Hybrid approach to elastic-wave imaging of heterogeneous fractures

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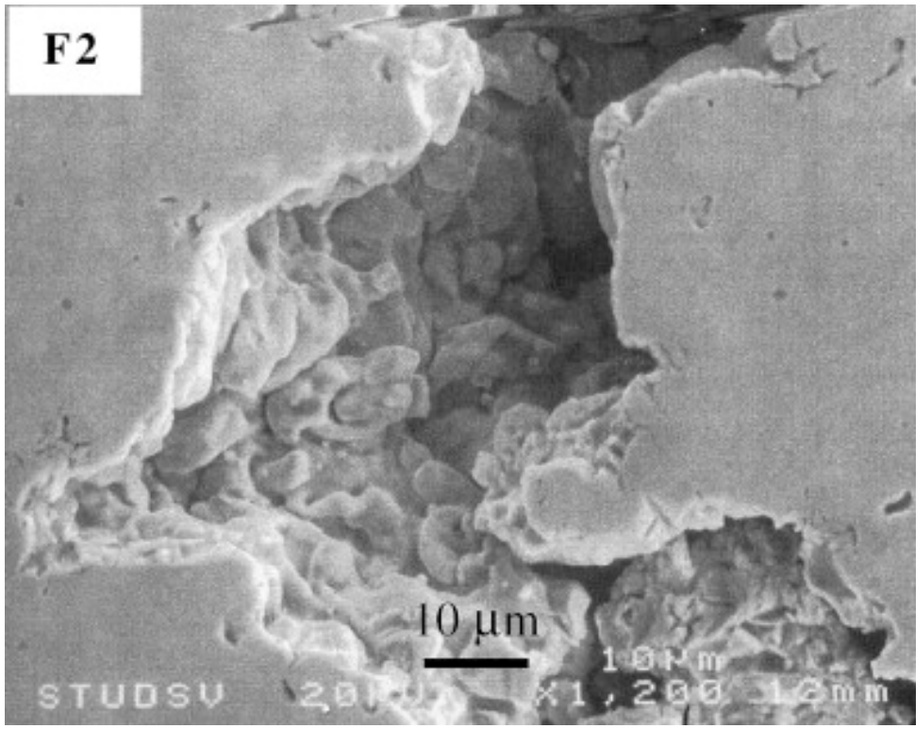
***Ecole Polytechnique (CMAP)**

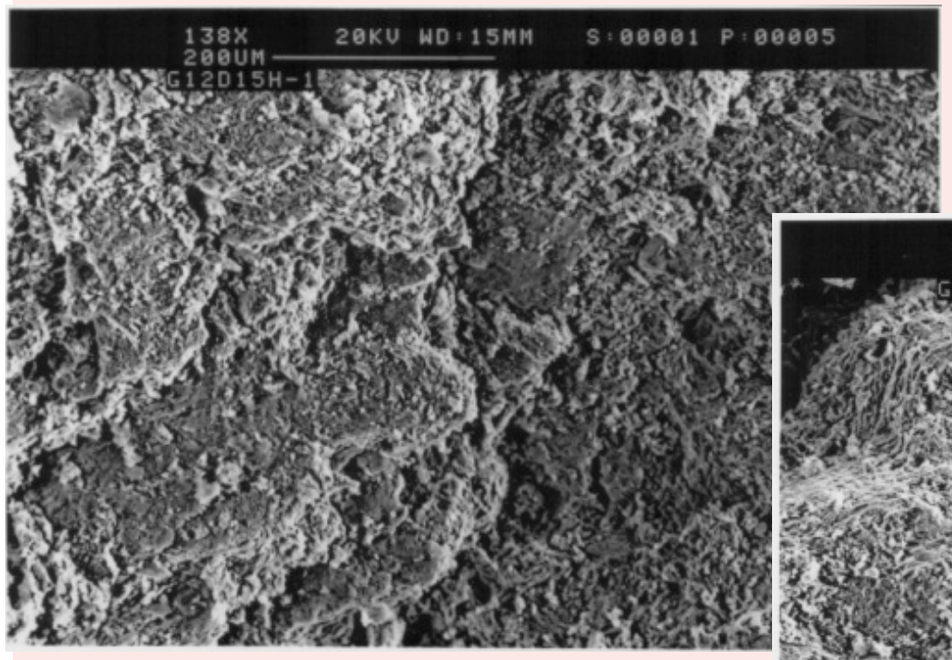
Heterogeneous fractures

Fracture surface topography

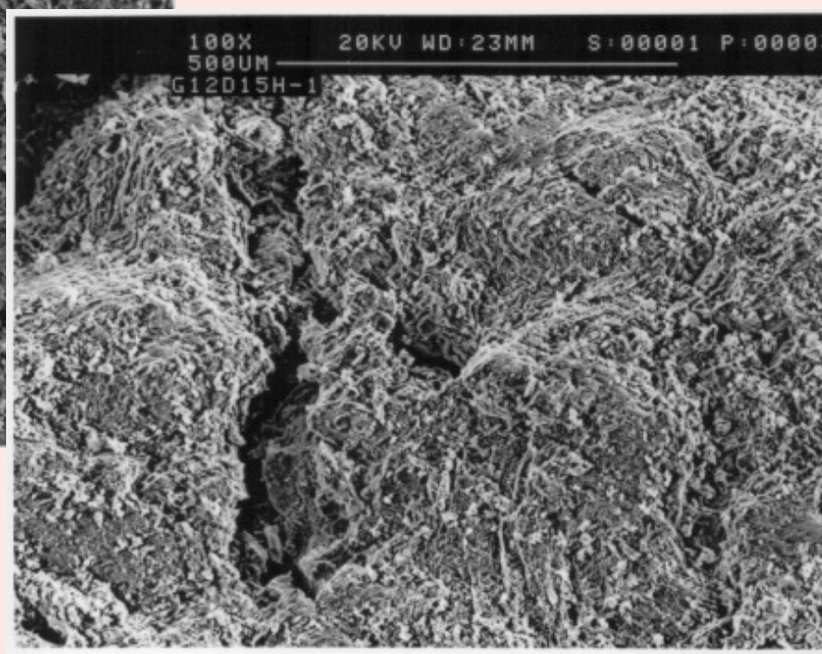


C. Pecorari, Int. J. Eng. Sc.(2008)

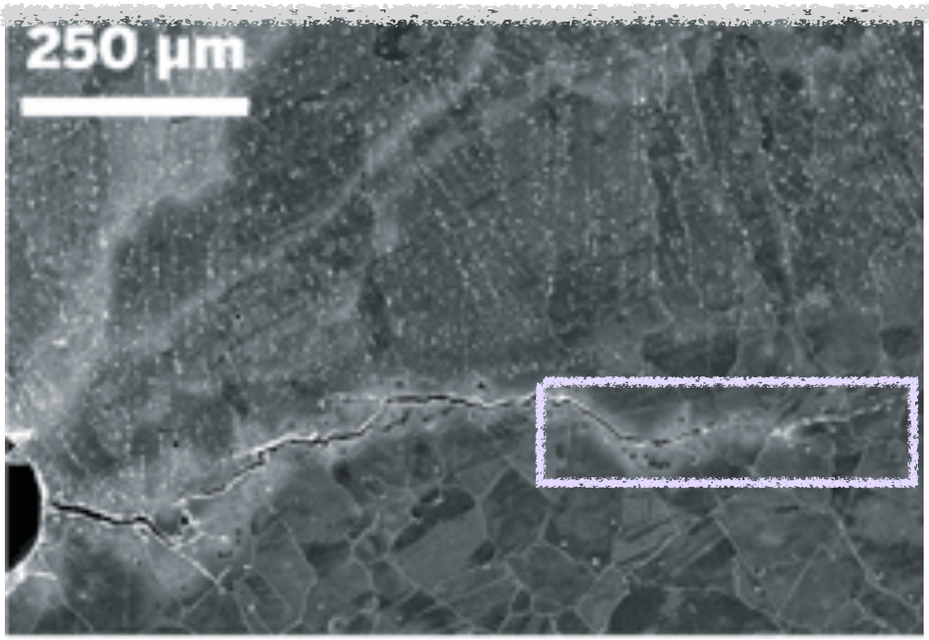
Micrographs of the fracture surface of IM1-24 graphite



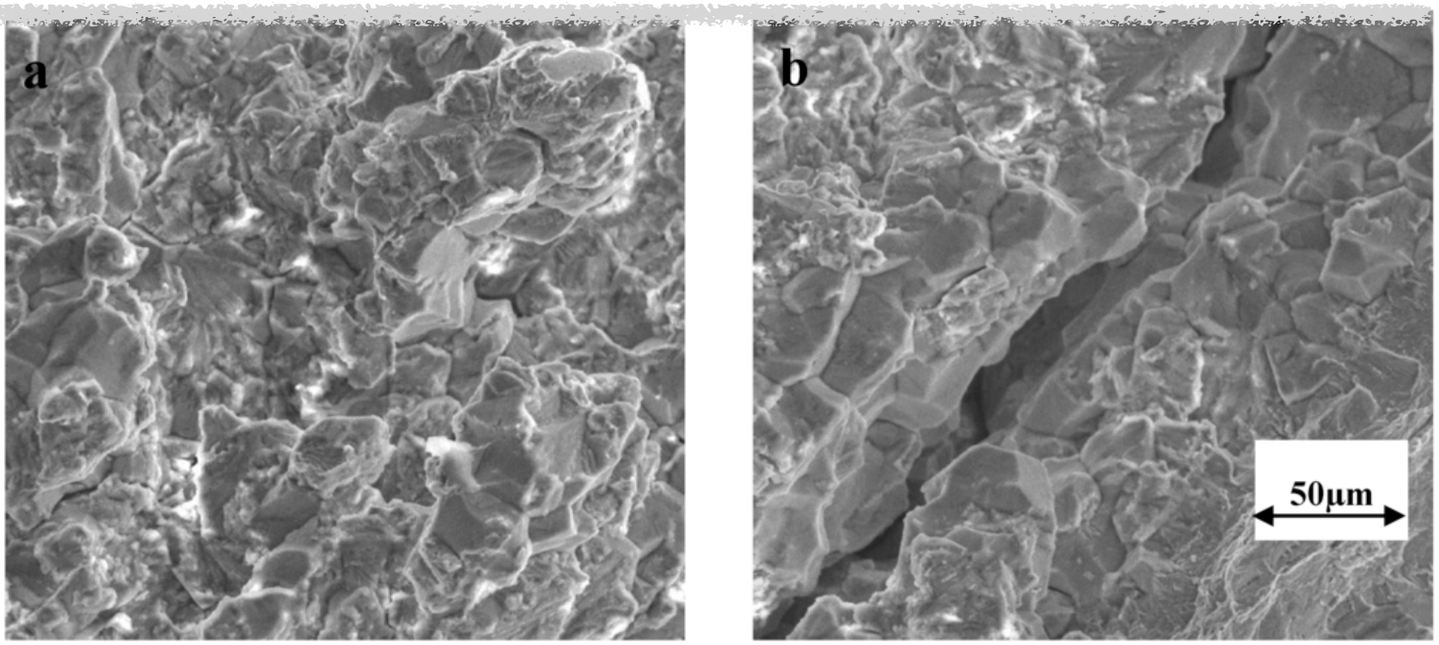
M. J. Holt



Stress corrosion cracking in nuclear reactors



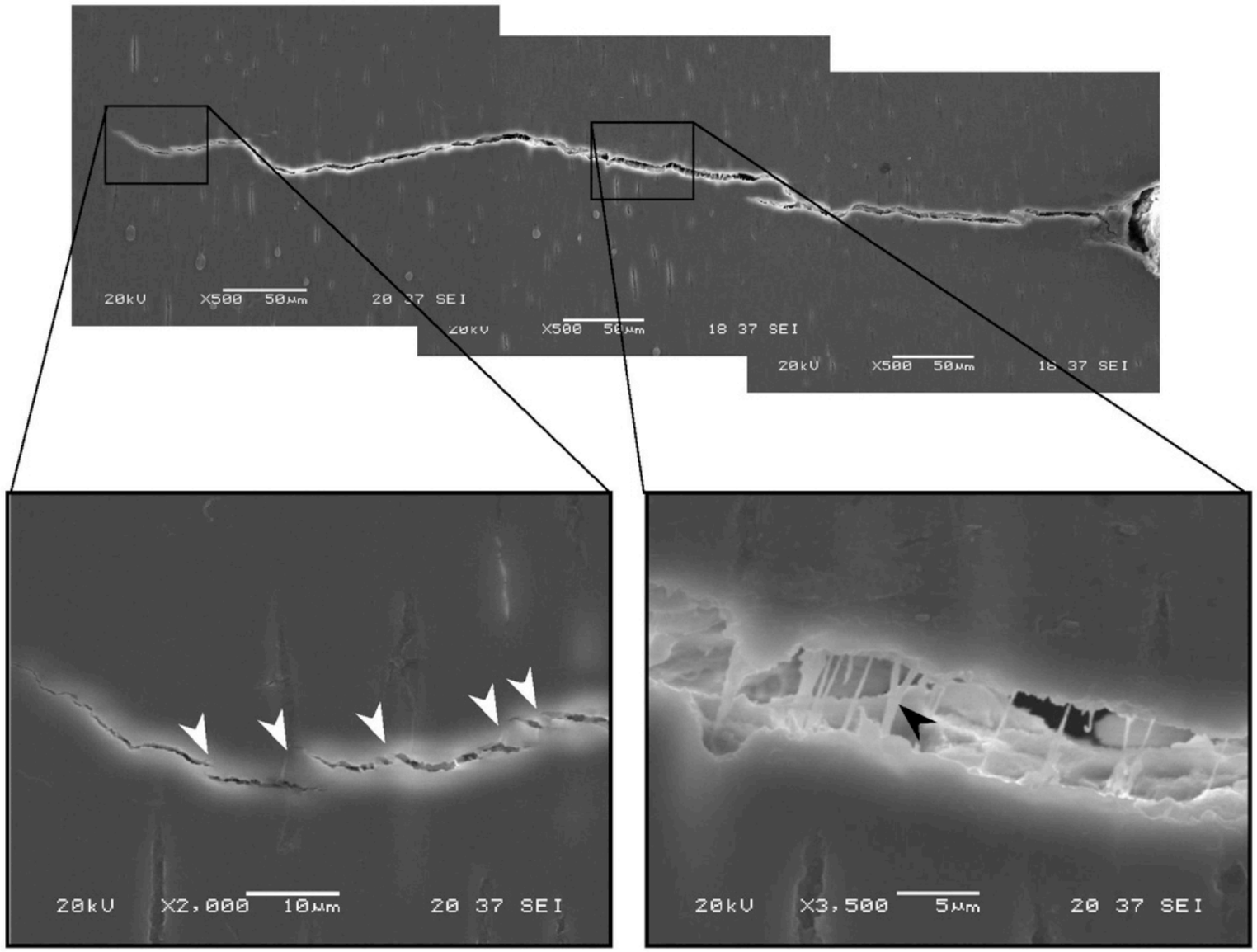
B. Alexandreanu, Argonne National Lab



R. K. Singh Raman, Materials (2014)

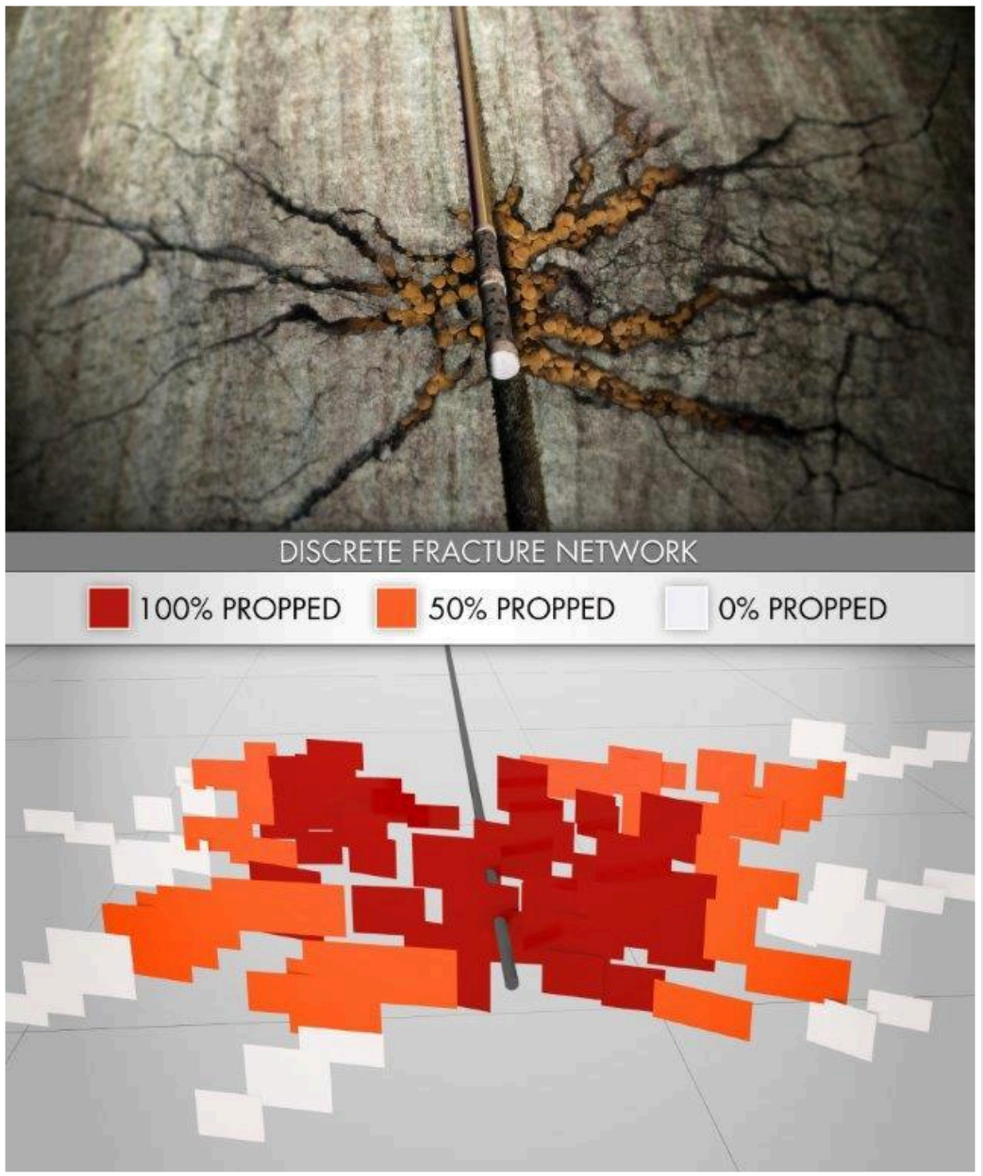
Heterogeneous fractures

Fracture propagation mechanisms

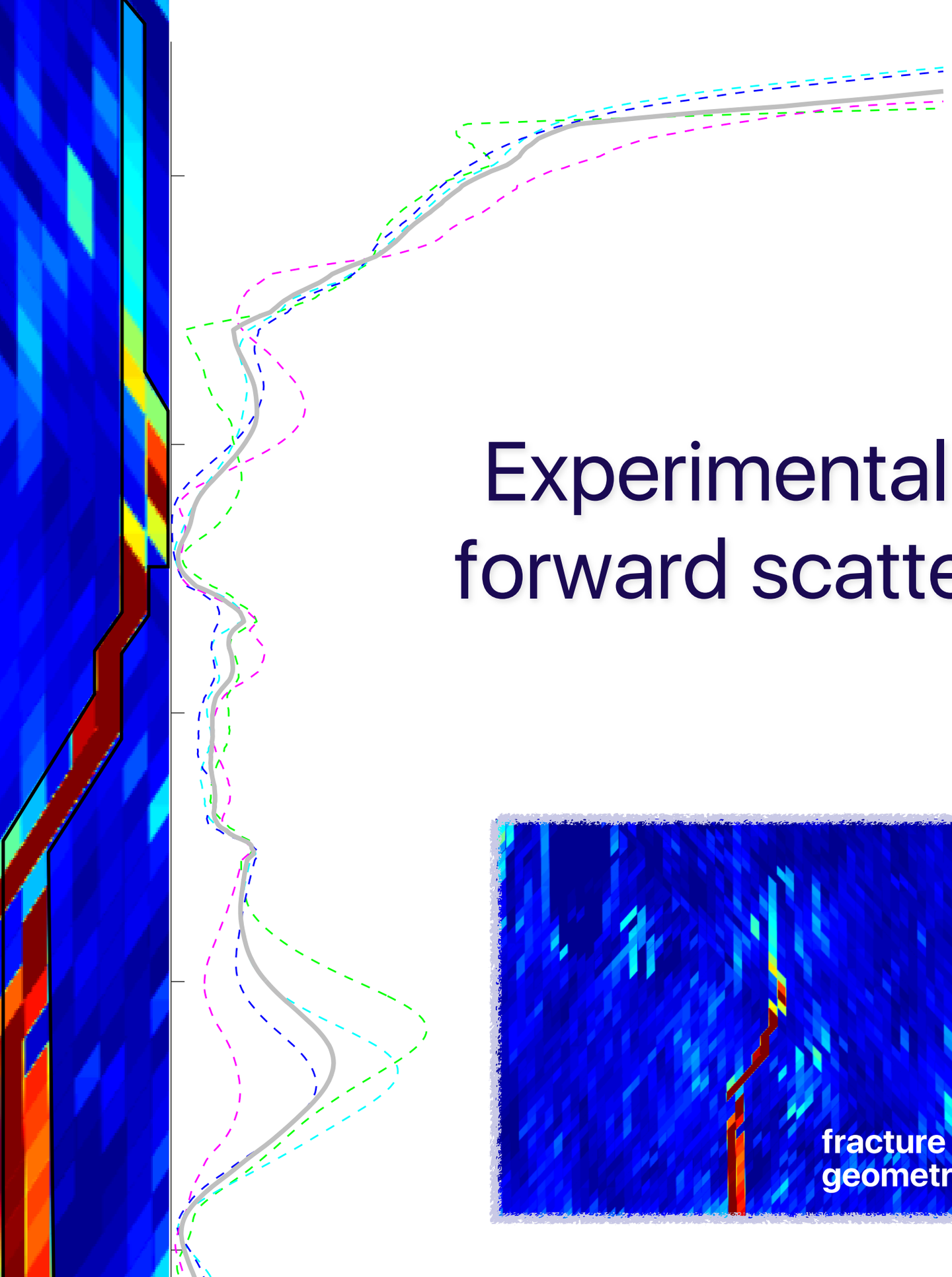


D. Bajaj, J. Biomed. Mater. Res. B (2008)

Proppant injection process



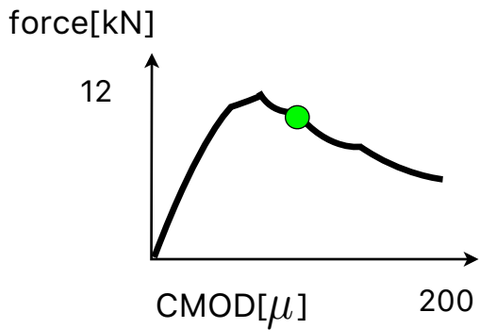
Microseismic Inc. (2013)



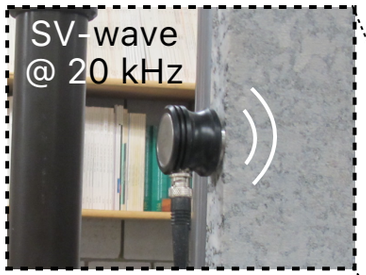
Experimental study of the forward scattering problem

