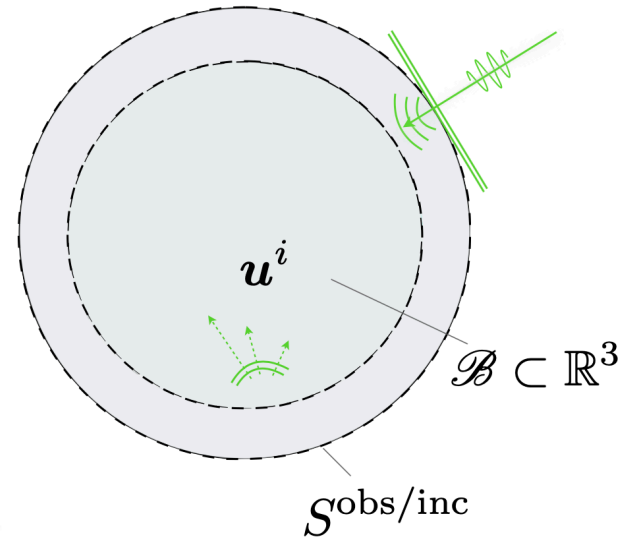
background

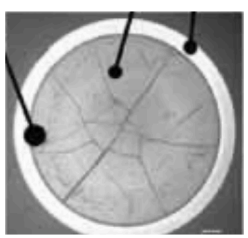


evolved specimen



baseline model





aged nuclear fuel

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v}_\kappa) + \rho\omega^2\boldsymbol{v}_\kappa = \boldsymbol{0} \quad \text{in} \quad \mathscr{B}\backslash\{\Gamma_\circ \cup \Gamma_\kappa\},$$

$$\boldsymbol{n}_\circ\cdot\boldsymbol{C}:\nabla(\boldsymbol{v}_\kappa + \boldsymbol{u}^i) = \boldsymbol{K}_\circ(\boldsymbol{\xi})[\![\boldsymbol{v}_\kappa]\!] \quad \text{on} \quad \Gamma_\circ,$$

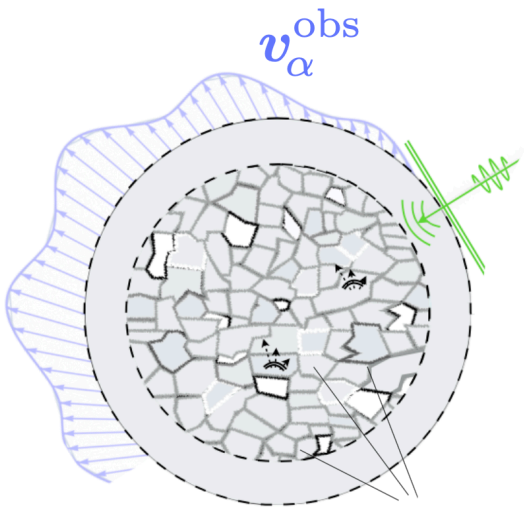
$$\boldsymbol{n}_\kappa\cdot\boldsymbol{C}:\nabla(\boldsymbol{v}_\kappa + \boldsymbol{u}^i) = \boldsymbol{K}_\kappa(\boldsymbol{\xi})[\![\boldsymbol{v}_\kappa]\!] \quad \text{on} \quad \Gamma_\kappa,$$

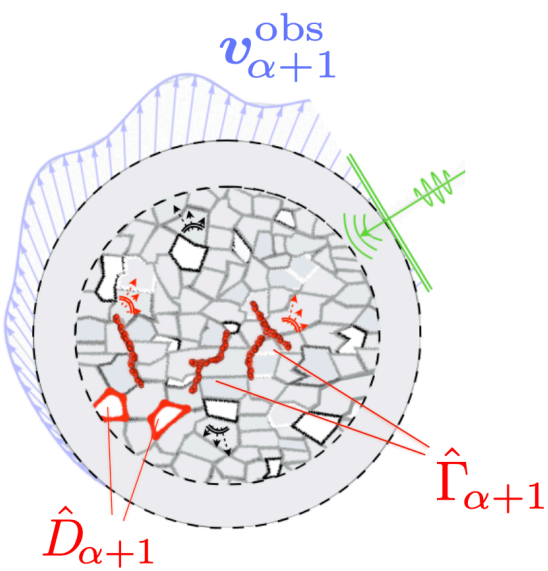
$$\boldsymbol{n}\cdot\boldsymbol{C}:\nabla(\boldsymbol{v}_\kappa + \boldsymbol{u}^i) = \boldsymbol{0} \quad \text{on} \quad \partial\mathscr{B}_t,$$