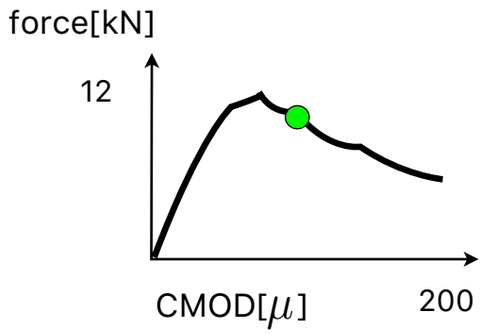
3-point bending configuration

SLDV measurements

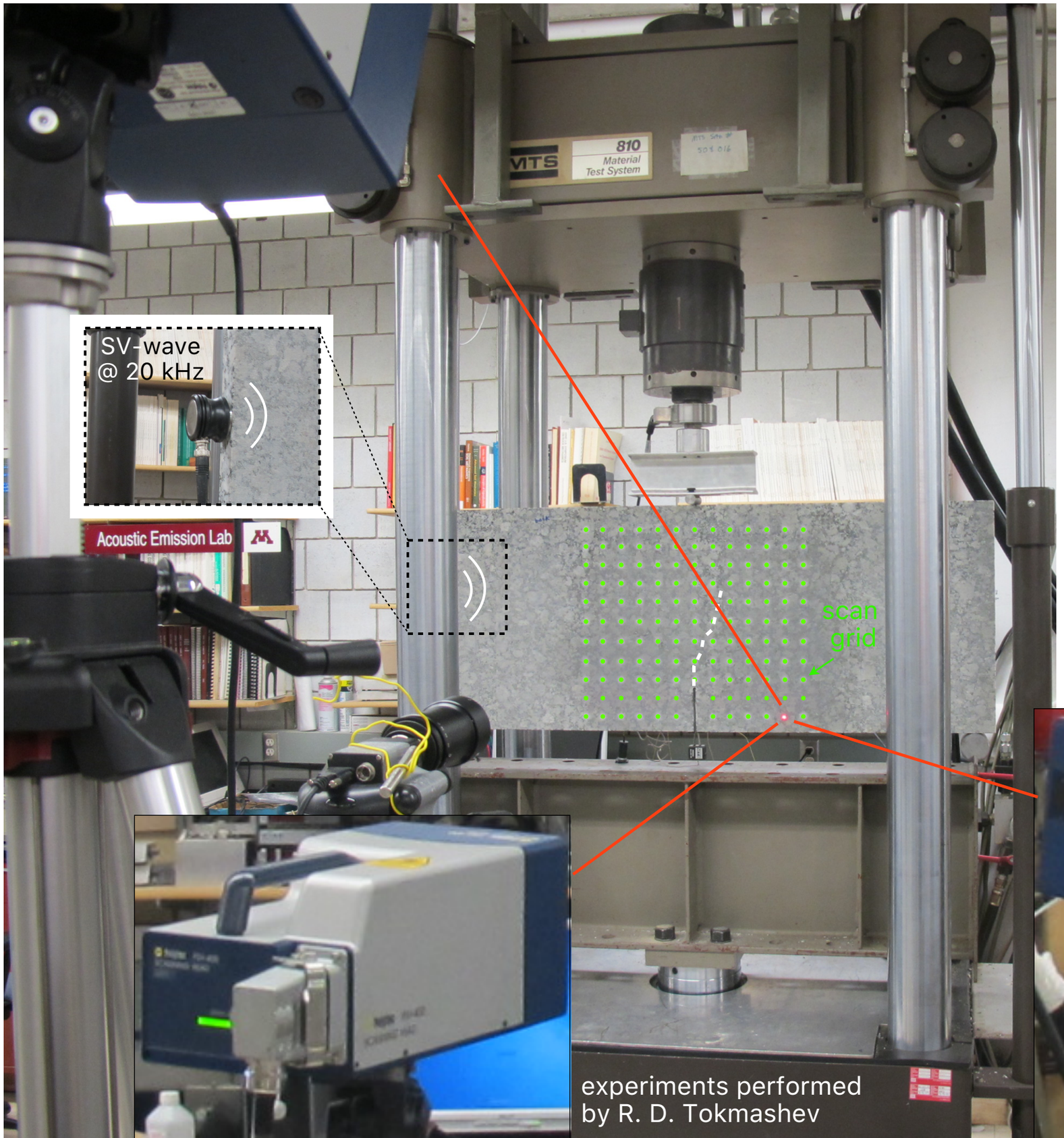


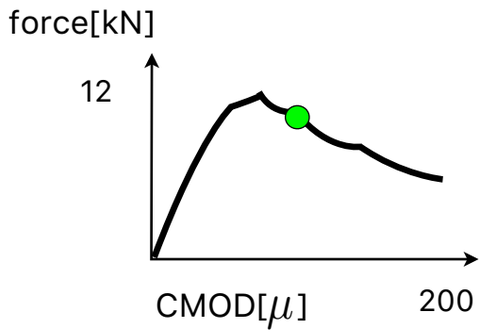
SLDV measurements



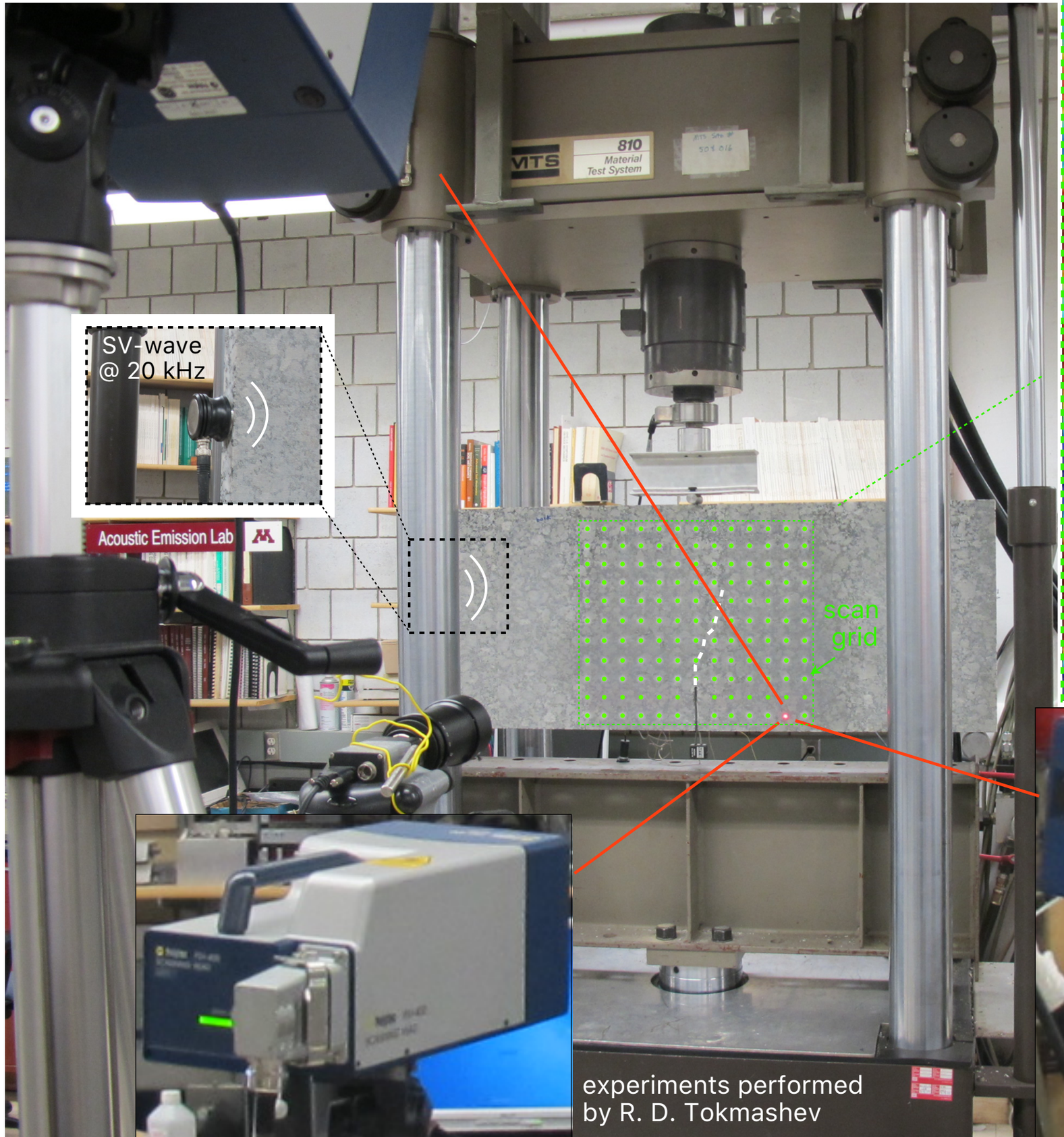


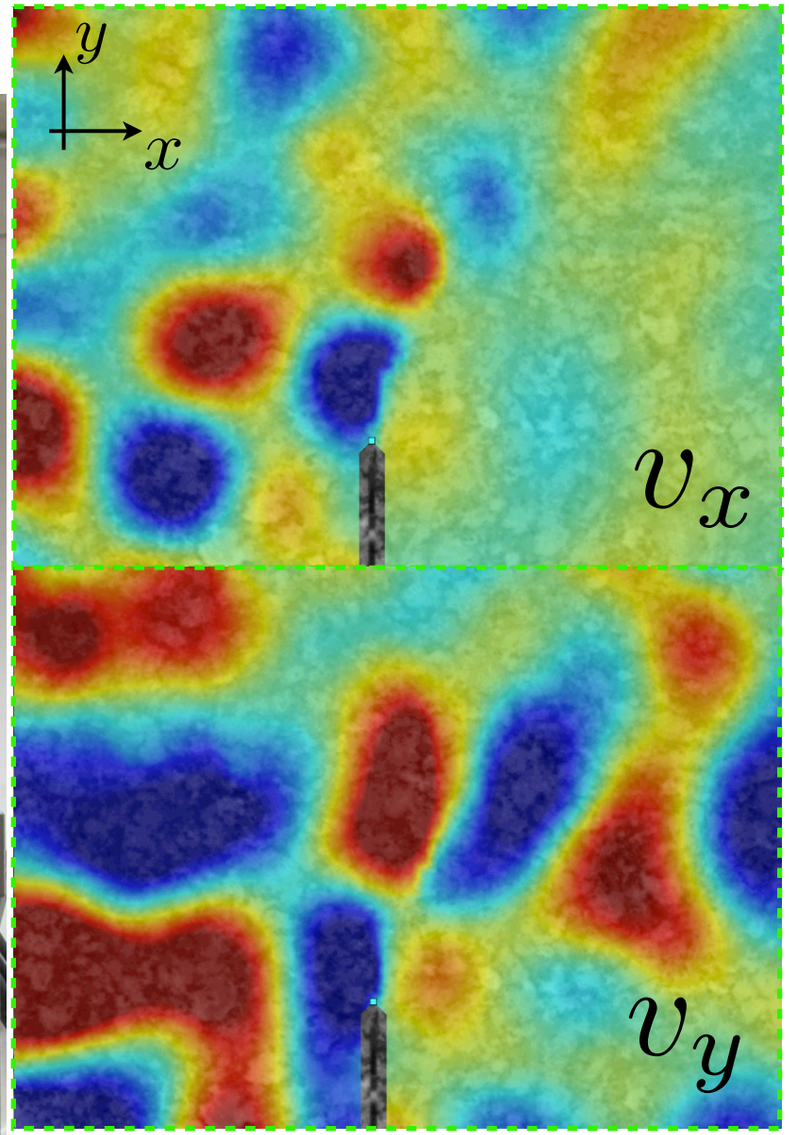
SLDV measurements





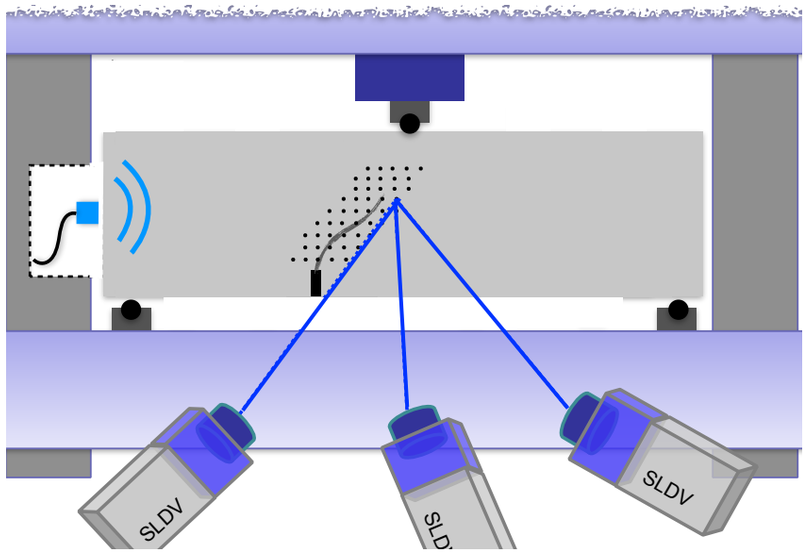
SLDV measurements



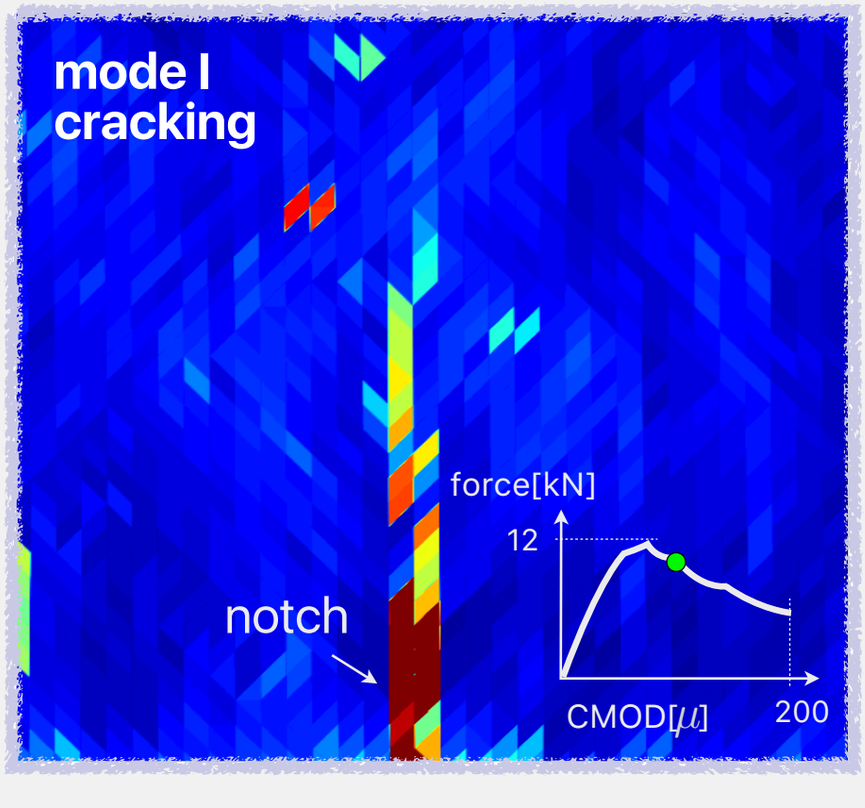


Geometric reconstruction

absolute value of velocity jump along x and y
summed and averaged over time



65% post peak



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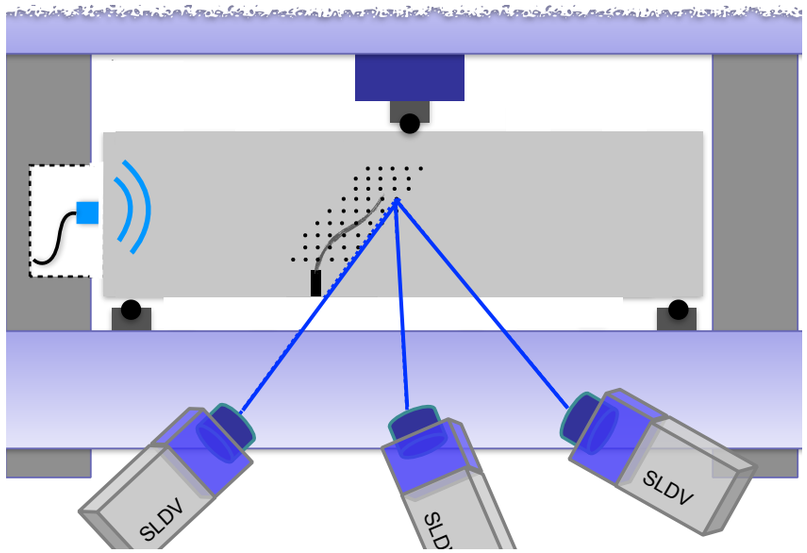
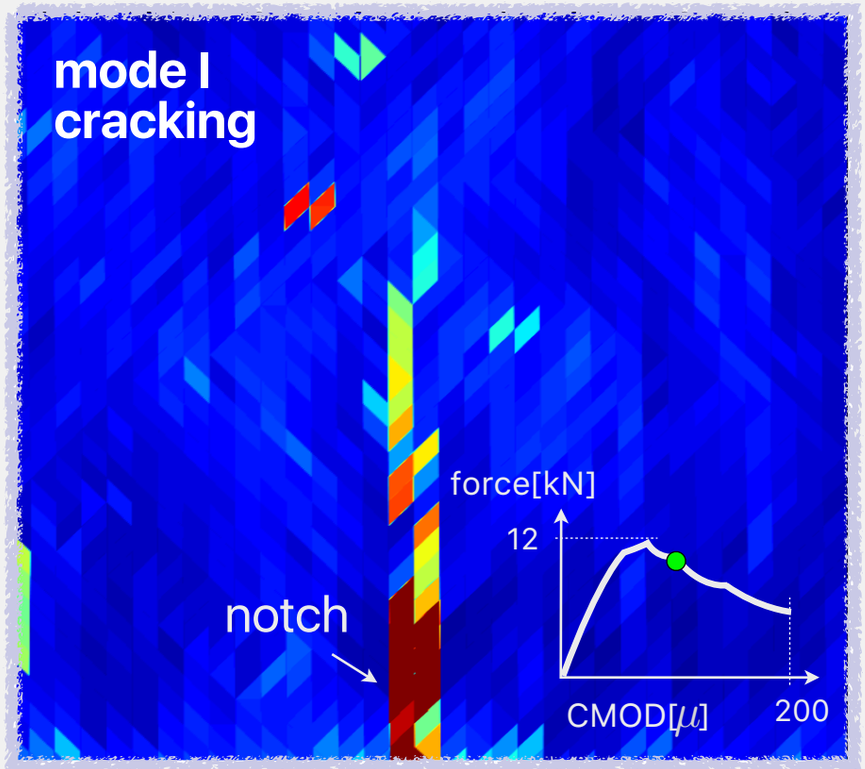
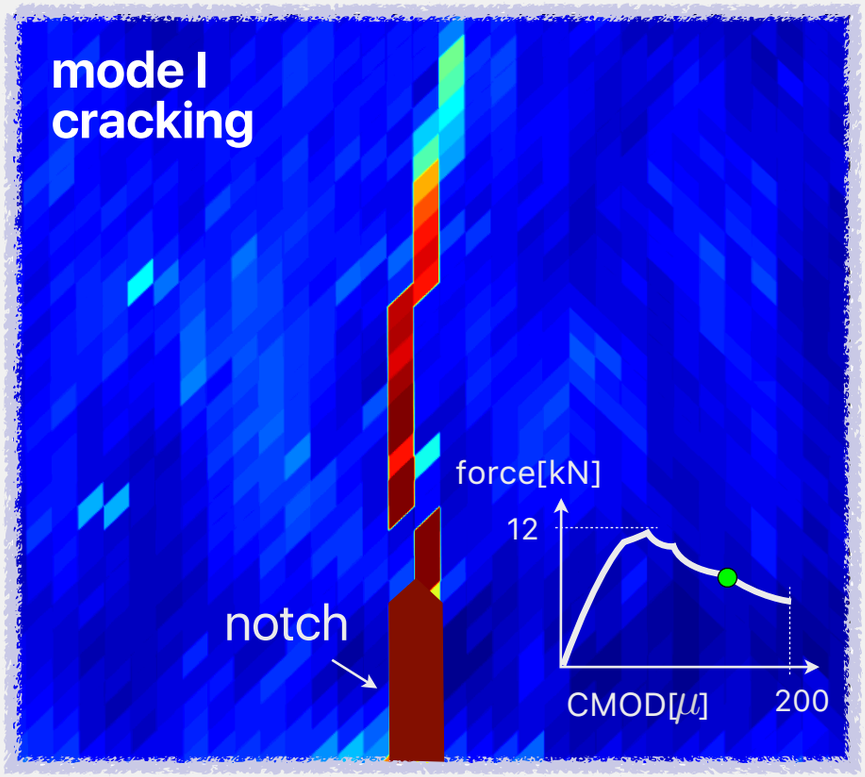


65% post peak



50% post peak



Geometric reconstruction

absolute value of velocity jump along x and y
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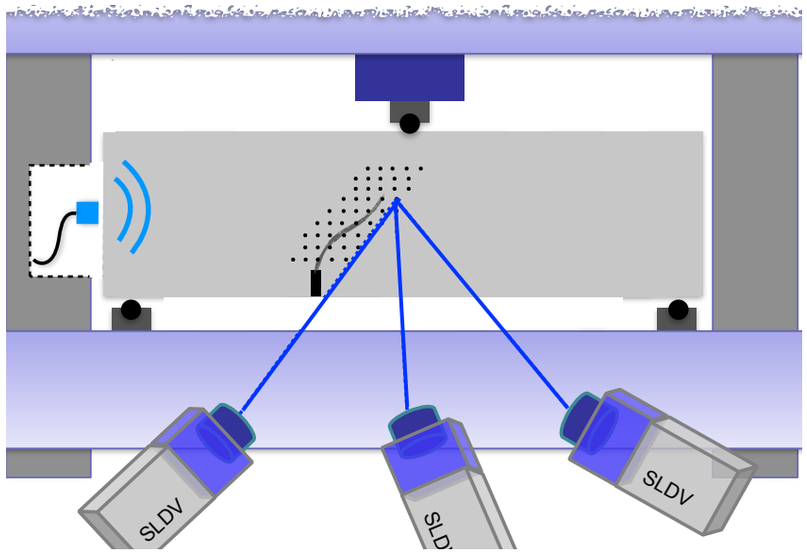
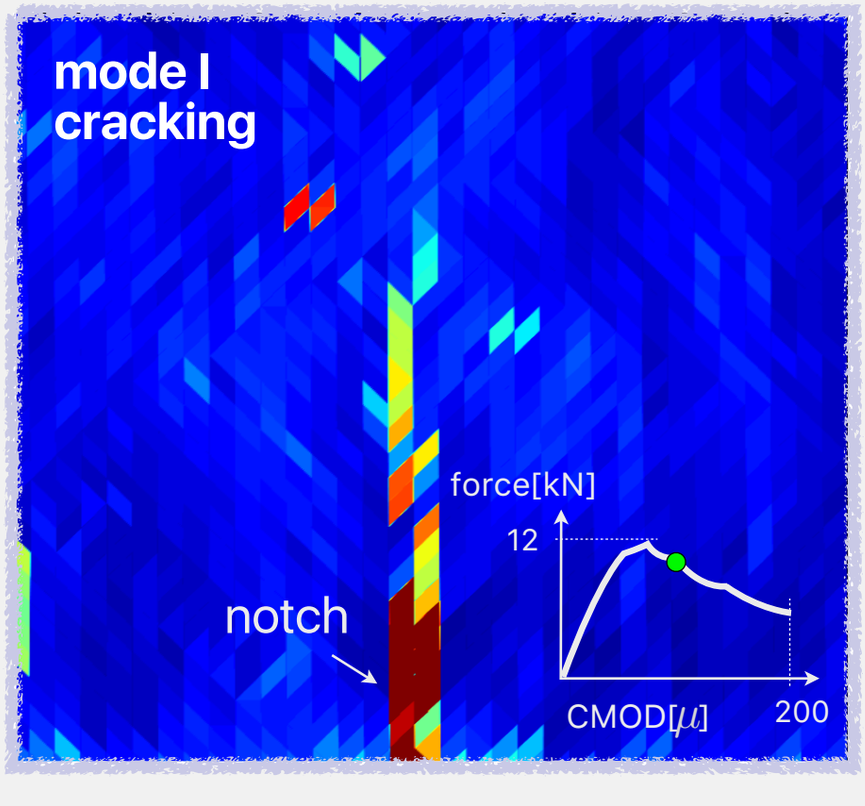
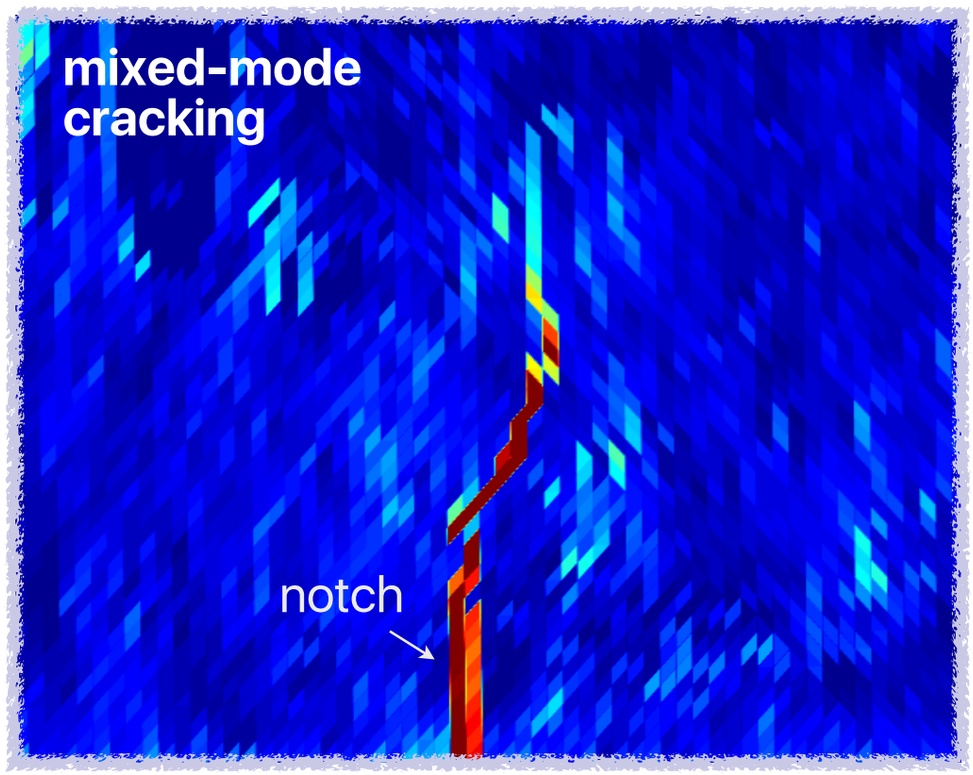


65% post peak



70% post peak



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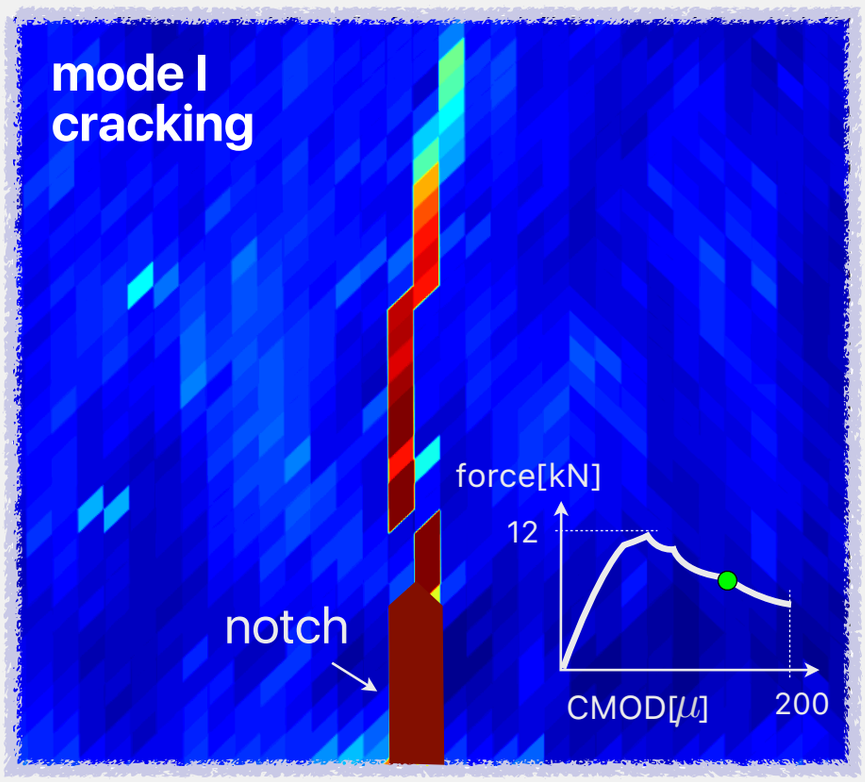


Signal processing displacement

SLDV velocity field

↓

denoising in time

band-pass filtering
@ **every** scan point

↓

numerical integration

displacement field

↓

fixing integration error

high-pass filtering
@ **every** scan point

↓

displacement field

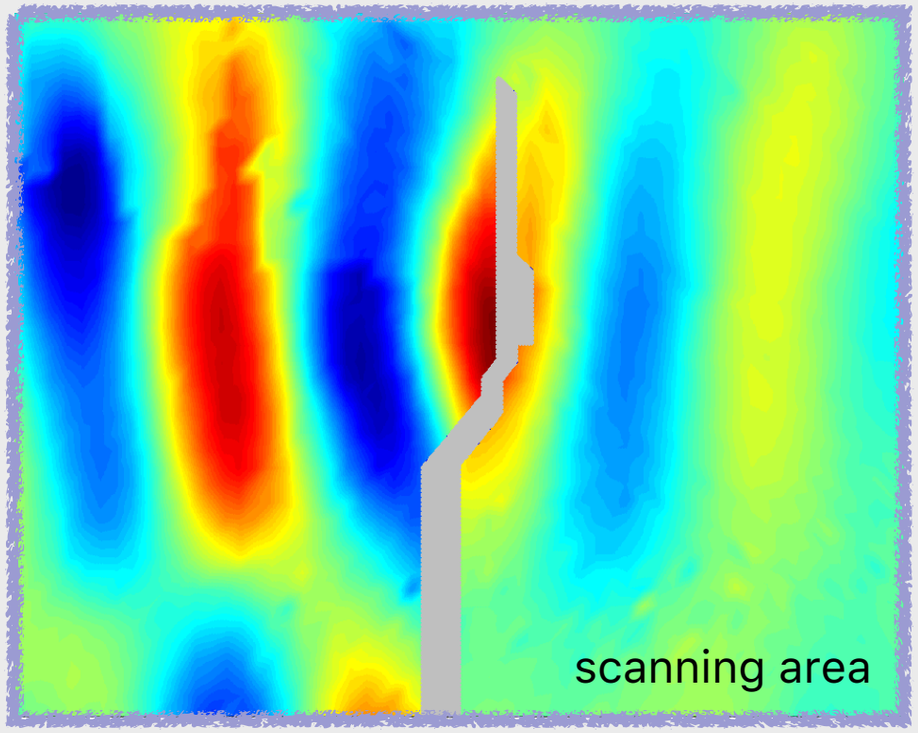
↓

denoising in space

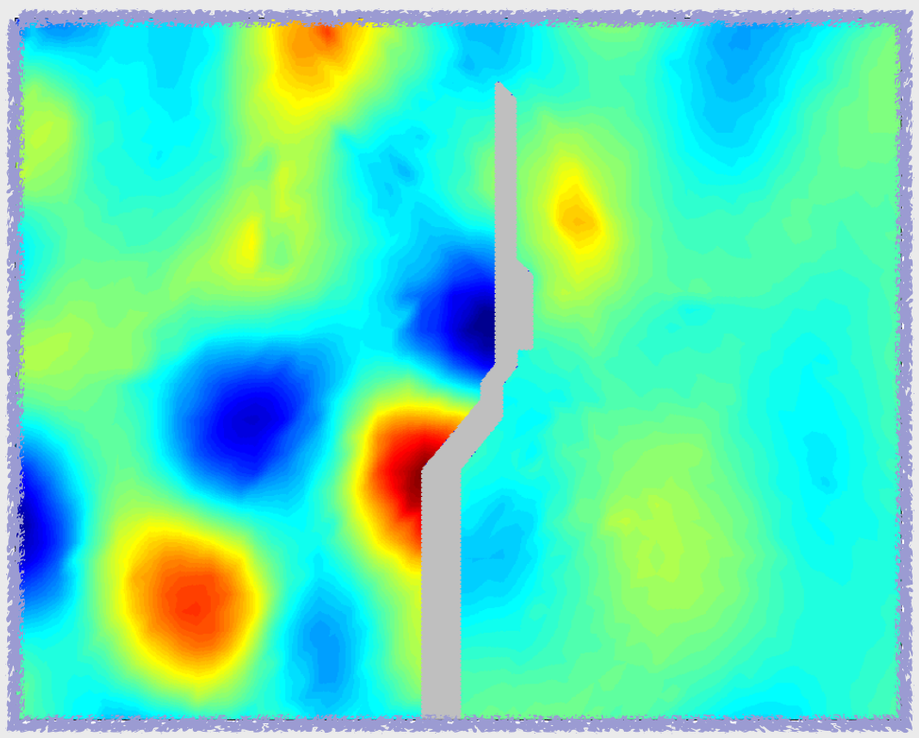
Fourier series fitting in both x and y directions (up to 7 harmonics)

before

displacement field in y direction

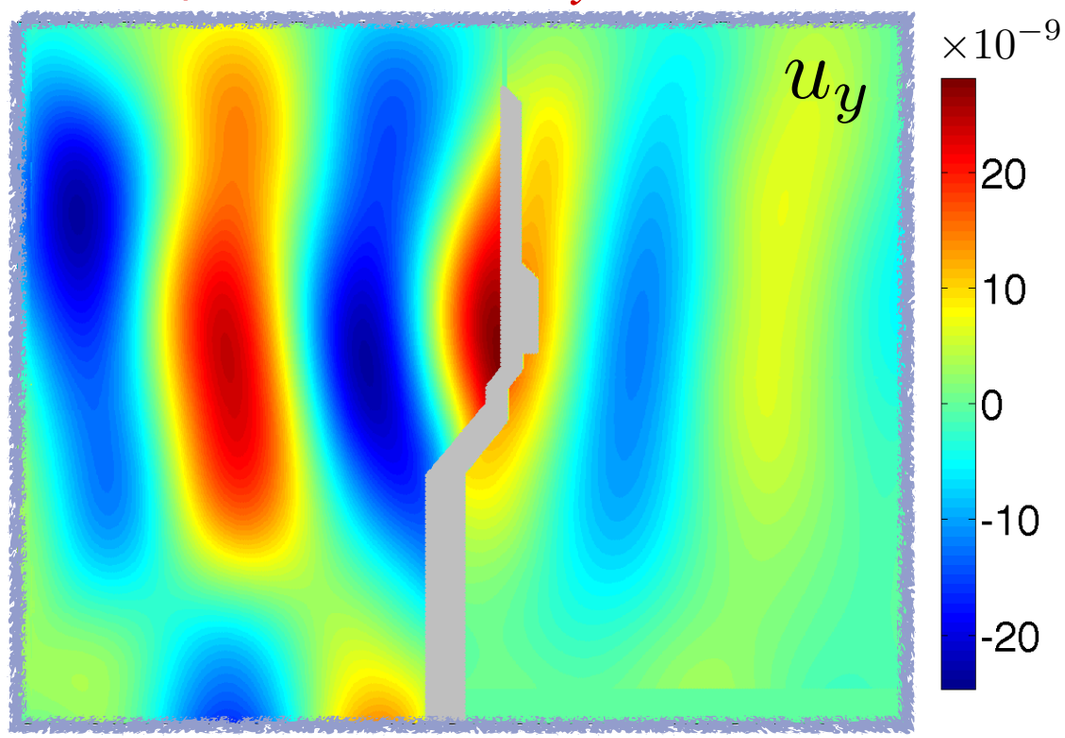


displacement field in x direction

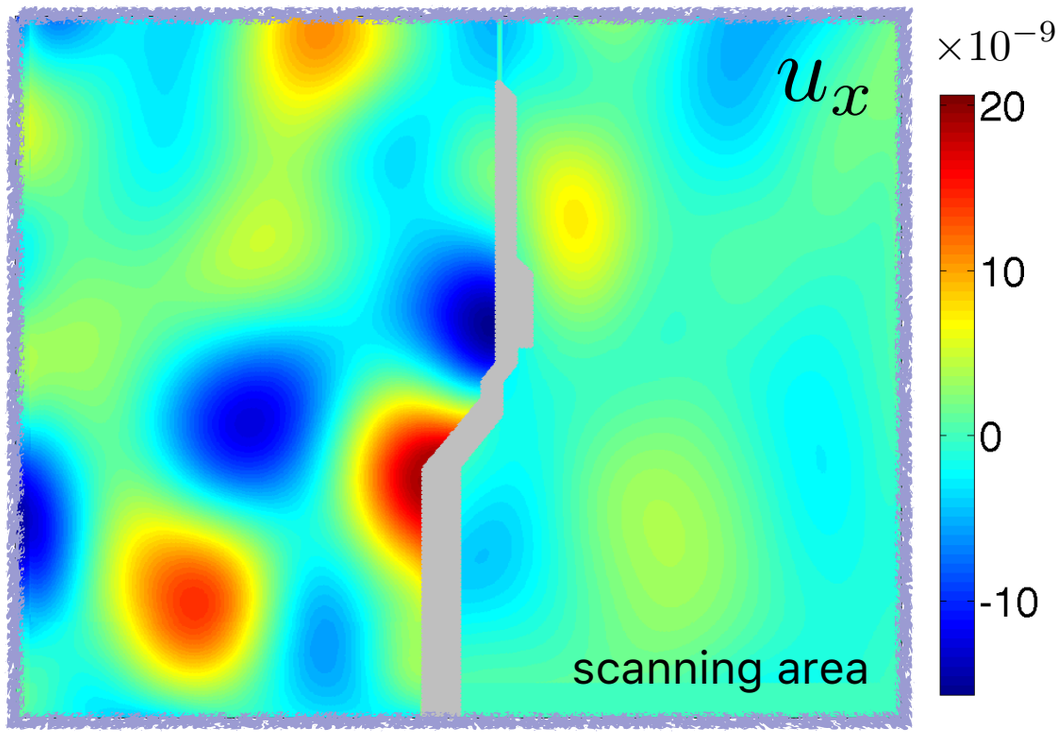


after

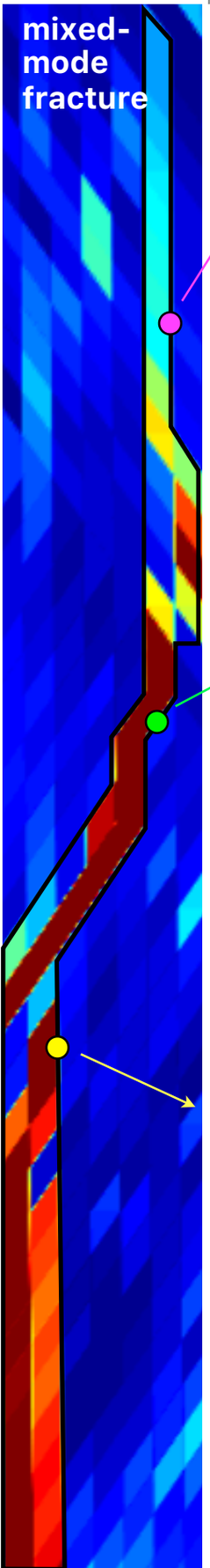
displacement field in y direction

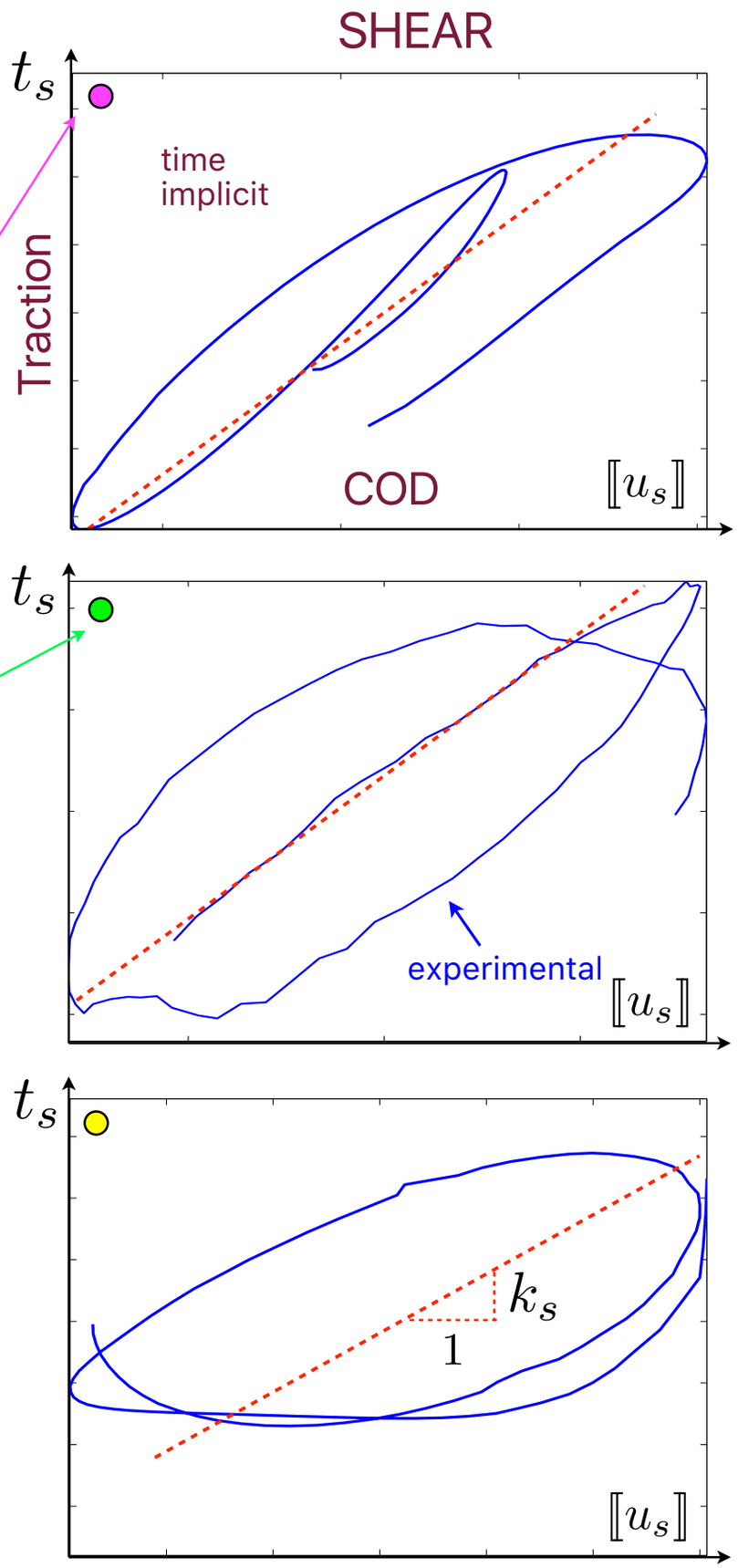


displacement field in x direction

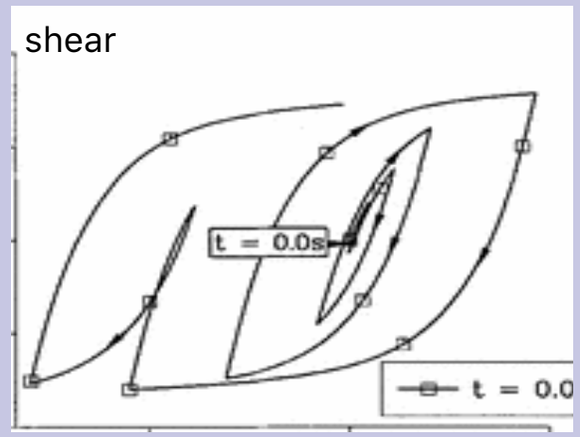


Contact law@ fracture interface

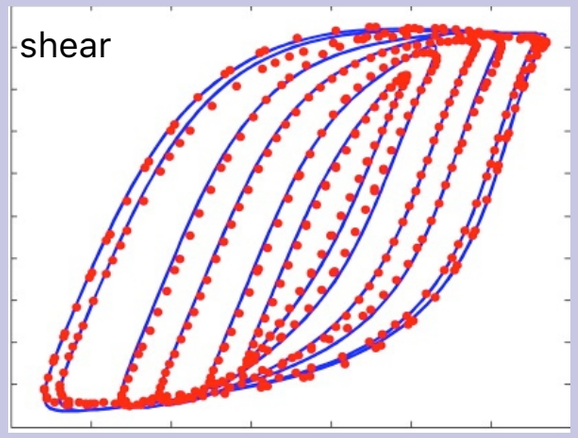




reported hystereses behavior at a frictional contact

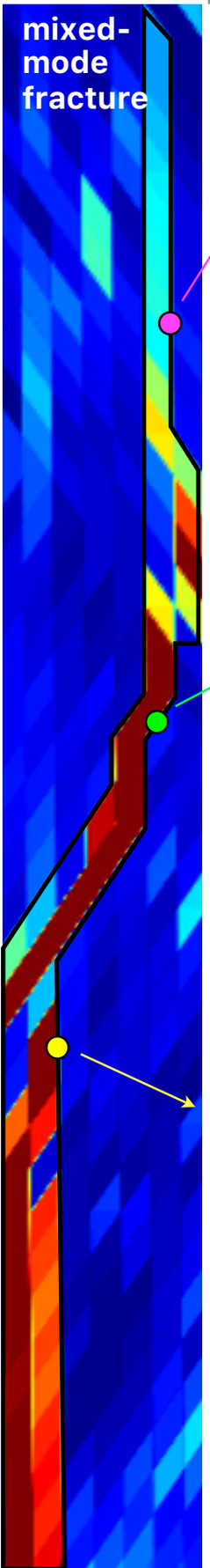


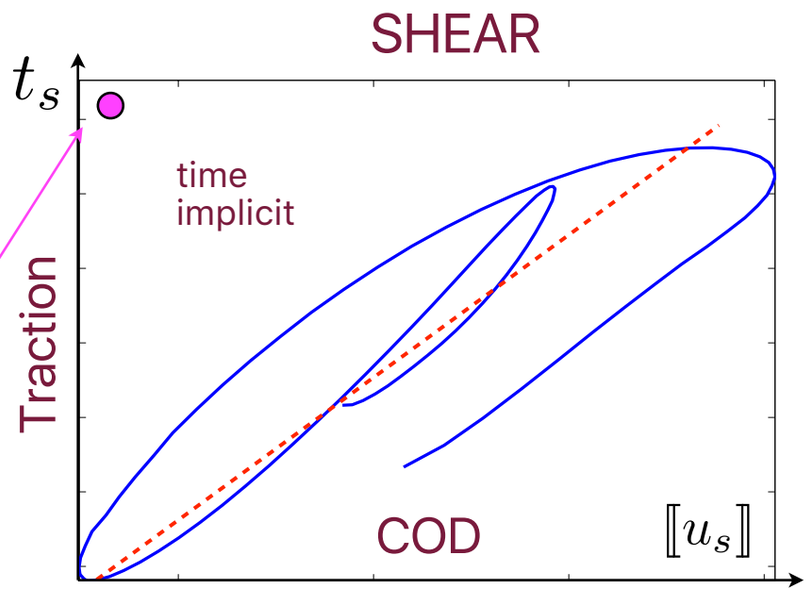
Gaul & Lenz, Acta Mechanica (1997)