$$\boldsymbol{v}_\kappa + \boldsymbol{u}^i = \boldsymbol{0} \quad \text{on} \quad \partial\mathscr{B}_u,$$

3-tier differential evolution indicator

sequential sensing ⇨ distilling signature of internal scatterers ⇨ reconstruction of microstructural evolution





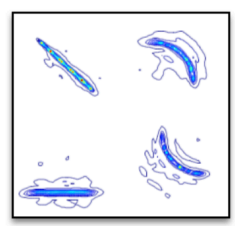
non-iterative minimization of the cost functional

$$v_\alpha^{\mathrm{obs}}\,,\; v_{\alpha+1}^{\mathrm{obs}}$$

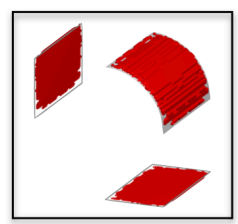
$$\Downarrow \quad \mathrm{argmin}_{g_\alpha} \mathfrak{J}_\alpha^{\delta,\gamma}$$

$$g_\alpha\,,\; g_{\alpha+1}$$

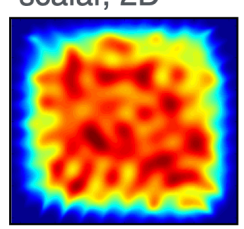
scalar, 2D



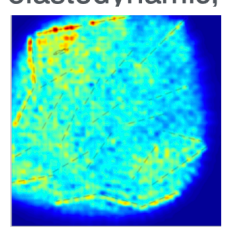
elastodynamic, 3D



scalar, 2D



elastodynamic, 3D



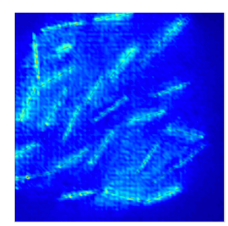
differential indicator

$$g_\alpha\,,\; g_{\alpha+1}$$

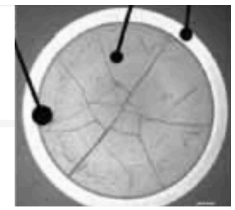
$$\Downarrow \quad I_\alpha^{\mathscr{D}}$$

$$\hat{\Gamma}_{\alpha+1}\,,\; \hat{D}_{\alpha+1}$$

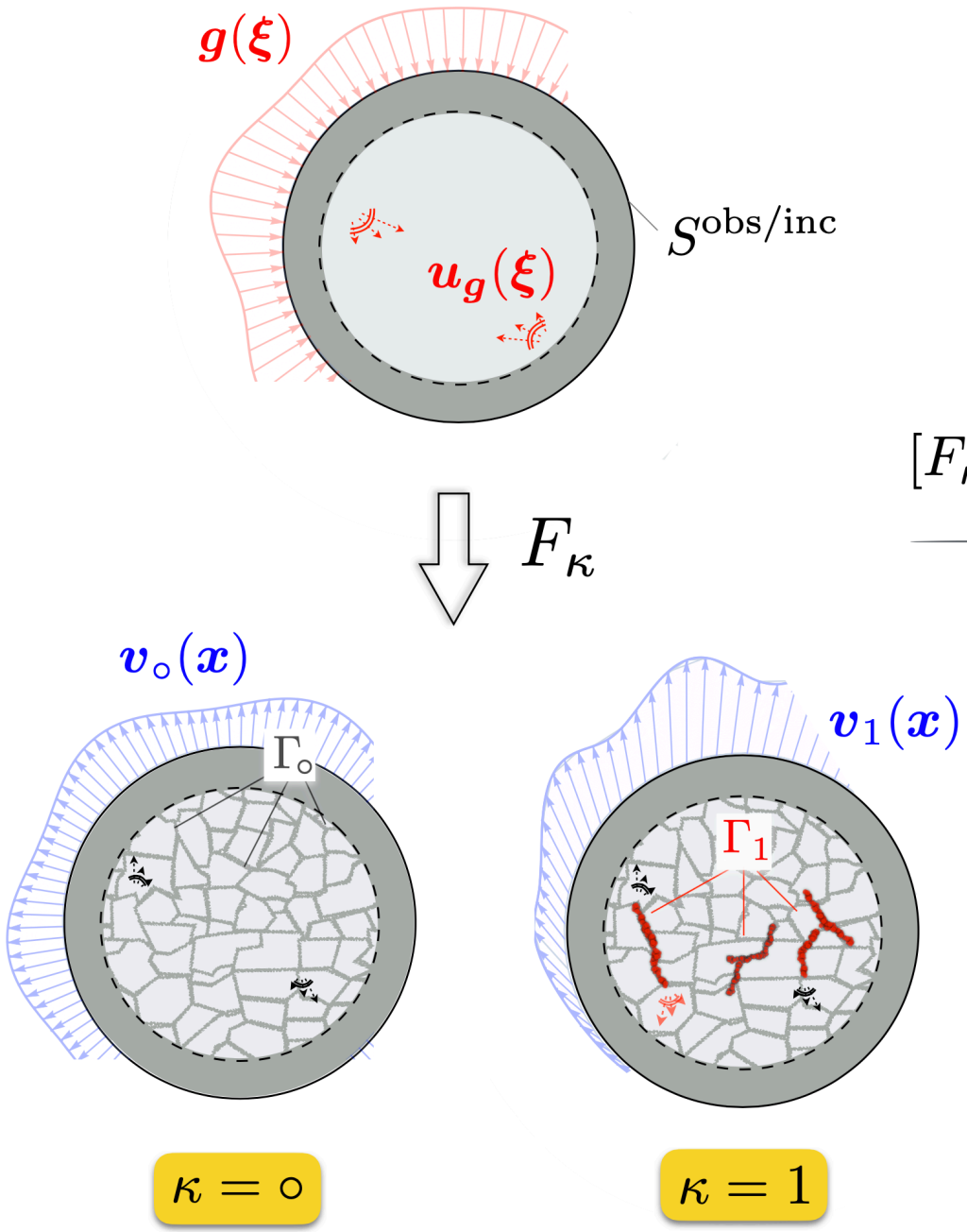
elastodynamic, 3D



Scattering operator



aged nuclear fuel



$$v_\kappa(x) := F_\kappa\, g(\xi)$$

$$[F_\kappa\, g](x) := \int_{S^{\mathrm{inc}}} g(\xi) \cdot W_\kappa^{\mathrm{exp}}(\xi, x)\, \mathrm{d}S_\xi$$