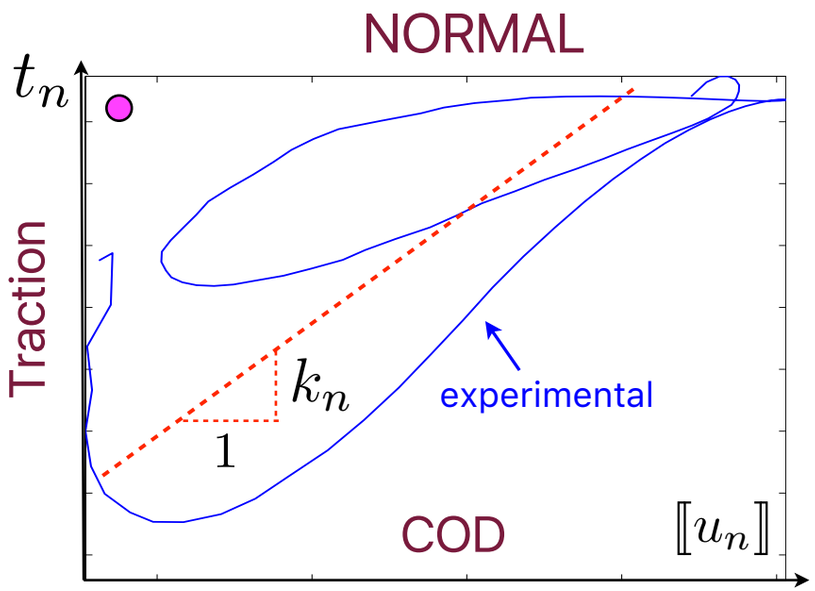


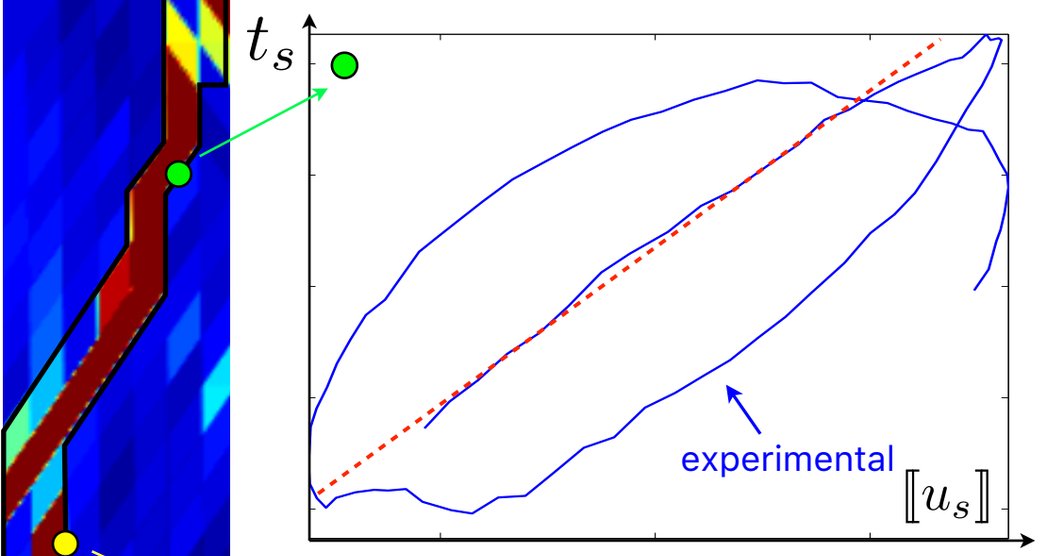
Ahmadian et. al, MSSP (2010)

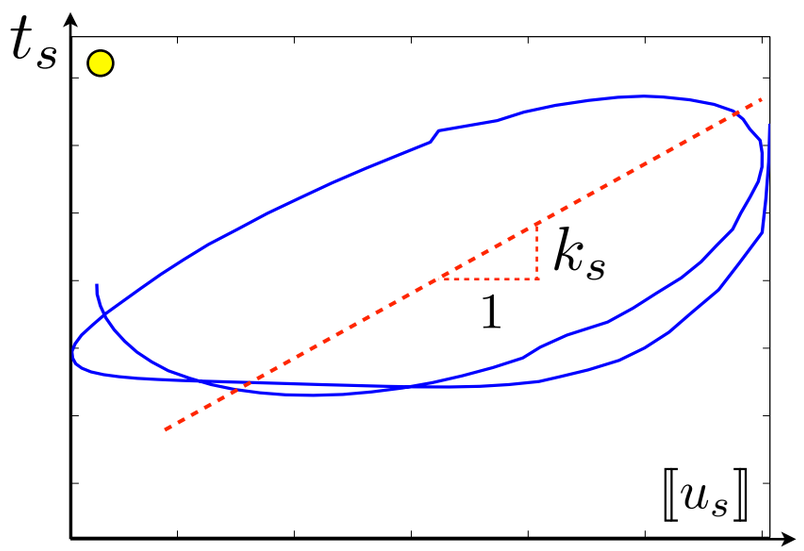
Contact law@ fracture interface

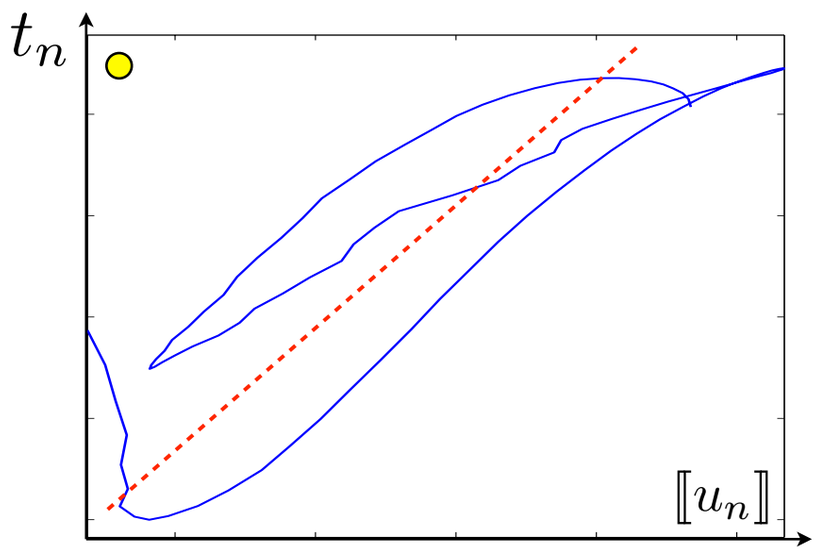












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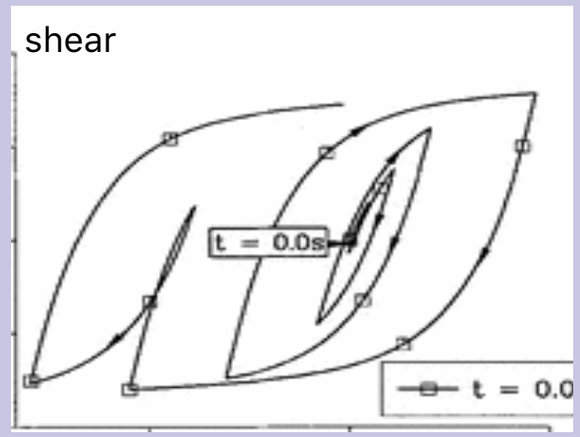
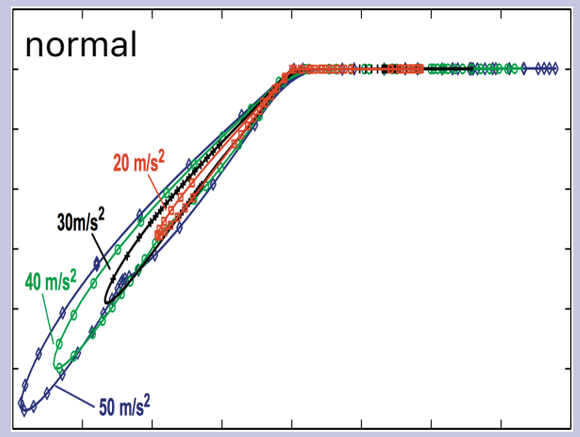


Gaul & Lenz, Acta Mechanica (1997)

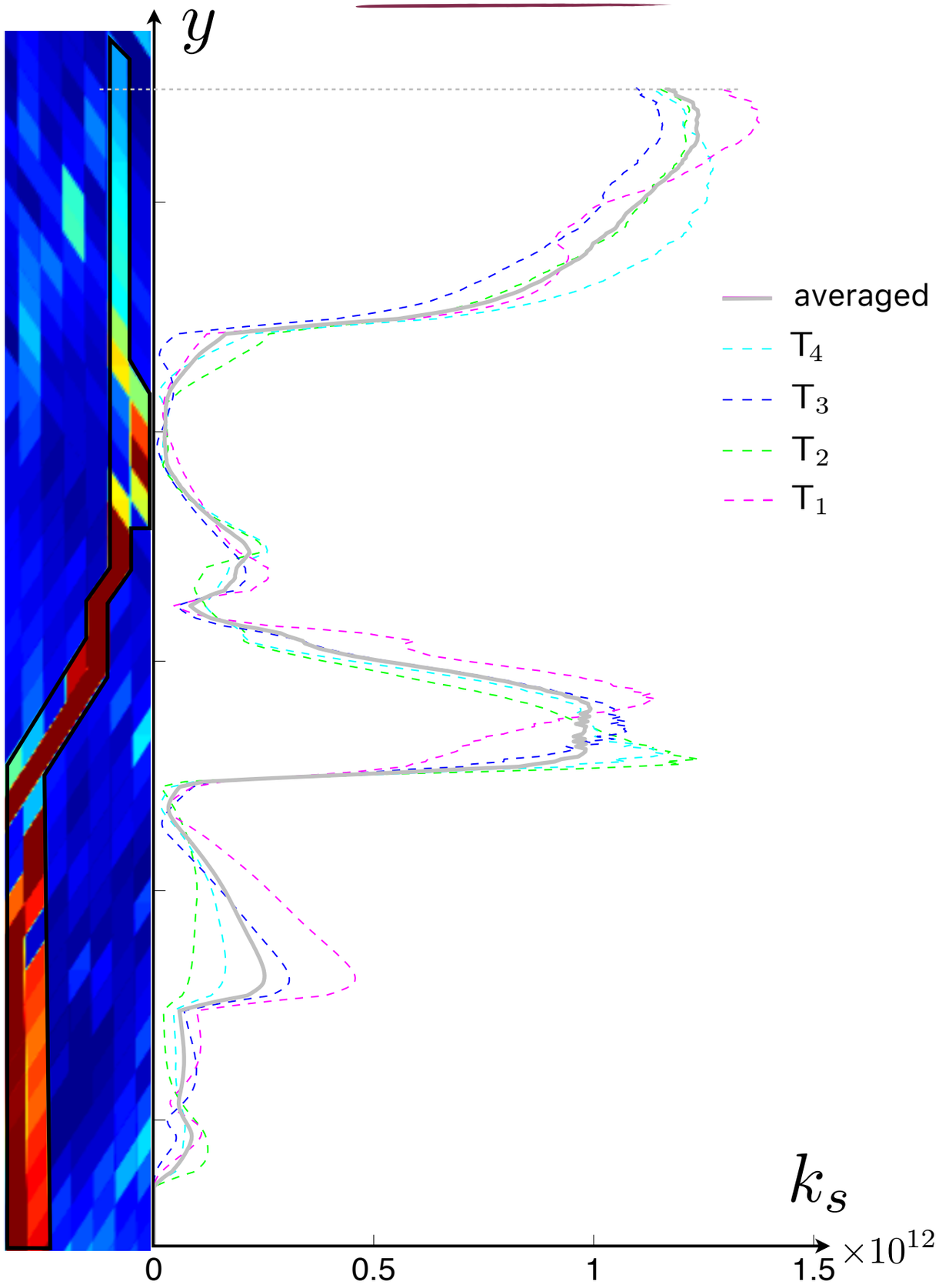
Ahmadian et. al, MSSP (2010)



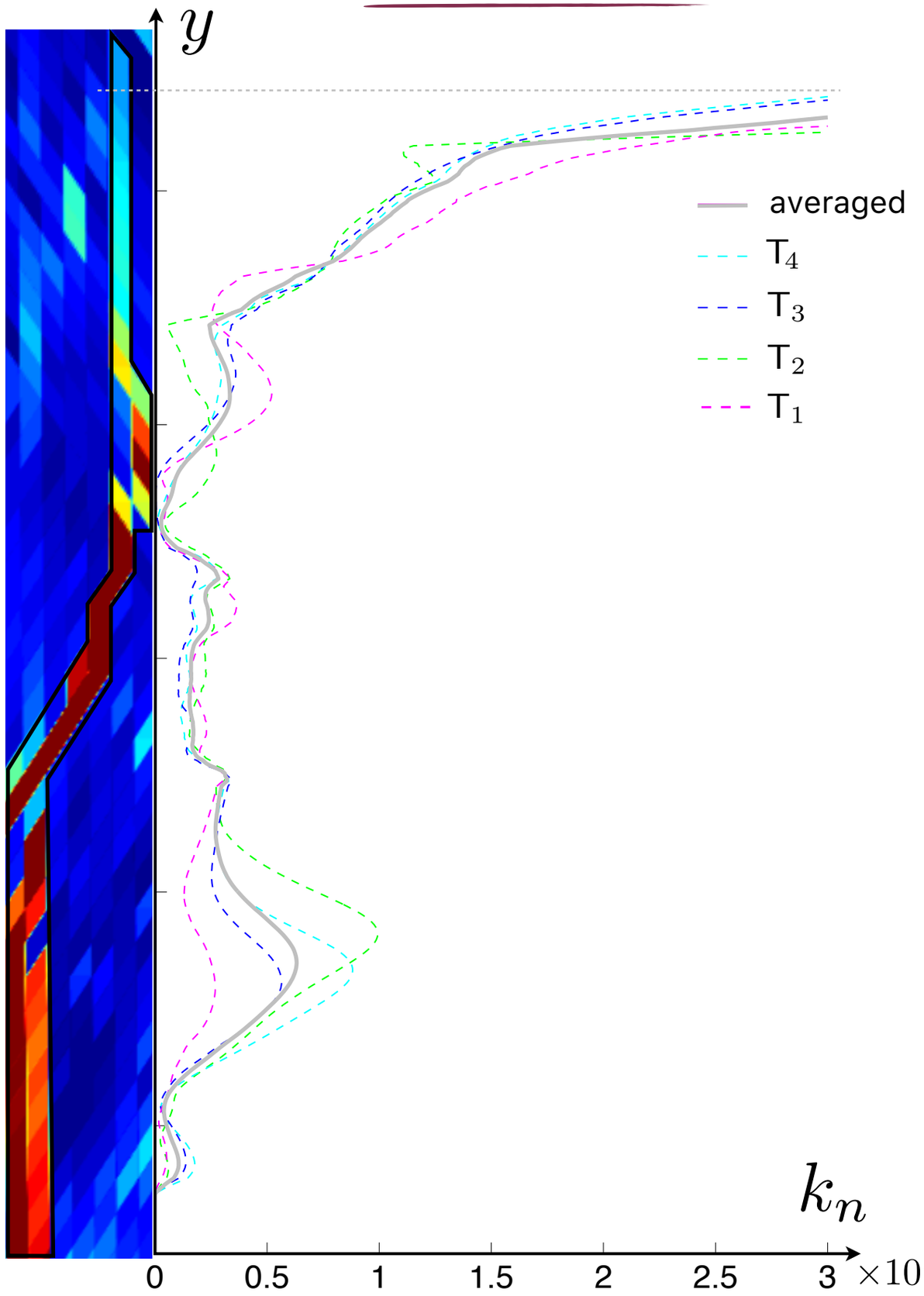
Jalali et. al, MSSP (2011)