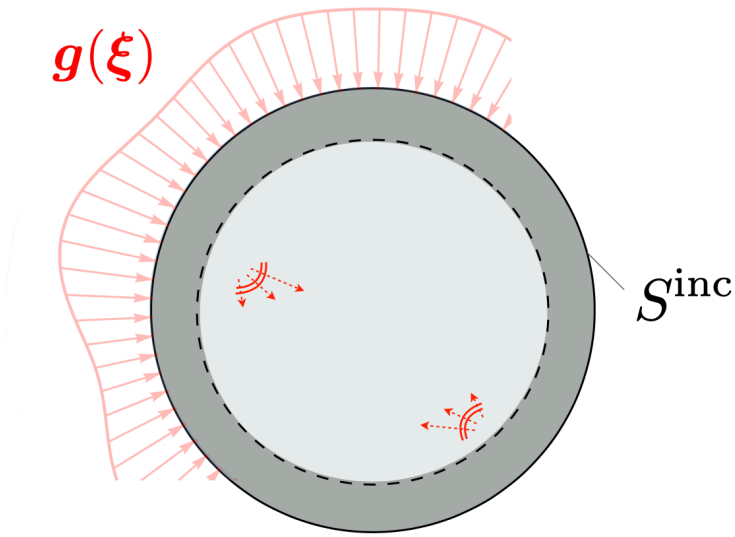
Physical experiment

$$W_\kappa^{\mathrm{exp}}(\xi, x)$$

Scattered field measured at x

due to excitation at ξ

Distilled signatures



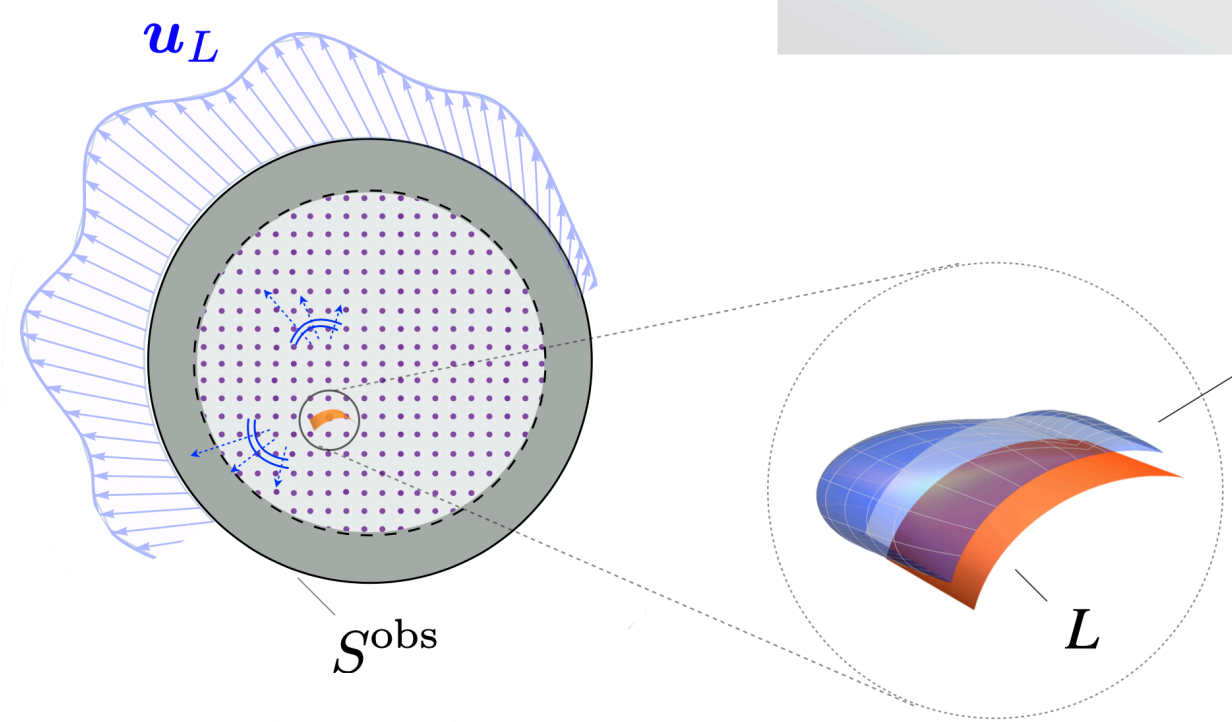
$$\boldsymbol{u}_L = F_\kappa \, \boldsymbol{g}_\kappa$$

IDEA

$$\boldsymbol{u}_L(\boldsymbol{x}) := \int_{S^{\text{inc}}} \boldsymbol{g}_\kappa(\boldsymbol{\xi}) \cdot \boldsymbol{W}_\kappa^{\text{exp}}(\boldsymbol{\xi}, \boldsymbol{x}) \, \mathrm{d}S_{\boldsymbol{\xi}}$$

experiment

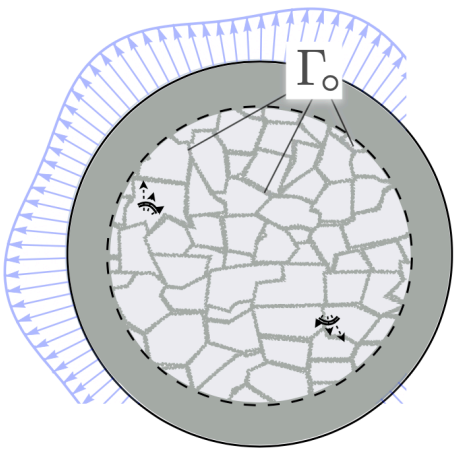
synthetic wavefront adjustment



$$[\![\boldsymbol{u}_L]\!] \in \tilde{H}^{1/2}(L)^3$$

fracture opening displacement (FOD)