Interfacial Stiffness mixed-mode

shear stiffness

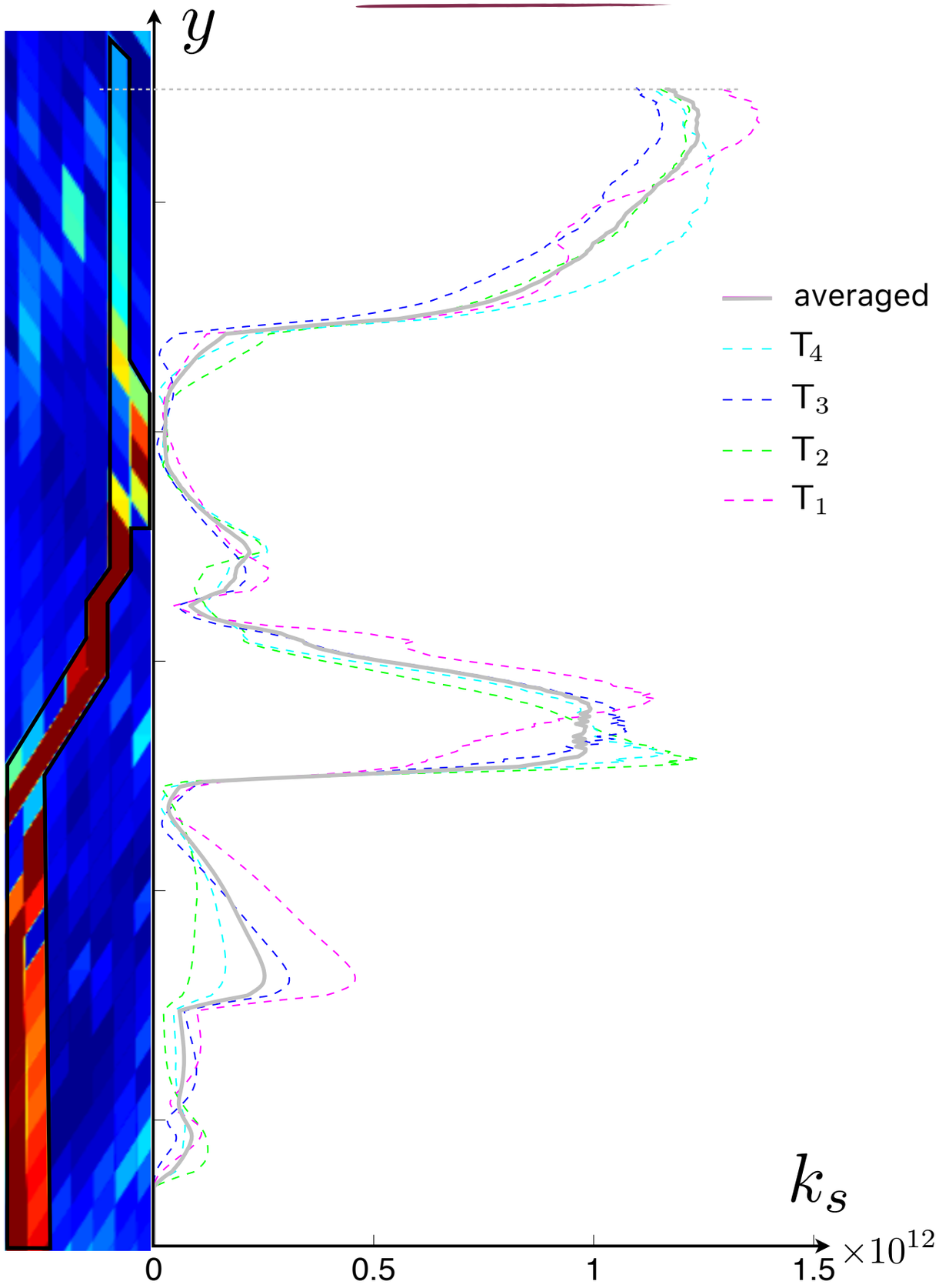


normal stiffness

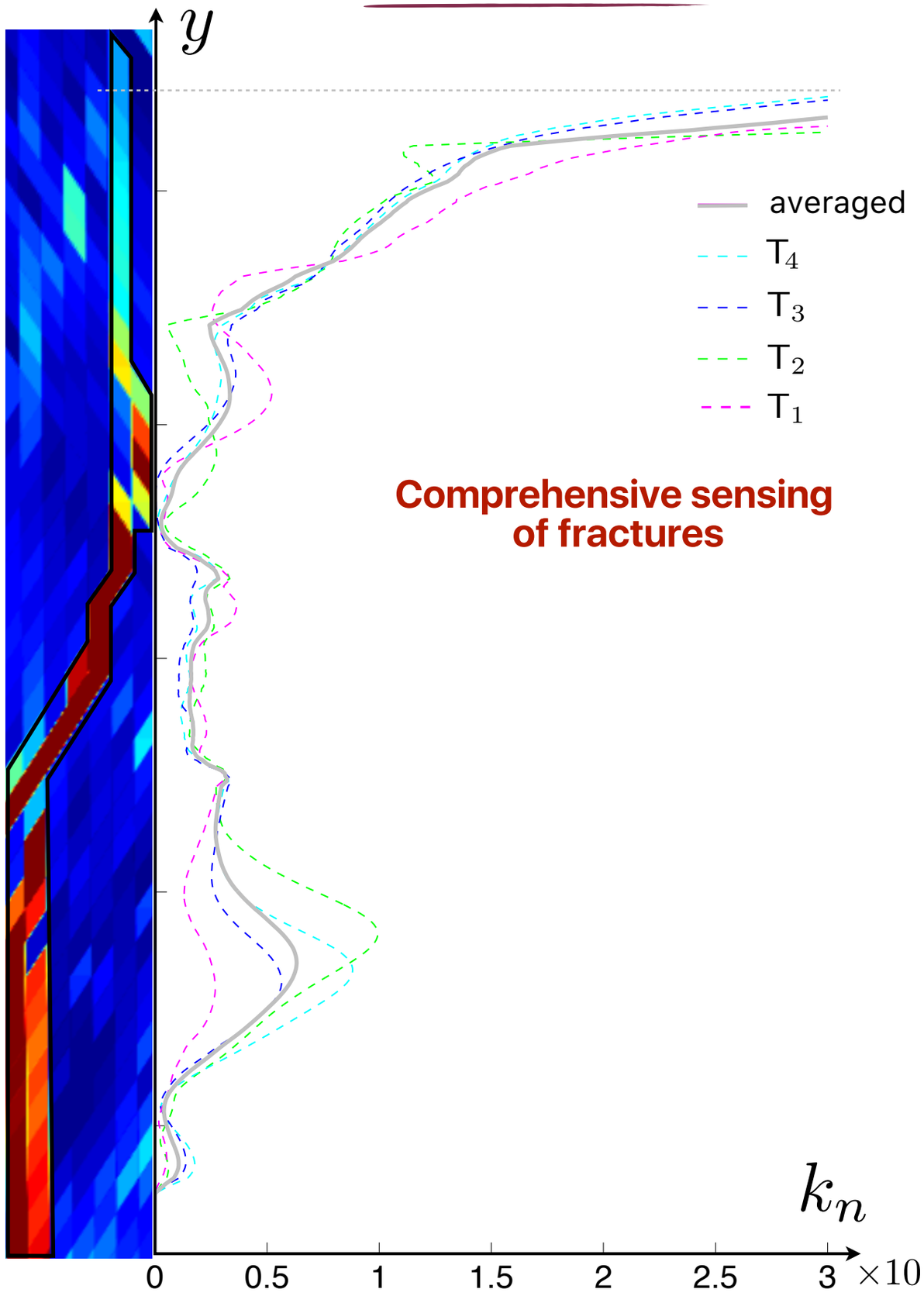


Interfacial Stiffness mixed-mode

shear stiffness



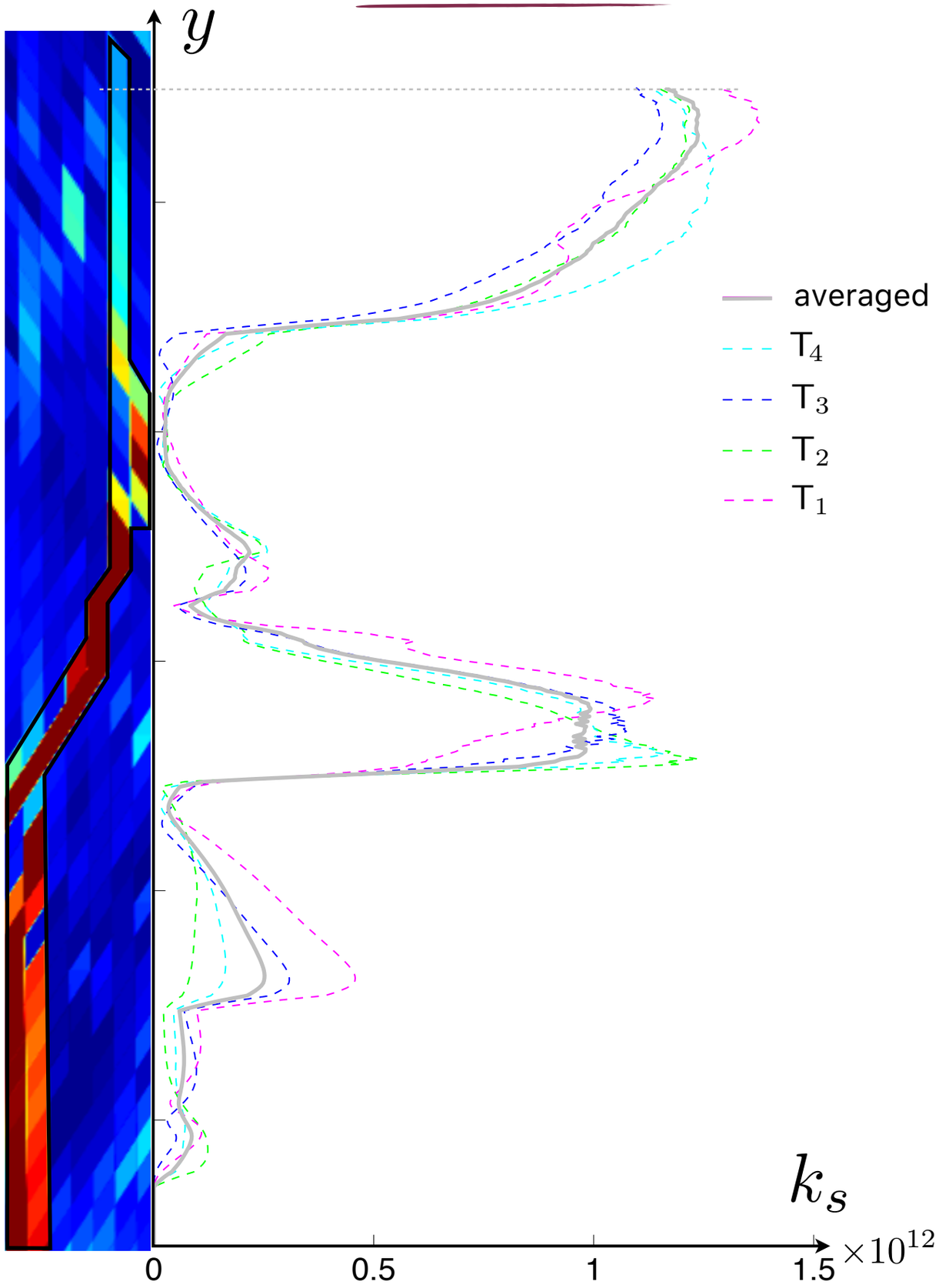
normal stiffness



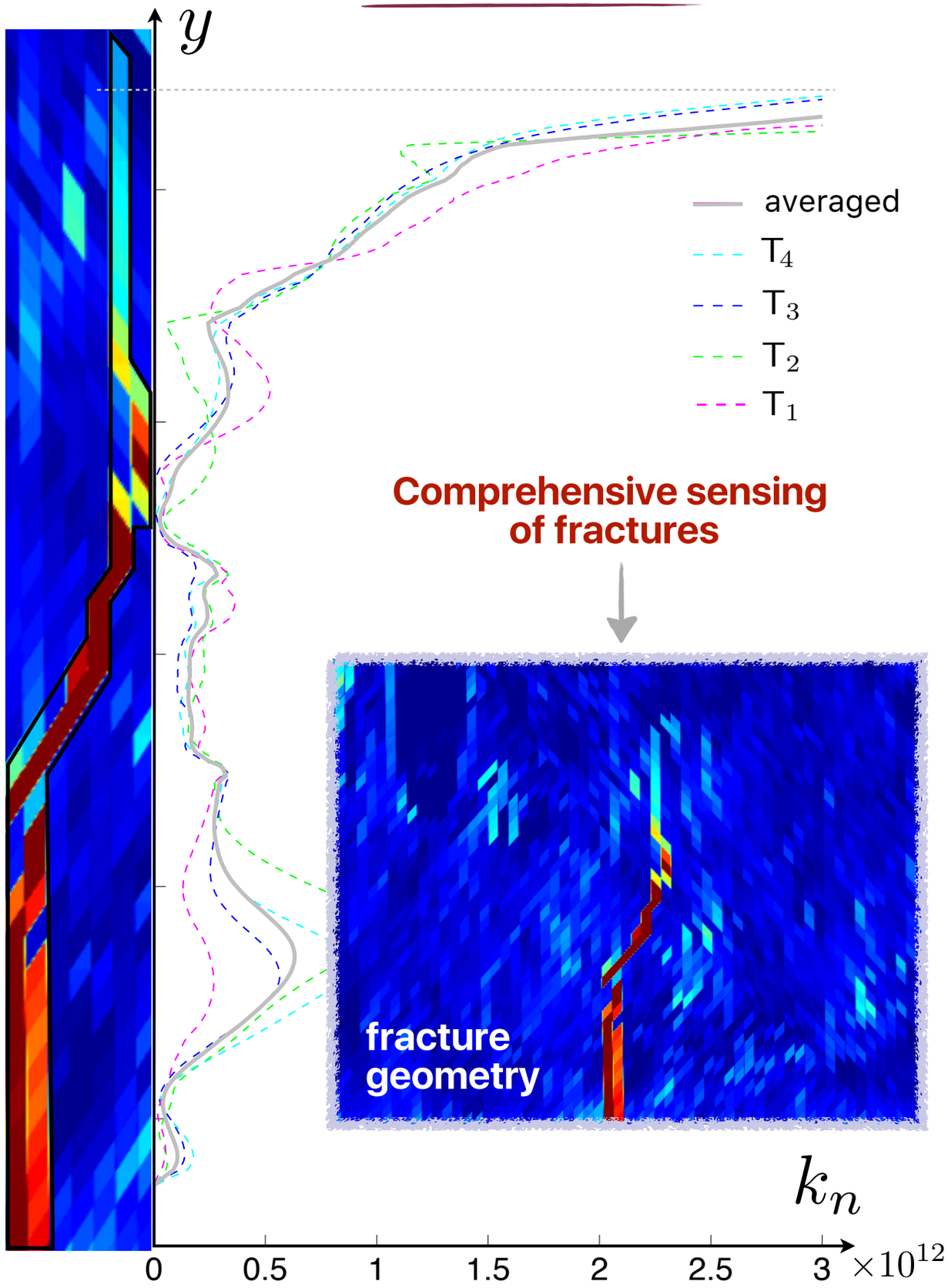
Comprehensive sensing of fractures

Interfacial Stiffness mixed-mode

shear stiffness

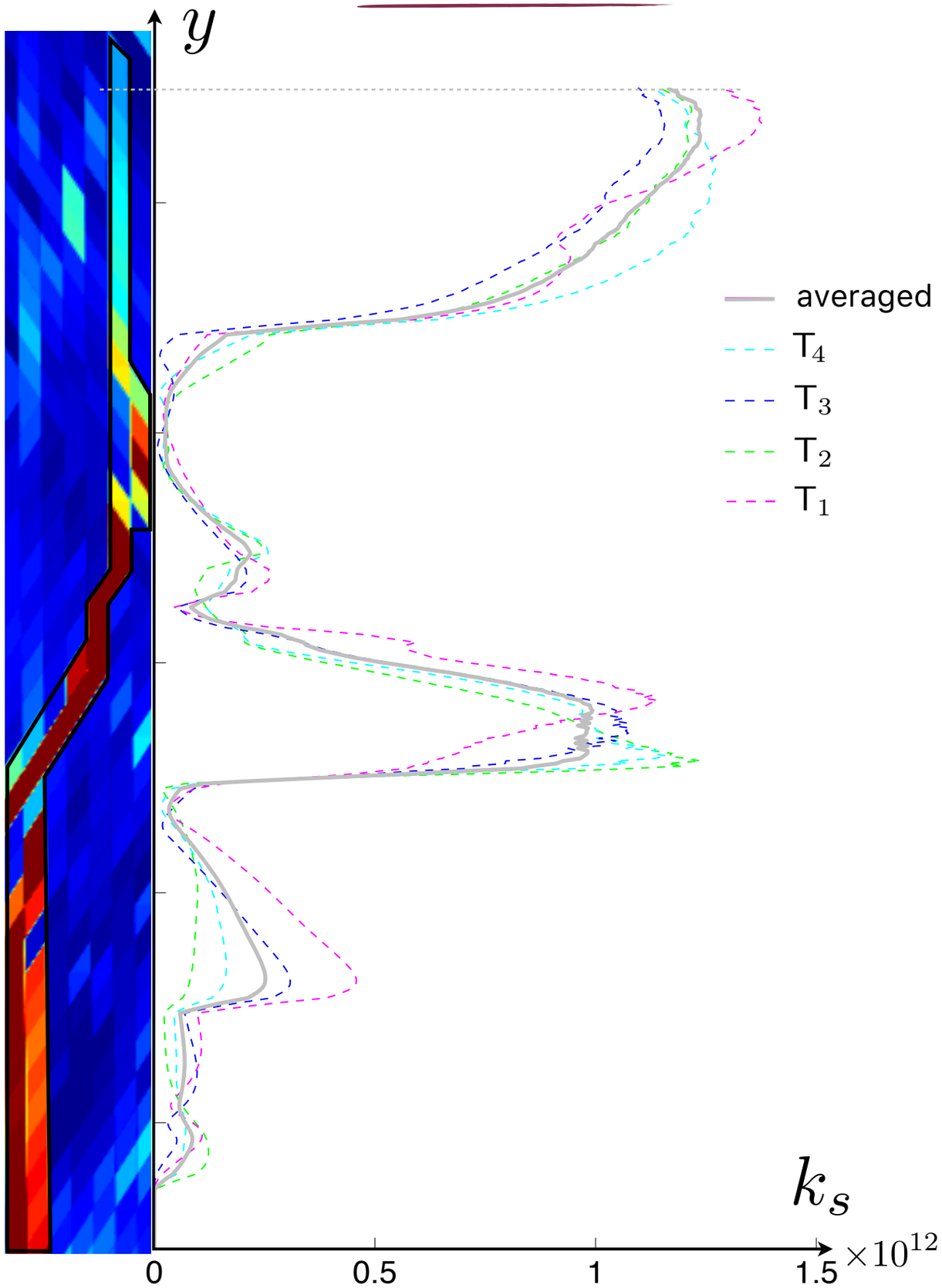


normal stiffness

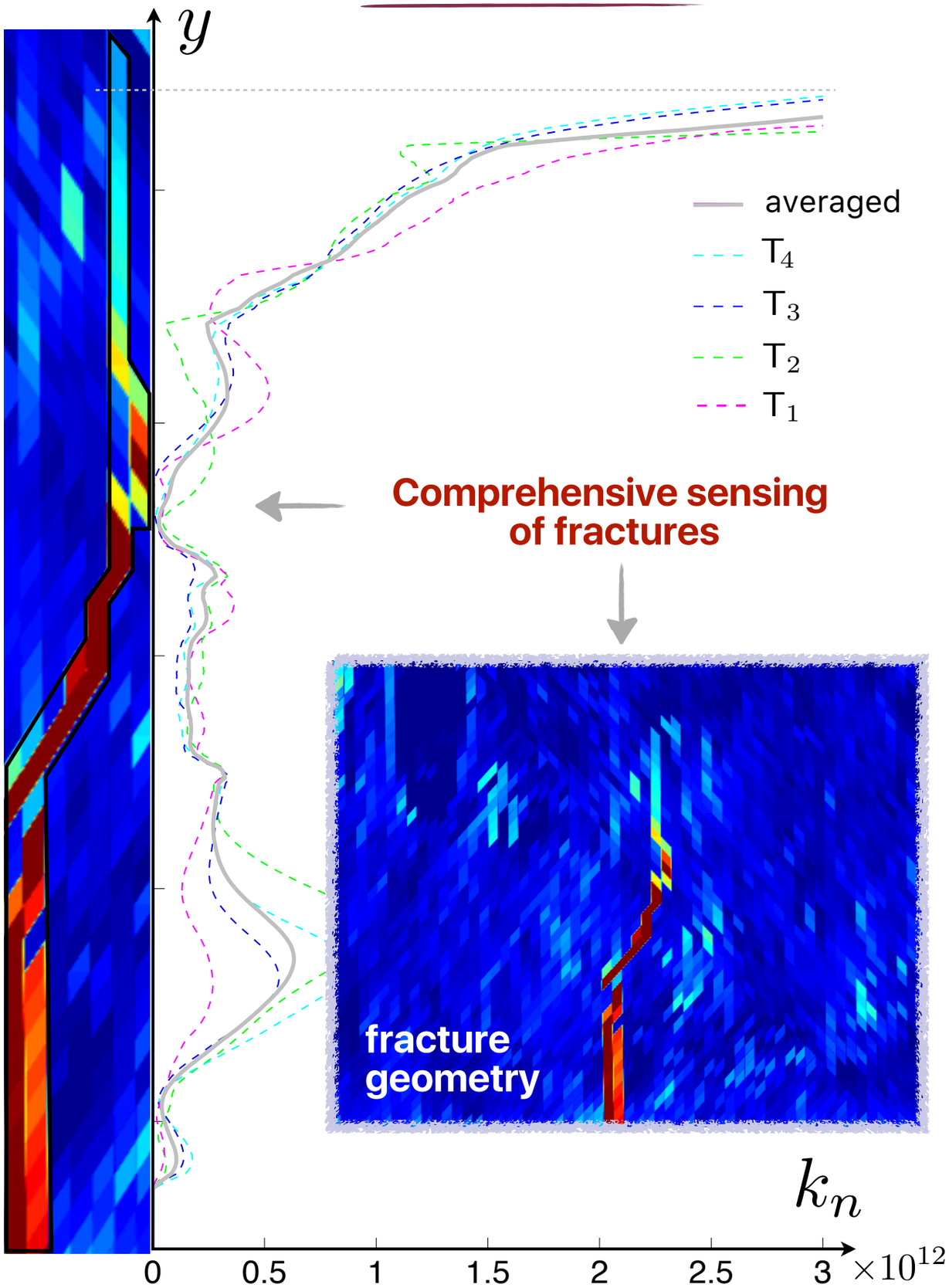


Interfacial Stiffness mixed-mode

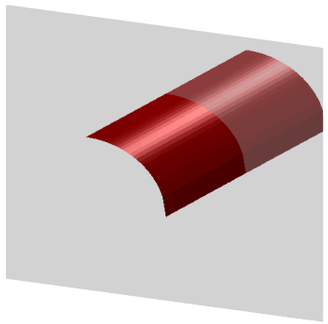
shear stiffness

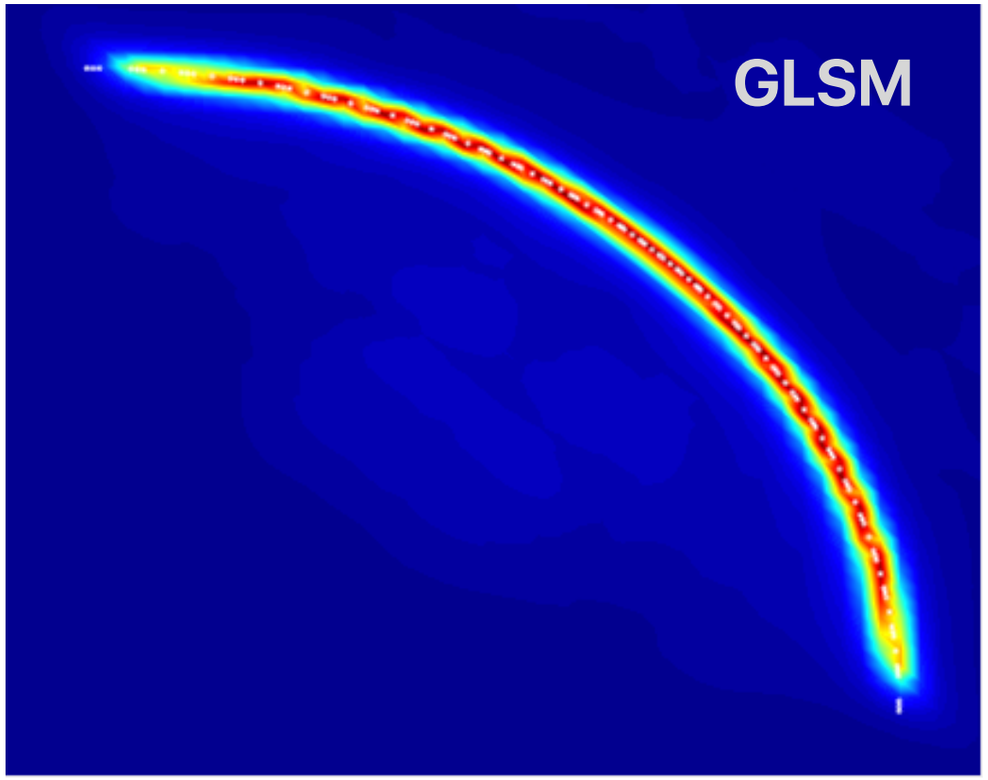


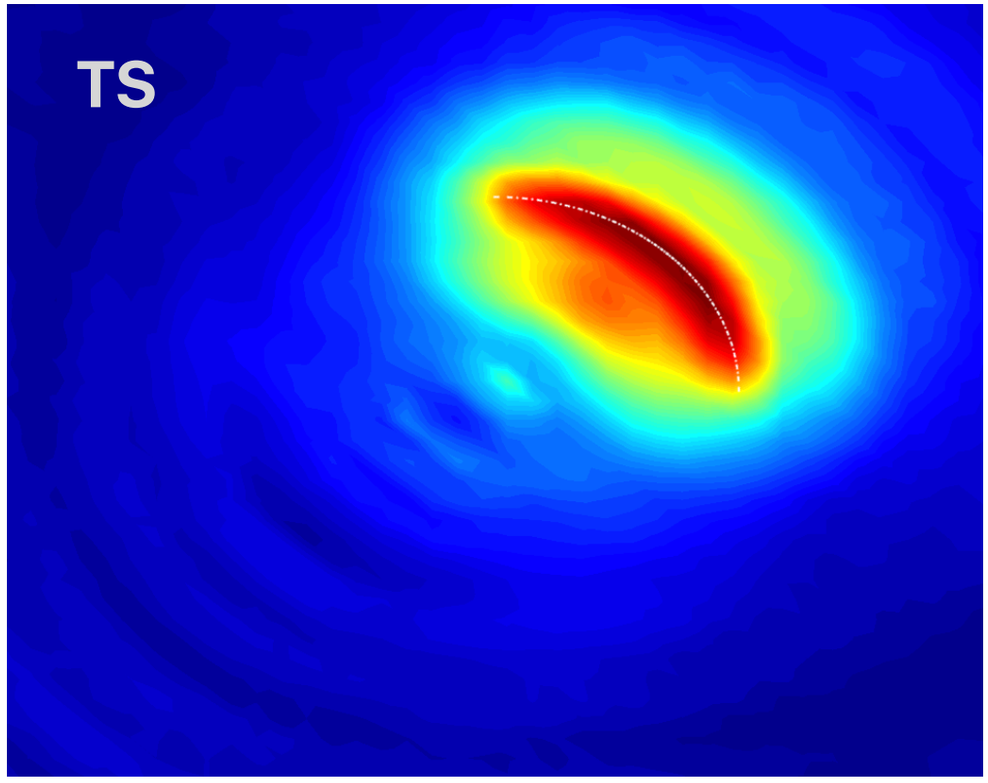
normal stiffness



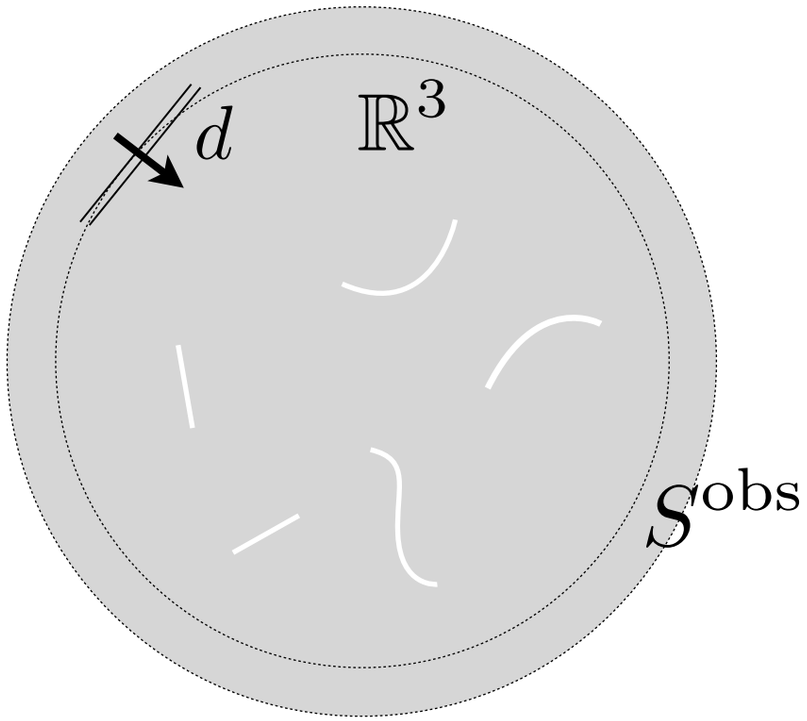
Analytical developments toward a robust 3D imaging platform







Generalized Linear Sampling Method



1998-
Colton, Kirsch, Kress, Monk, Arens, Guzina, Nintcheu Fata, Madyarov , Bellis, Bonnet, ...