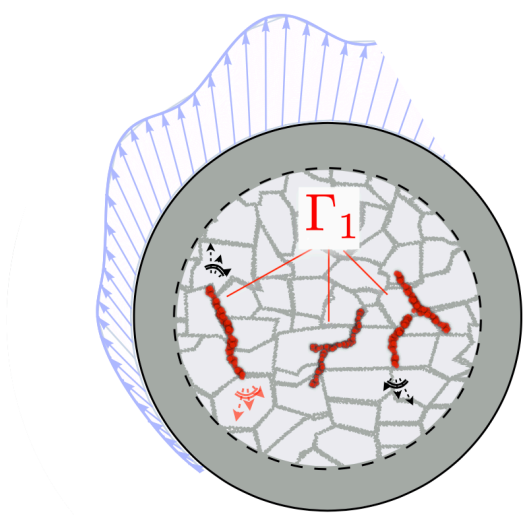
Differential Evolution Indicator



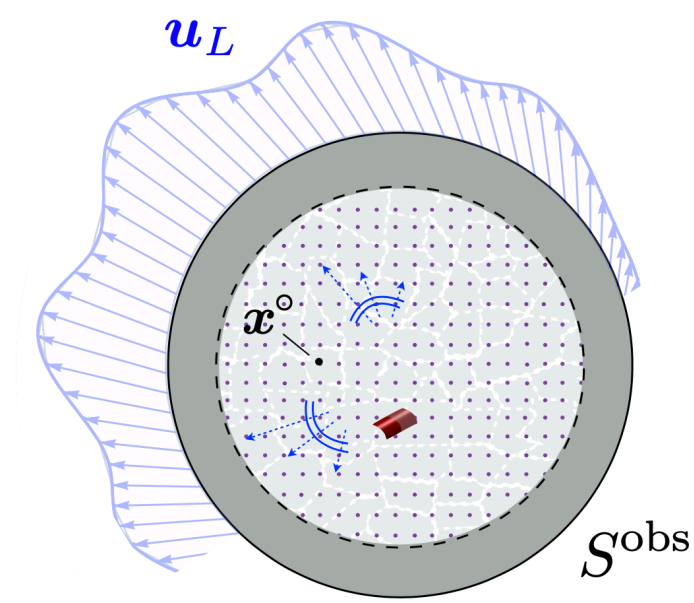
$$J_{\alpha,\delta}^{\mathrm{GLSM}} = \|F_\kappa \boldsymbol{g} - \boldsymbol{u}_L\|^2 + \alpha\left(\delta\|\boldsymbol{g}\|^2 + (F_\sharp \boldsymbol{g}, \boldsymbol{g})\right)$$

$$\mathscr{I}_{\mathrm{DLSM}} = \frac{1}{\sqrt{(F_\sharp^1 \boldsymbol{g}_1, \boldsymbol{g}_1)(1 + (F_\sharp^1 \boldsymbol{g}_1, \boldsymbol{g}_1)(F_\sharp^\circ (\boldsymbol{g}_1 - \boldsymbol{g}_\circ), (\boldsymbol{g}_1 - \boldsymbol{g}_\circ)))}}$$

$\kappa = \circ$



$$\boldsymbol{u}_L = F_\kappa \boldsymbol{g}_\kappa$$

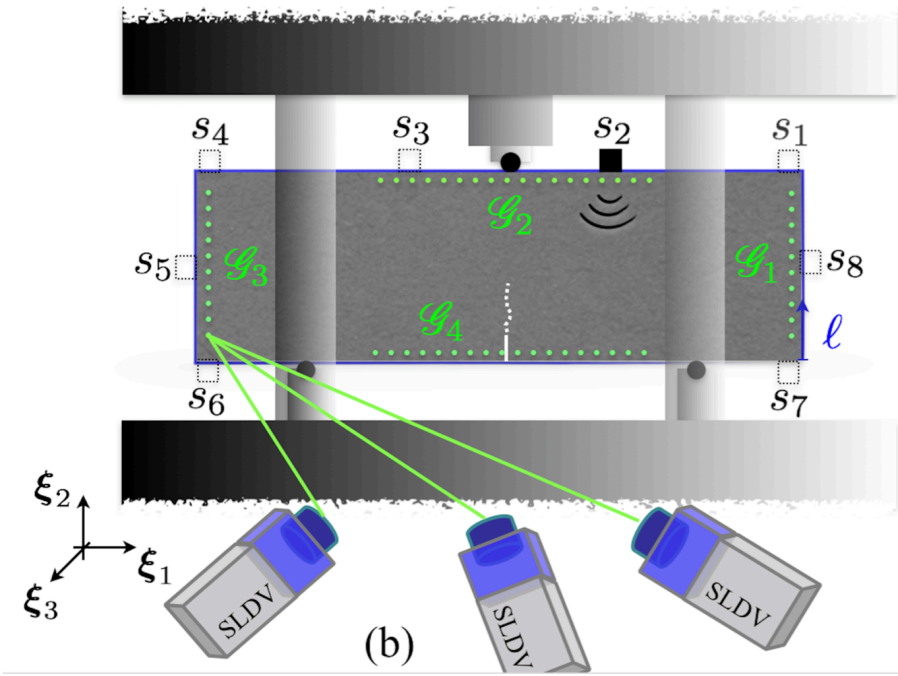


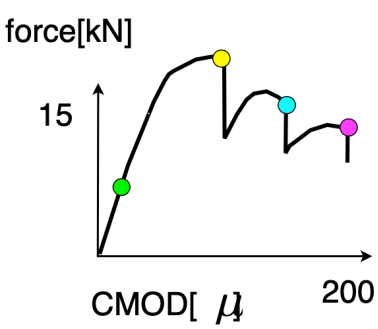
$\kappa = 1$

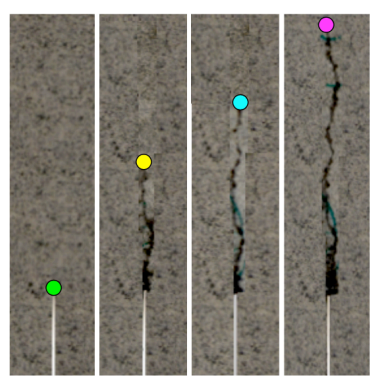
F. Pourahmadian, H. Haddar, Differential tomography of micromechanical evolution in elastic materials of unknown micro/macrostructure, SIAM Journal on Imaging Sciences 13 (3) (2020) 1302–1330.