Obstacles

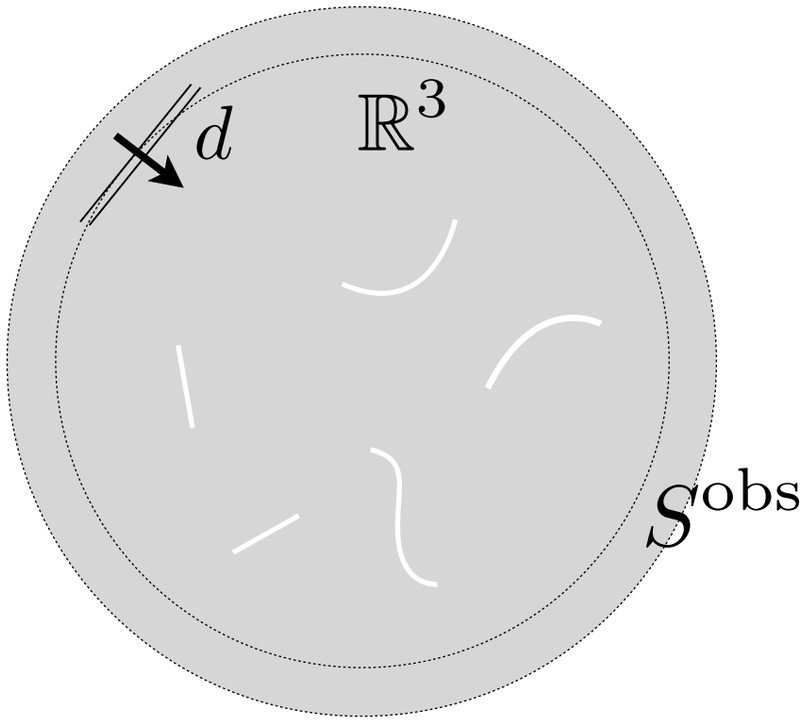
2003-
Cakoni, Haddar, Boukari, Kress, Ritter, Potthast, Monch, Park, Bourgeois, ...

Cracks

2014-
Haddar, Audibert, Cakoni, ...

GLSM

Generalized Linear Sampling Method



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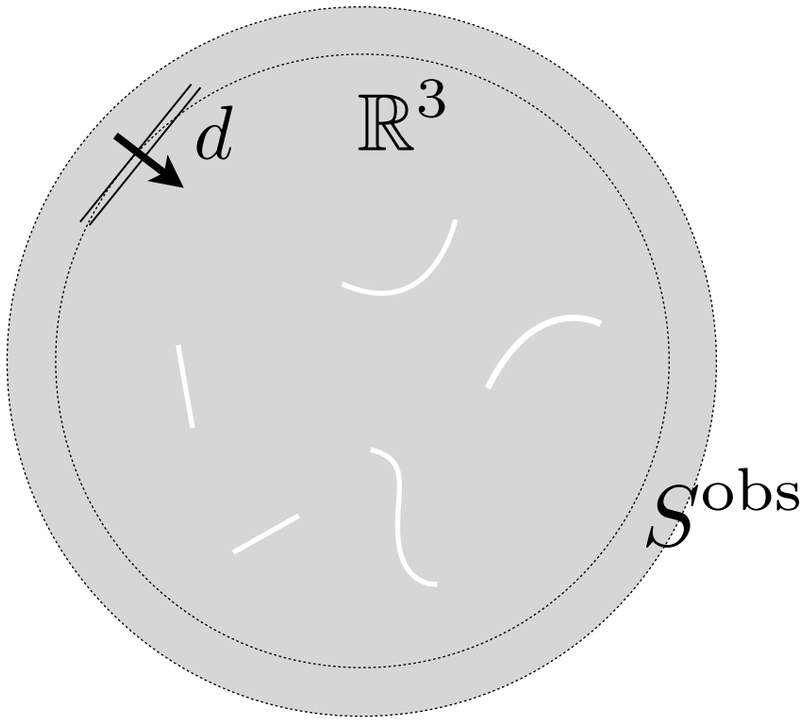
2014-
Haddar, Audibert, Cakoni, ... — **GLSM**

Far field pattern

$$\hat{\boldsymbol{\xi}} = \frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|},\ |\boldsymbol{\xi}| \to \infty$$

$$\boldsymbol{v}(\boldsymbol{\xi}) \;=\; \frac{e^{\mathrm{i}k_p|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|(\lambda+2\mu)}\,\boldsymbol{v}_p^{\infty}(\hat{\boldsymbol{\xi}}) \;+\; \frac{e^{\mathrm{i}k_s|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|\mu}\,\boldsymbol{v}_s^{\infty}(\hat{\boldsymbol{\xi}})$$

Generalized Linear Sampling Method



1998-
Colton, Kirsch, Kress, Monk, Arens, Guzina, Nintcheu Fata, Madyarov , Bellis, Bonnet, ... — Obstacles

2003-
Cakoni, Haddar, Boukari, Kress, Ritter, Potthast, Monch, Park, Bourgeois, ... — Cracks

2014-
Haddar, Audibert, Cakoni, ... — **GLSM**

Far field pattern

$$\hat{\boldsymbol{\xi}} = \frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|},\ |\boldsymbol{\xi}| \to \infty$$

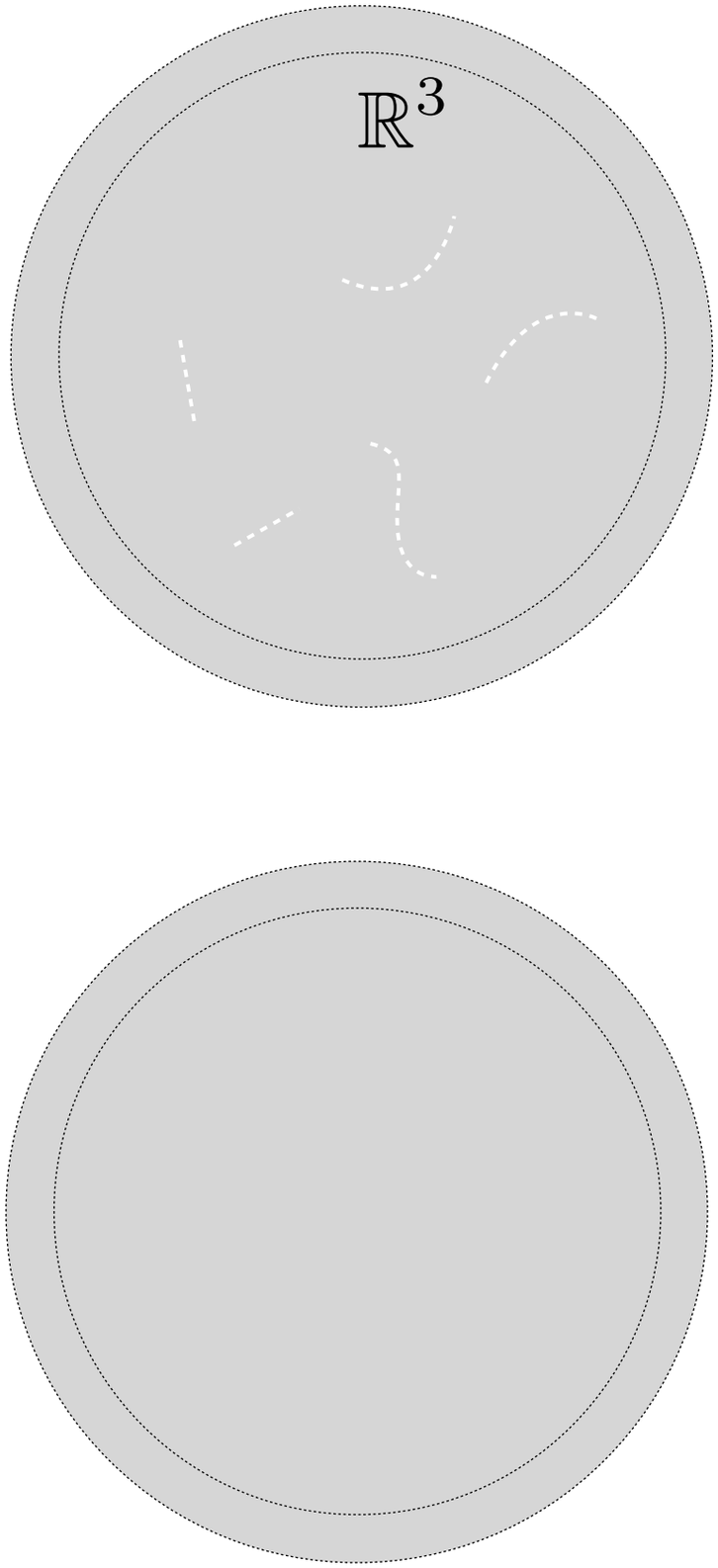
$$\boldsymbol{v}(\boldsymbol{\xi}) \;=\; \frac{e^{\mathrm{i}k_p|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|(\lambda+2\mu)}\boldsymbol{v}_p^\infty(\hat{\boldsymbol{\xi}}) \;+\; \frac{e^{\mathrm{i}k_s|\boldsymbol{\xi}|}}{4\pi|\boldsymbol{\xi}|\mu}\boldsymbol{v}_s^\infty(\hat{\boldsymbol{\xi}})$$

Far field operator

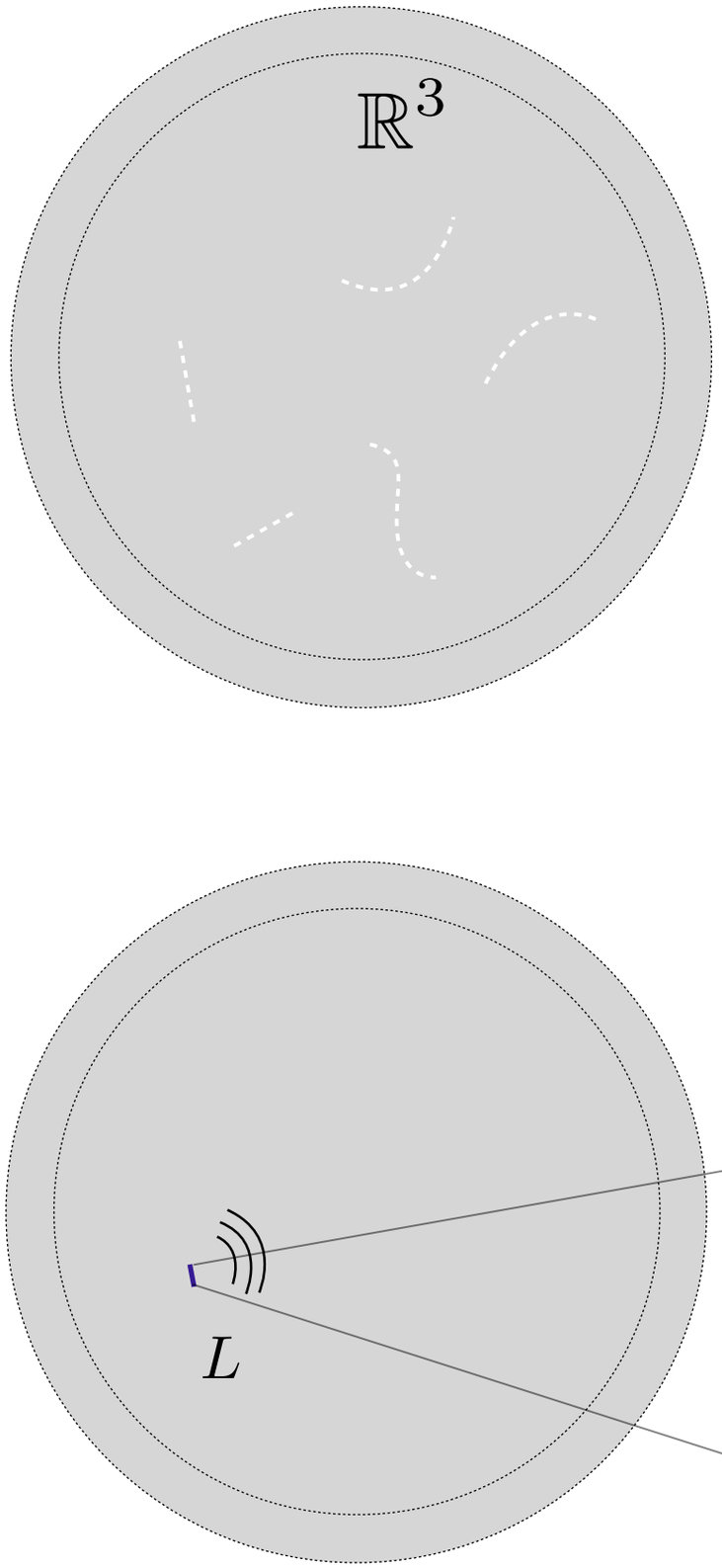
$$\boldsymbol{v}^\infty \;:=\; F(\boldsymbol{g}) \;:=\; \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}(\boldsymbol{d}) \cdot \boldsymbol{W}^\infty(\boldsymbol{d}, \hat{\boldsymbol{\xi}})\, \mathrm{d}S_{\boldsymbol{d}}$$

$(\boldsymbol{v}_p^\infty, \boldsymbol{v}_s^\infty)$ $(\boldsymbol{g}_p, \boldsymbol{g}_s)$ unit sphere measurements

GLSM IDEA

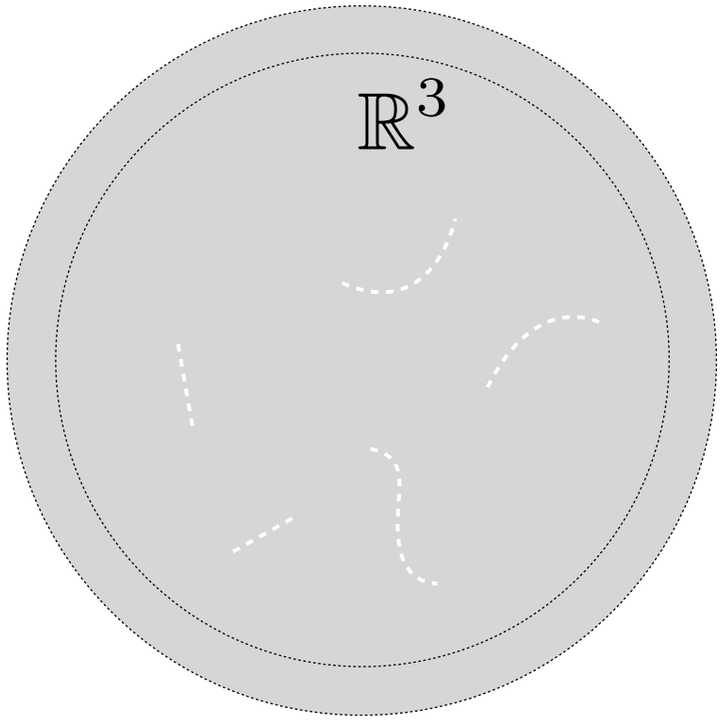


GLSM IDEA

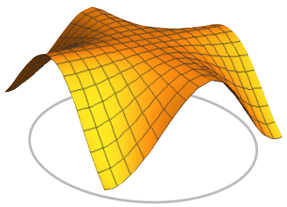


smooth
non-intersecting

GLSM IDEA



$\boldsymbol{a} \in \tilde{H}^{\frac{1}{2}}(L)$



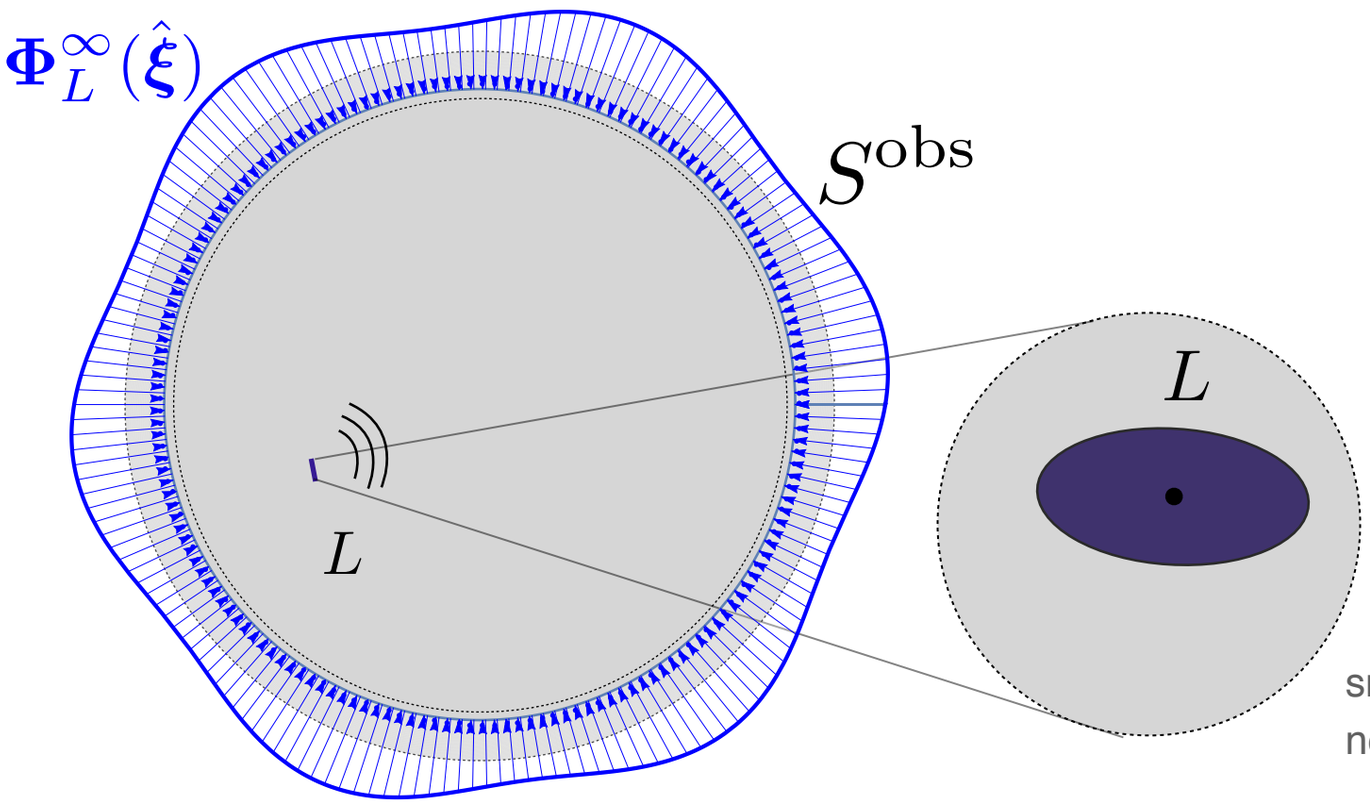
L

$$\boldsymbol{\Phi}_L^{\infty}[\boldsymbol{a}](\hat{\boldsymbol{\xi}}) = -\Big(ik_p\,\hat{\boldsymbol{\xi}} \int_L \left\{\lambda(\boldsymbol{a}\cdot\boldsymbol{n}) + 2\mu(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\right\} e^{-ik_p\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\,\mathrm{d}S_{\boldsymbol{y}},$$

$$ik_s\,\hat{\boldsymbol{\xi}} \times \int_L \left\{\mu(\boldsymbol{a}\times\hat{\boldsymbol{\xi}})(\boldsymbol{n}\cdot\hat{\boldsymbol{\xi}}) + \mu(\boldsymbol{n}\times\hat{\boldsymbol{\xi}})(\boldsymbol{a}\cdot\hat{\boldsymbol{\xi}})\right\} e^{-ik_s\hat{\boldsymbol{\xi}}\cdot\boldsymbol{y}}\,\mathrm{d}S_{\boldsymbol{y}}\Big)$$

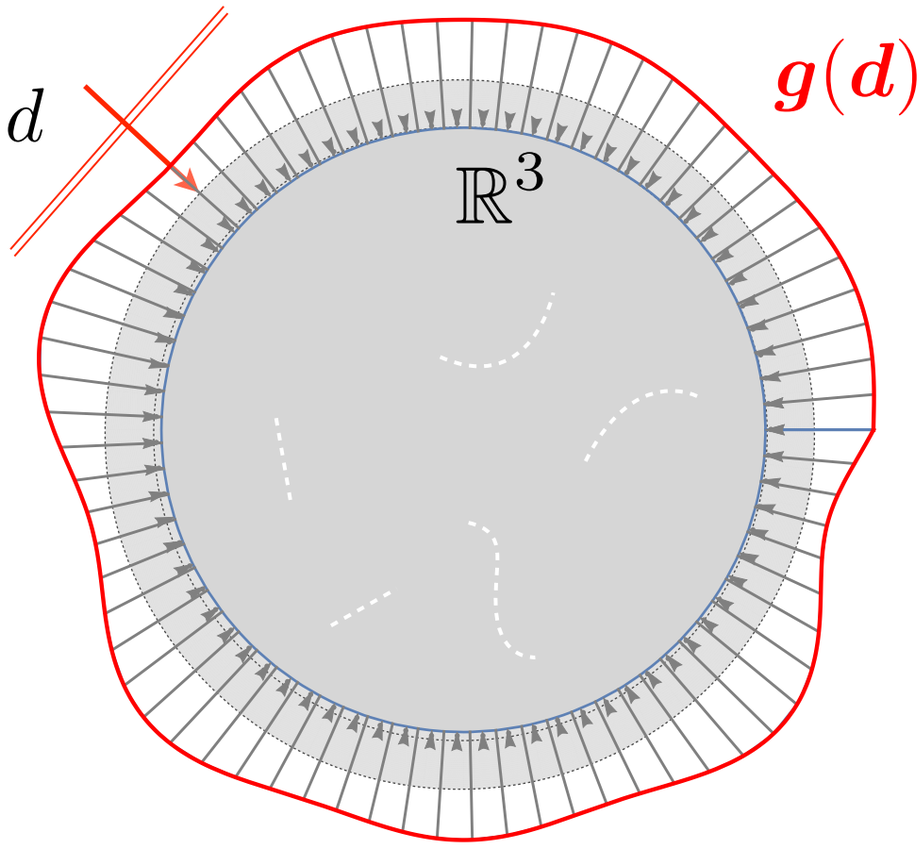
P patten

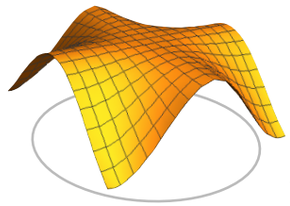
S patten



smooth
non-intersecting

GLSM IDEA





$$\Phi_L^\infty[a](\hat{\xi}) = -\Big(ik_p\,\hat{\xi} \int_L \{\lambda(a\cdot n) + 2\mu(n\cdot\hat{\xi})(a\cdot\hat{\xi})\}\, e^{-ik_p\hat{\xi}\cdot y}\,\mathrm{d}S_y,$$

$$ik_s\,\hat{\xi}\times \int_L \{\mu(a\times\hat{\xi})(n\cdot\hat{\xi}) + \mu(n\times\hat{\xi})(a\cdot\hat{\xi})\}\, e^{-ik_s\hat{\xi}\cdot y}\,\mathrm{d}S_y\Big)$$

P patten

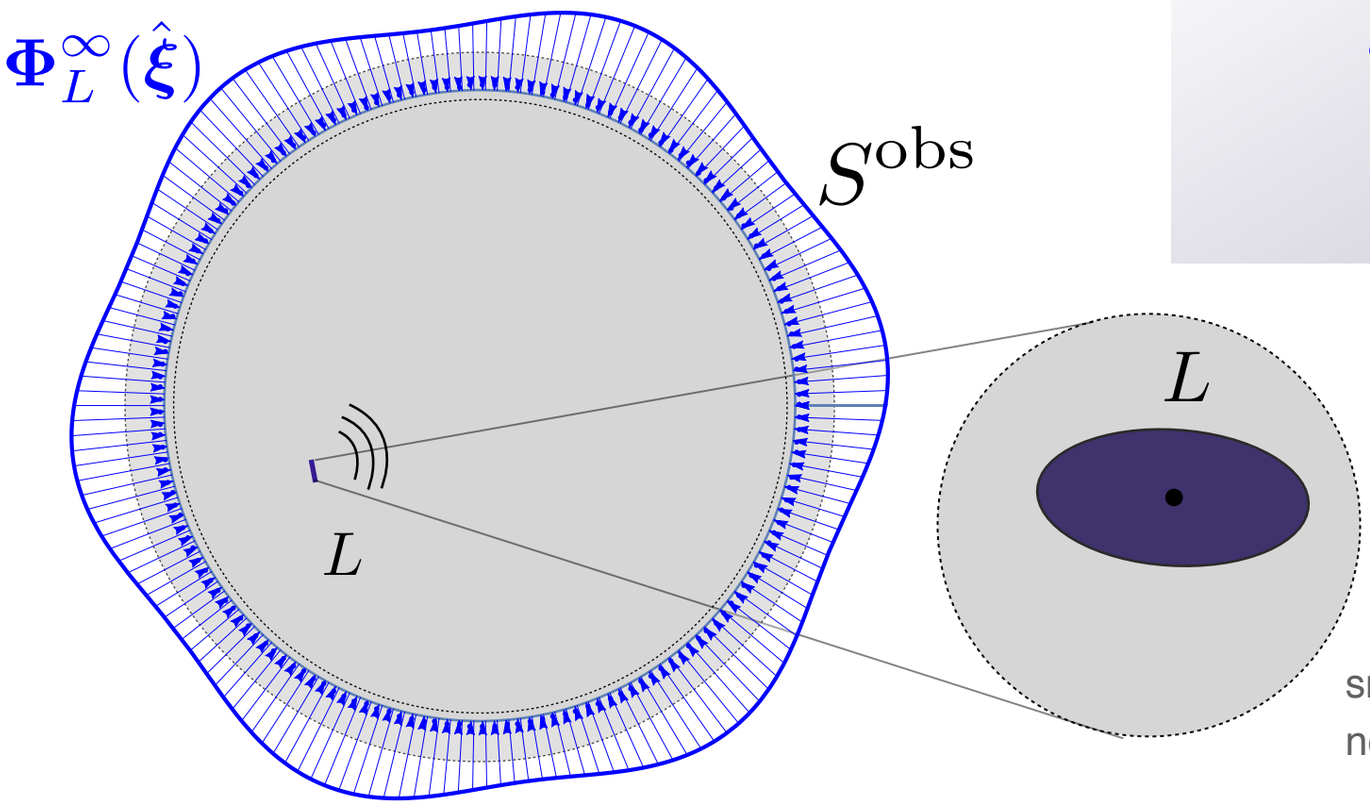
S patten

IDEA

$$\Phi_L^\infty(\hat{\xi}) = \int_{\Omega_d} g(d)\cdot W^\infty(d,\hat{\xi})\,\mathrm{d}S_d$$

experiments

synthetic rearrangement of the sources



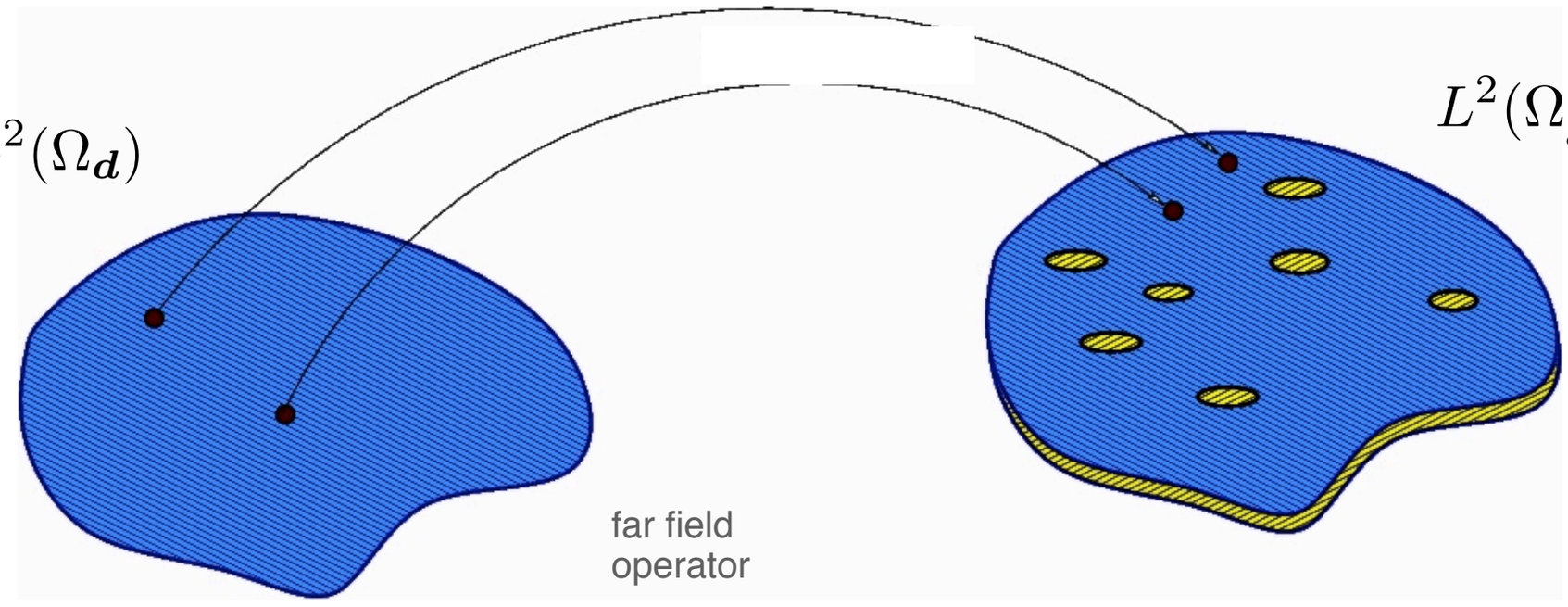
smooth
non-intersecting

$$Fg = \Phi_L^\infty(\hat{\xi})$$

Ill-posedness

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$

$L^2(\Omega_{\boldsymbol{d}}) \times L^2(\Omega_{\boldsymbol{d}})$

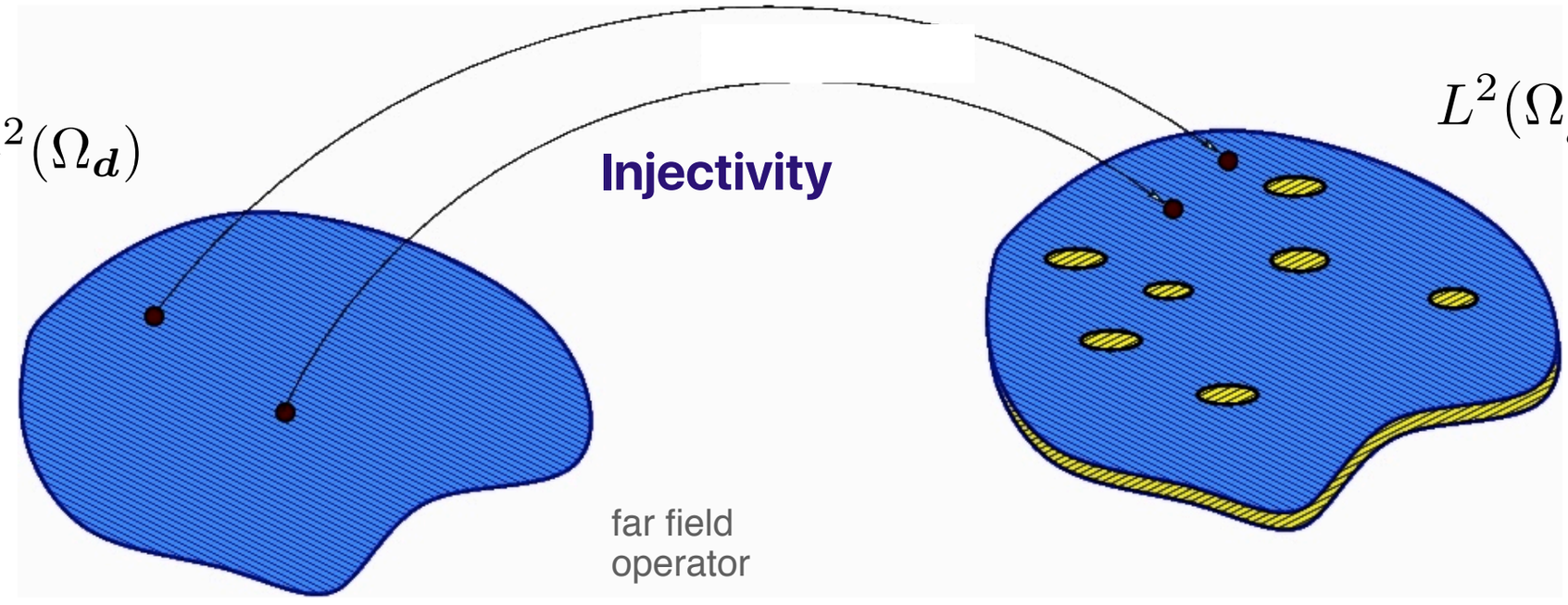


$L^2(\Omega_{\hat{\boldsymbol{\xi}}}) \times L^2(\Omega_{\hat{\boldsymbol{\xi}}})$

$$F : L^2(\Omega_{\boldsymbol{d}}) \times L^2(\Omega_{\boldsymbol{d}}) \to L^2(\Omega_{\hat{\boldsymbol{\xi}}}) \times L^2(\Omega_{\hat{\boldsymbol{\xi}}})$$

Ill-posedness

$$Fg = \Phi_L^\infty(\hat{\xi})$$



$$F : L^2(\Omega_d) \times L^2(\Omega_d) \to L^2(\Omega_{\hat{\xi}}) \times L^2(\Omega_{\hat{\xi}})$$

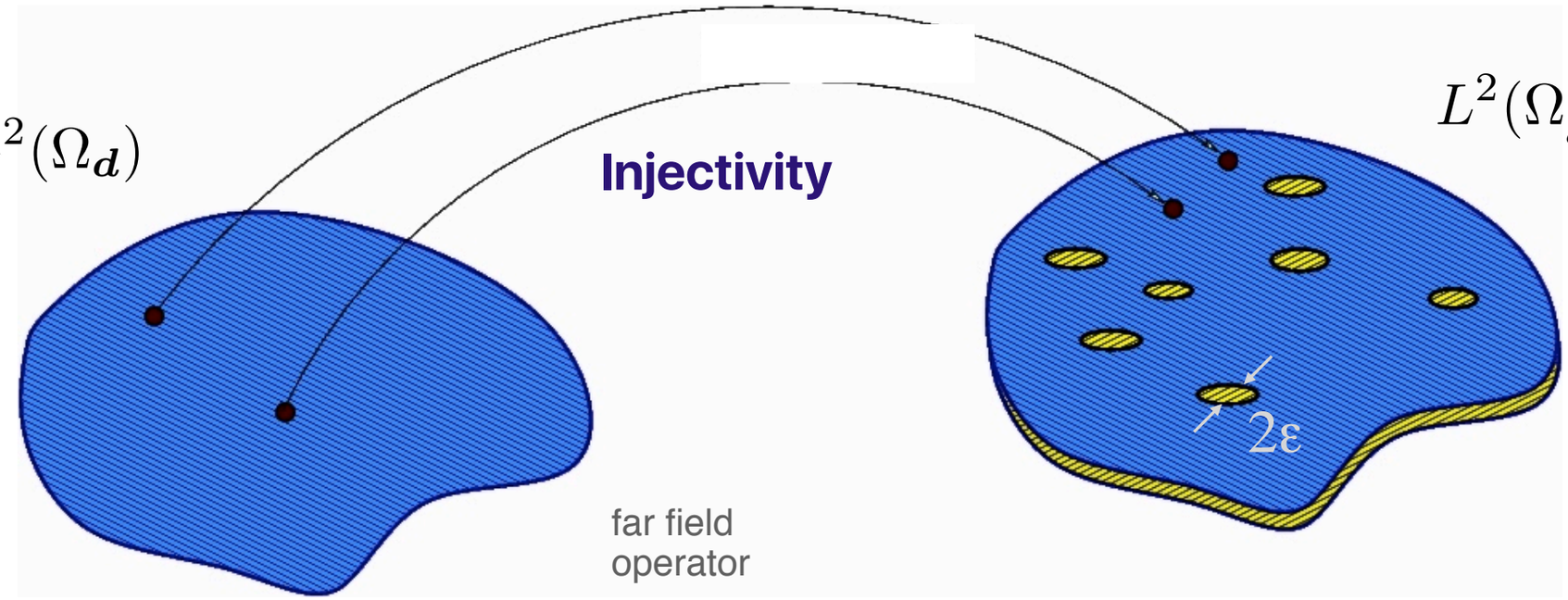
Hadamard (1923)

injective

Uniqueness $\quad Fg = 0 \quad \Rightarrow \quad g = 0$

Ill-posedness

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$



$$F : L^2(\Omega_{\boldsymbol{d}}) \times L^2(\Omega_{\boldsymbol{d}}) \to L^2(\Omega_{\hat{\boldsymbol{\xi}}}) \times L^2(\Omega_{\hat{\boldsymbol{\xi}}})$$

Hadamard (1923)

injective

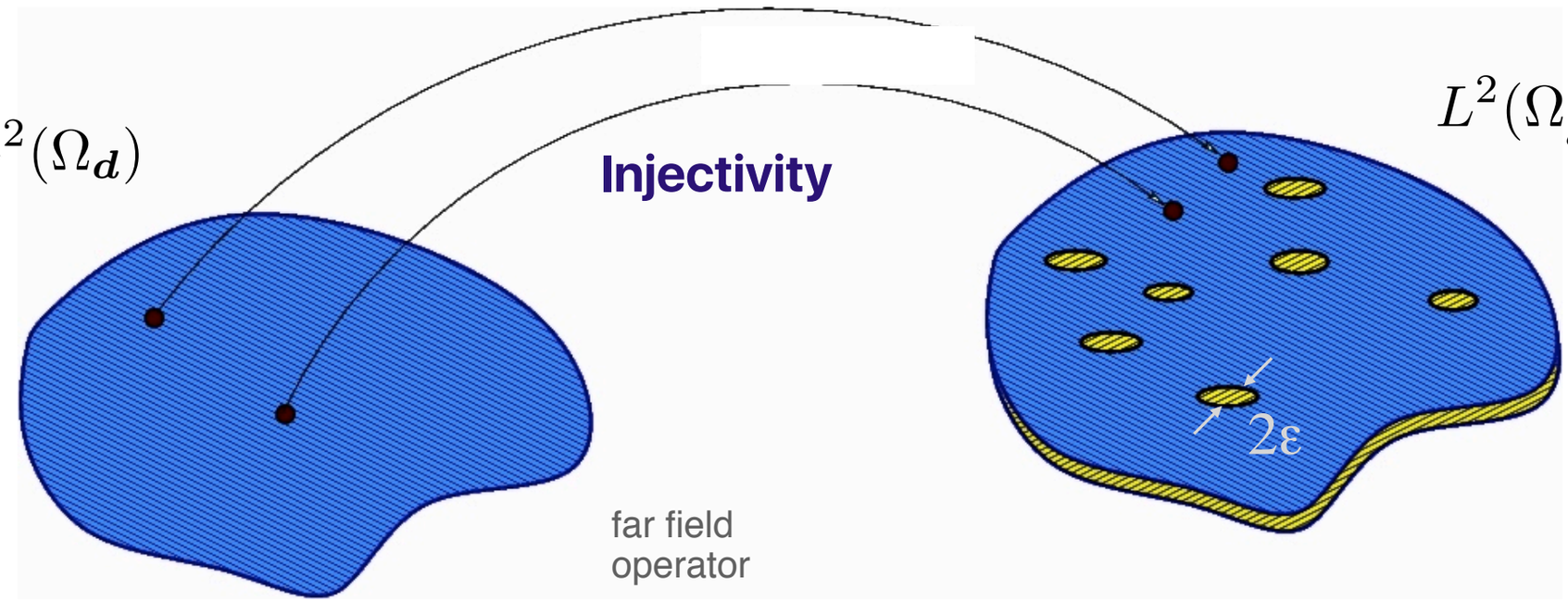
Uniqueness $\quad F\boldsymbol{g} = \mathbf{0} \;\Rightarrow\; \boldsymbol{g} = \mathbf{0}$

dense range

Existence $\quad \forall \varepsilon > 0, \quad \exists \boldsymbol{g}^{\varepsilon} : \; \| F\boldsymbol{g}^{\varepsilon} - \boldsymbol{\Phi}_L^{\infty} \|_{L^2(\Omega_{\hat{\boldsymbol{\xi}}})} < \varepsilon$

Ill-posedness

$$Fg = \Phi_L^\infty(\hat{\xi})$$



$$F : L^2(\Omega_d) \times L^2(\Omega_d) \to L^2(\Omega_{\hat{\xi}}) \times L^2(\Omega_{\hat{\xi}})$$

Hadamard (1923)

Uniqueness $Fg = 0 \quad \Rightarrow \quad g = 0$ (injective)

Existence $\forall \varepsilon > 0, \quad \exists g^\varepsilon : \ \| Fg^\varepsilon - \Phi_L^\infty \|_{L^2(\Omega_{\hat{\xi}})} < \varepsilon$ (dense range)

Stability $\nexists C > 0 : \ \| g \|_{L^2(\Omega_d)} \leqslant C \, \| \Phi_L^\infty \|_{L^2(\Omega_{\hat{\xi}})}$

compact operator of infinite dimension → no stability

Stability

$$Fg = \Phi_L^{\infty}(\hat{\xi})$$

1D Backwards heat equation

Fourier Coefficients of the input

$$\| T_{\circ} \|^2_{L^2(\ell)} = \frac{\pi}{2} \sum_{n=1}^{\infty} T_n^{\text{fin}} e^{2n^2 \tau}$$

norm of solution

Stability

$$Fg = \Phi_L^\infty(\hat{\xi})$$

1D Backwards heat equation

Fourier Coefficients of the input

$$\| T_\circ \|^2_{L^2(\ell)} = \frac{\pi}{2} \sum_{n=1}^{\infty} T_n^{\text{fin}} e^{2n^2 \tau}$$

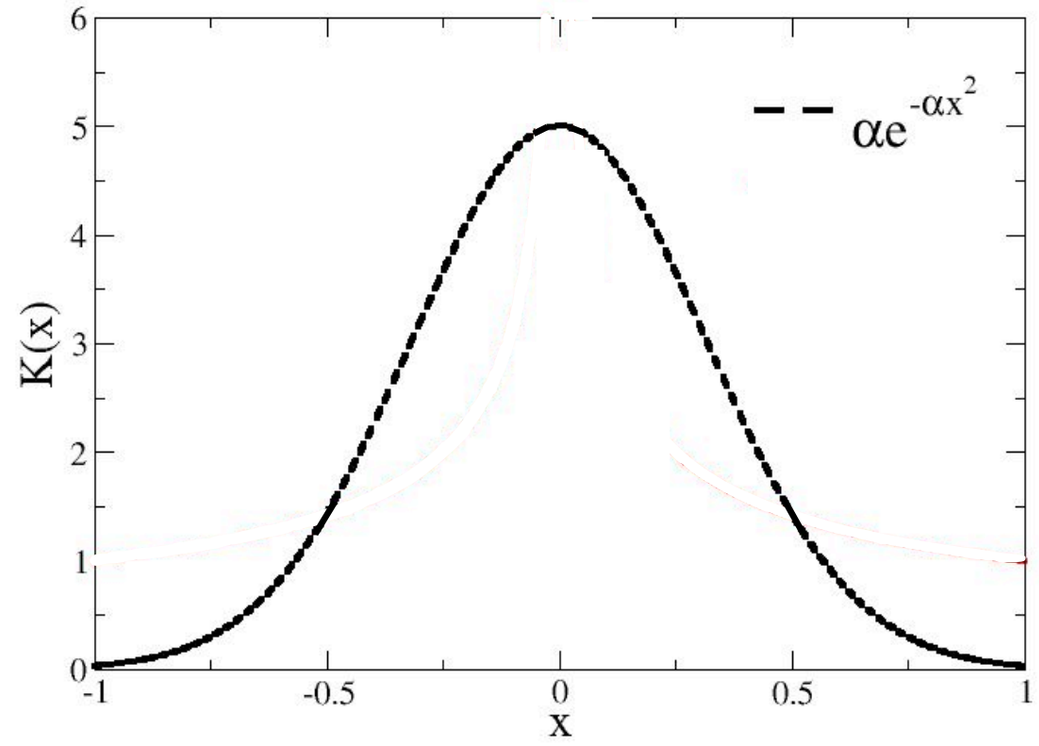
norm of solution

Heat conduction is irreversible

Stability

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$

$$\int_{-1}^{1} K(\xi - x)\cdot g(x)\,\mathrm{d}x \;=\; u(\xi)$$



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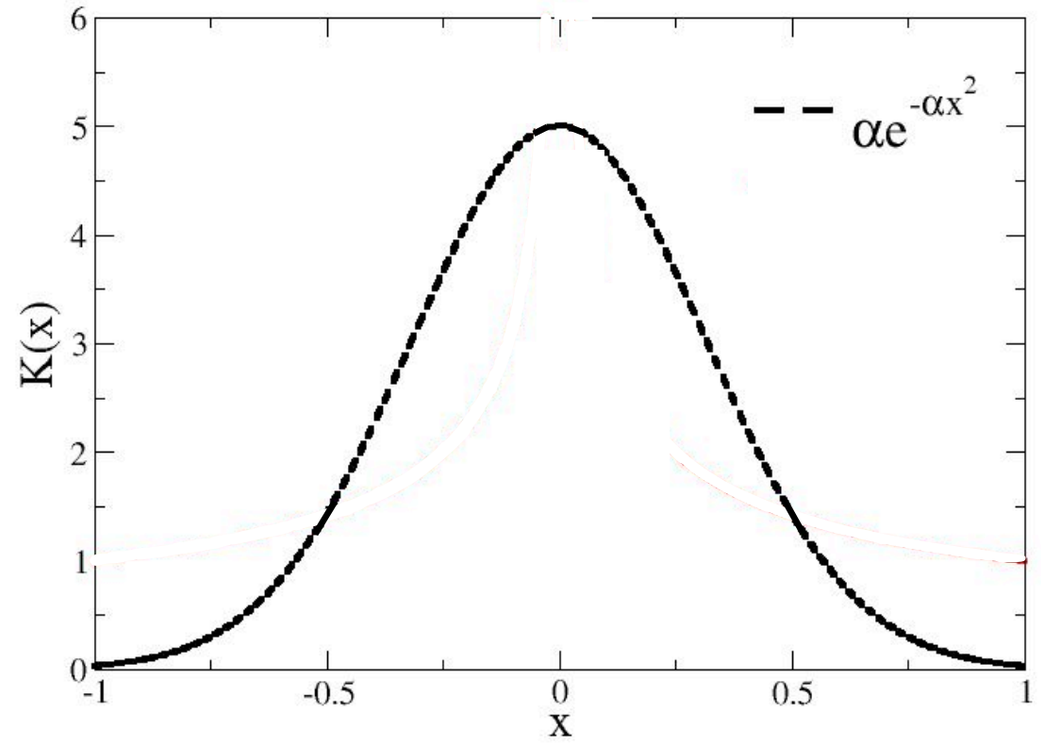
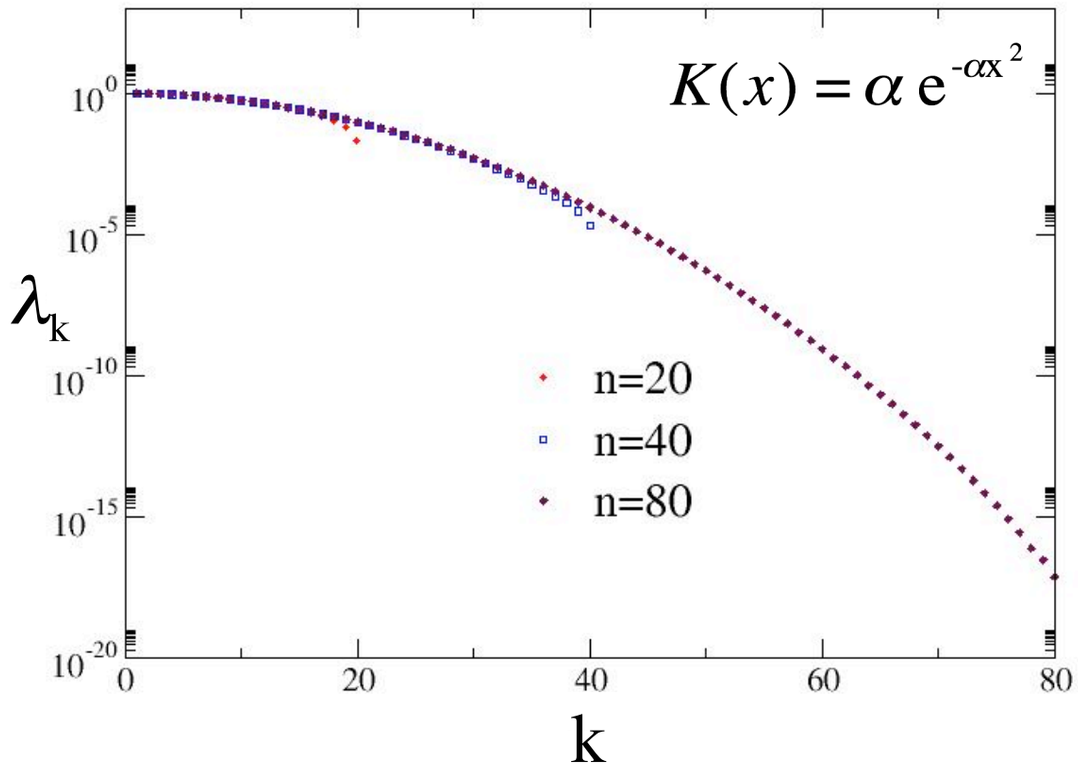