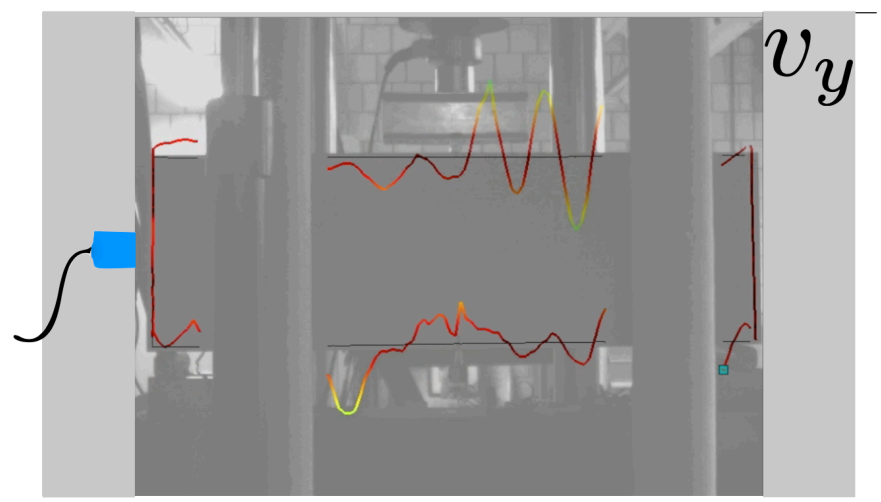
Measurements

Boundary Data



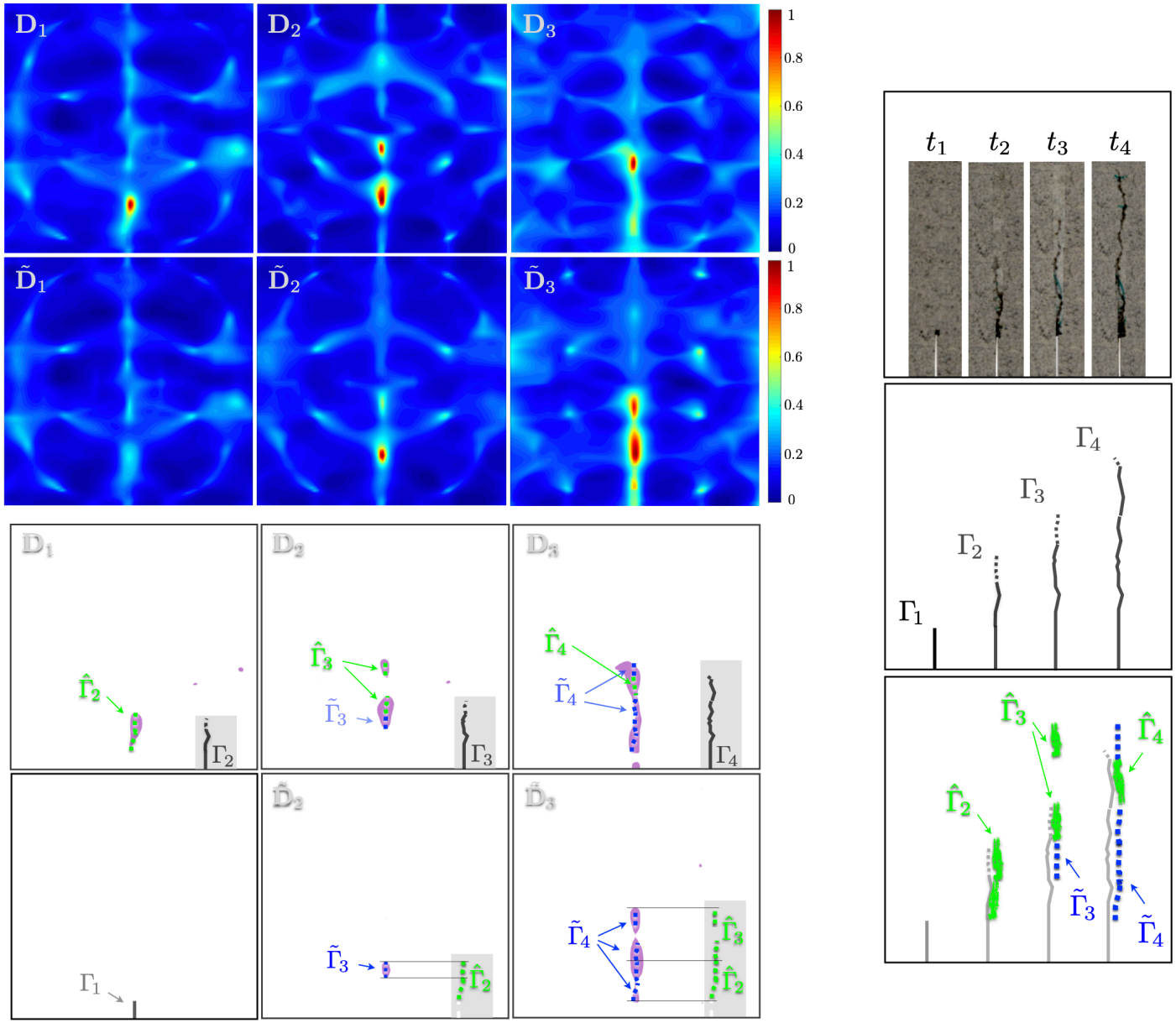






S-wave transducer
excitation frequency: 30 KHz

Differential Evolution Indicator



F. Pourahmadian, Experimental validation of differential evolution indicators for ultrasonic waveform tomography, arxiv preprint arXiv:2010.01813 (2020).