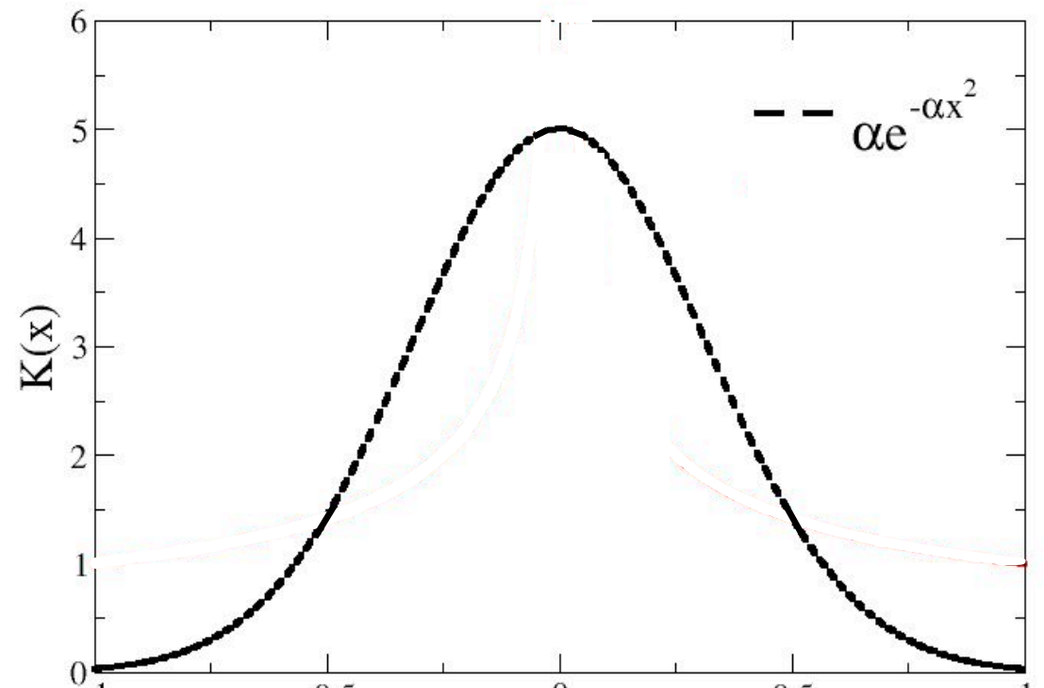
Stability

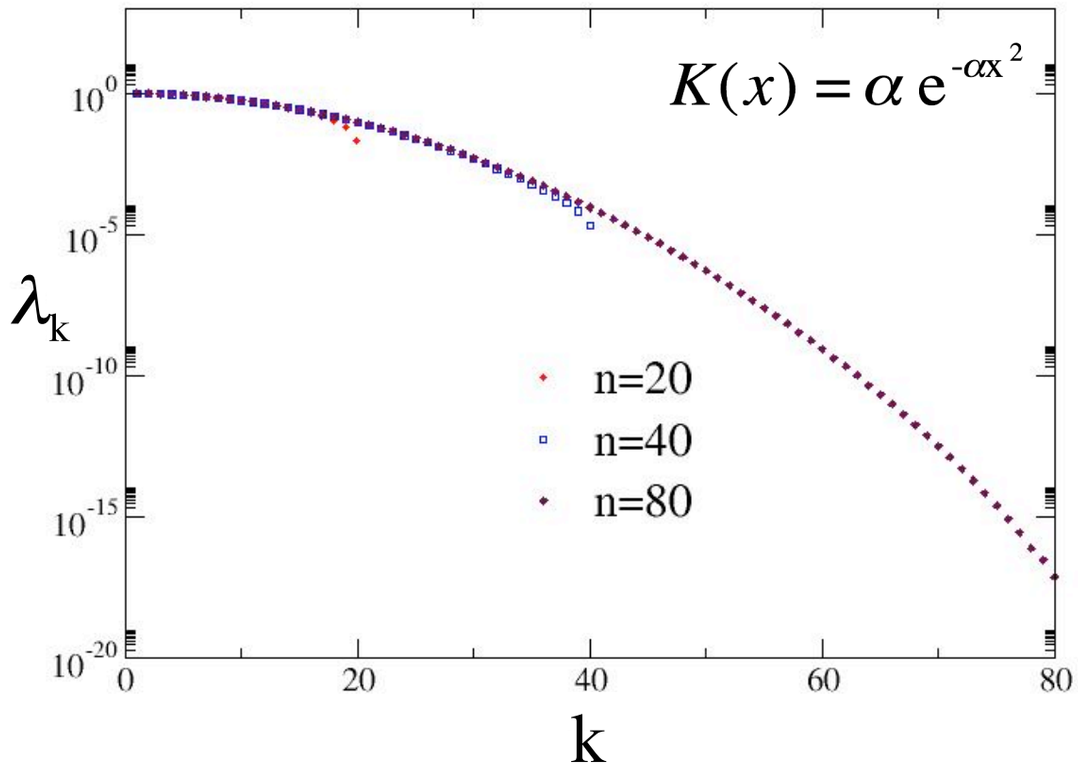
$$Fg = \Phi_L^{\infty}(\hat{\xi})$$

$$\int_{-1}^{1} K(\xi - x) \cdot g(x)\, \mathrm{d}x = u(\xi)$$

Tikhonov regularization

$$\min_{g \in L^2(\Omega_d)} \left\{ \| Fg - \Phi_L^{\infty} \|_{L^2(\Omega_{\hat{\xi}})} + \alpha \| g \|_{L^2(\Omega_d)} \right\}$$





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Tikhonov regularization

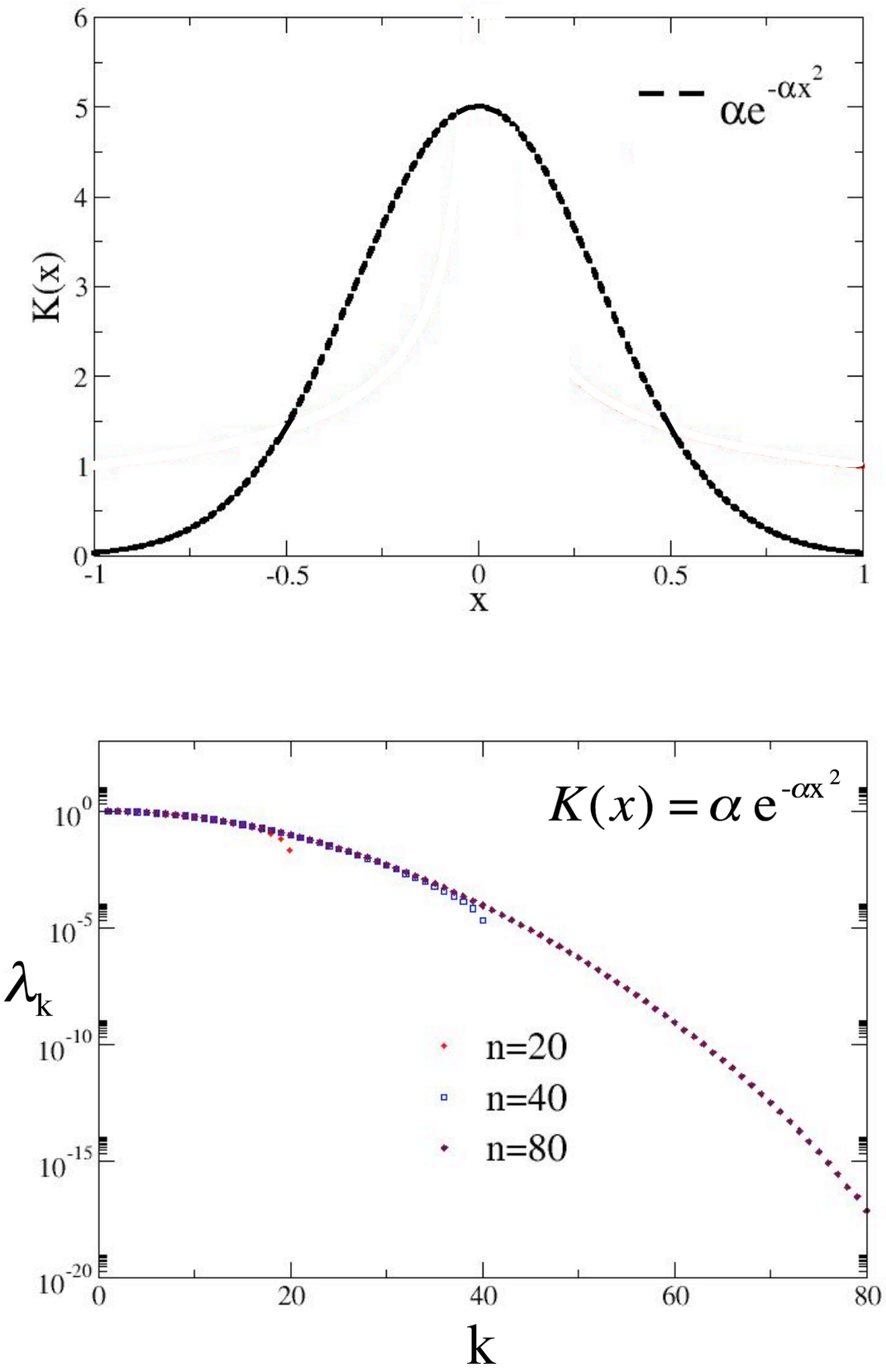
$$\min_{g \in L^2(\Omega_d)} \left\{ \| Fg - \Phi_L^{\infty} \|_{L^2(\Omega_{\hat{\xi}})} + \alpha \| g \|_{L^2(\Omega_d)} \right\}$$

Morozov discrepancy principle

$$\min_{g^{\alpha}} \left\{ \| g^{\alpha} \|_{L^2(\Omega_d)} : \ \| Fg^{\alpha} - \Phi_L^{\infty} \|_{L^2(\Omega_{\hat{\xi}})} \leqslant \delta \right\}$$

measurement noise

$$\lim_{\delta \to 0} \| g^{\delta} \|_{L^2(\Omega_d)} = \infty$$



Problem

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$

scattered field $\quad \mathbb{R}^3$

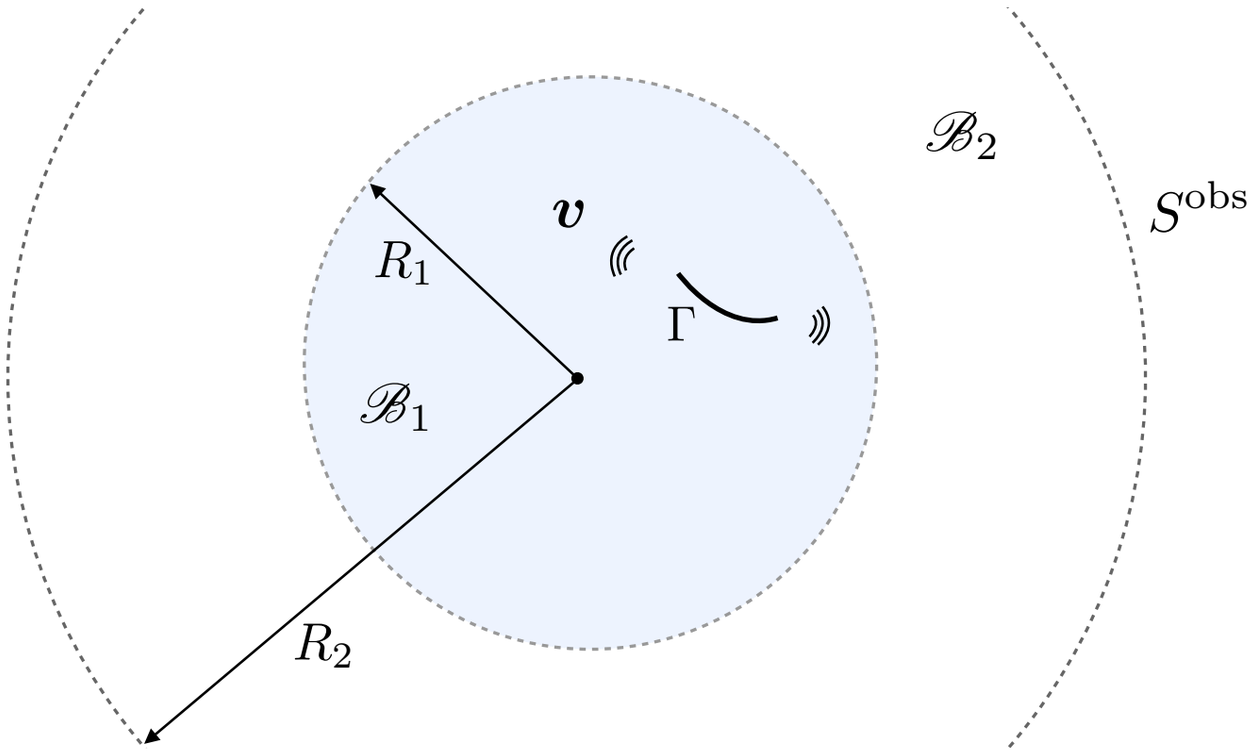
$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) \;+\; \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) \;=\; 0, \qquad \text{away from } \Gamma$$

$$\boldsymbol{n}\cdot\boldsymbol{C}:\nabla\boldsymbol{v} \;=\; \boldsymbol{K}(\boldsymbol{\xi})[\![\boldsymbol{v}]\!] \;-\; \boldsymbol{t}^f, \qquad \Gamma \text{ interface}$$

COD

Kupradze - Sommerfeld

$R_2 \to \infty$



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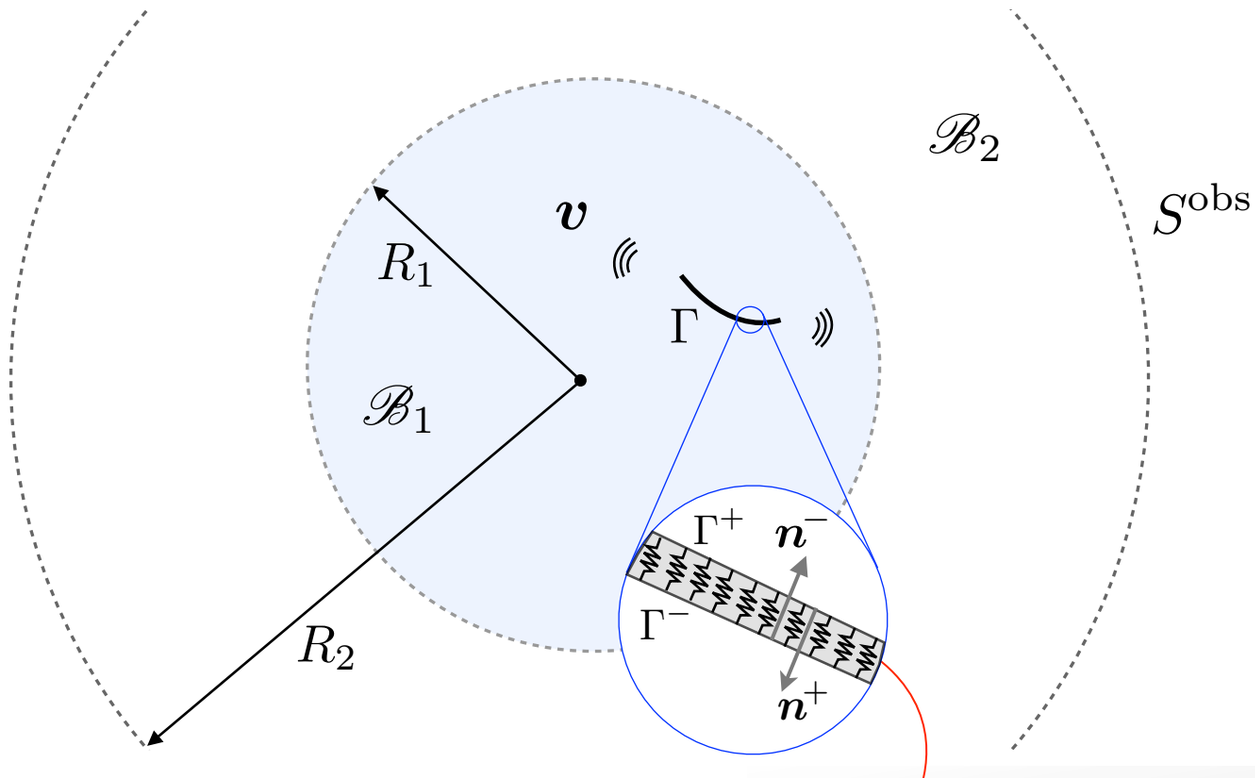
COD

Kupradze - Sommerfeld $\qquad R_2 \to \infty$



$\boldsymbol{K}(\boldsymbol{\xi})$

- heterogeneous
- complex
- symmetric

(reciprocity)

$\Im(\boldsymbol{K}) \leqslant 0,$

(uniqueness)

Problem

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scattered field $\quad \mathbb{R}^3$

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) \;+\; \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) \;=\; 0, \qquad \text{away from } \Gamma$$

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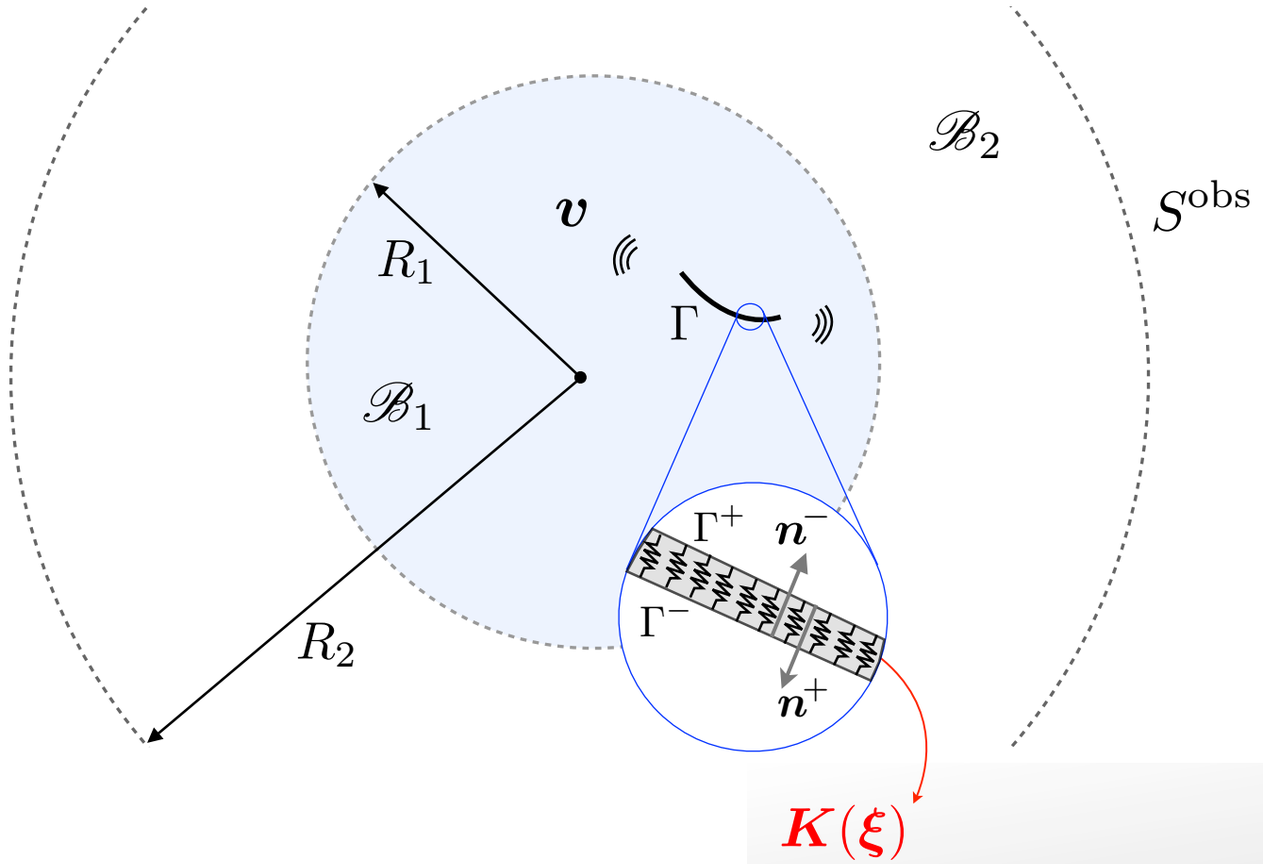
COD

Kupradze - Sommerfeld

$$R_2 \to \infty$$

dimensional platform $\quad \rho,\; c=\sqrt{\dfrac{\mu}{\rho}},\; R_1$



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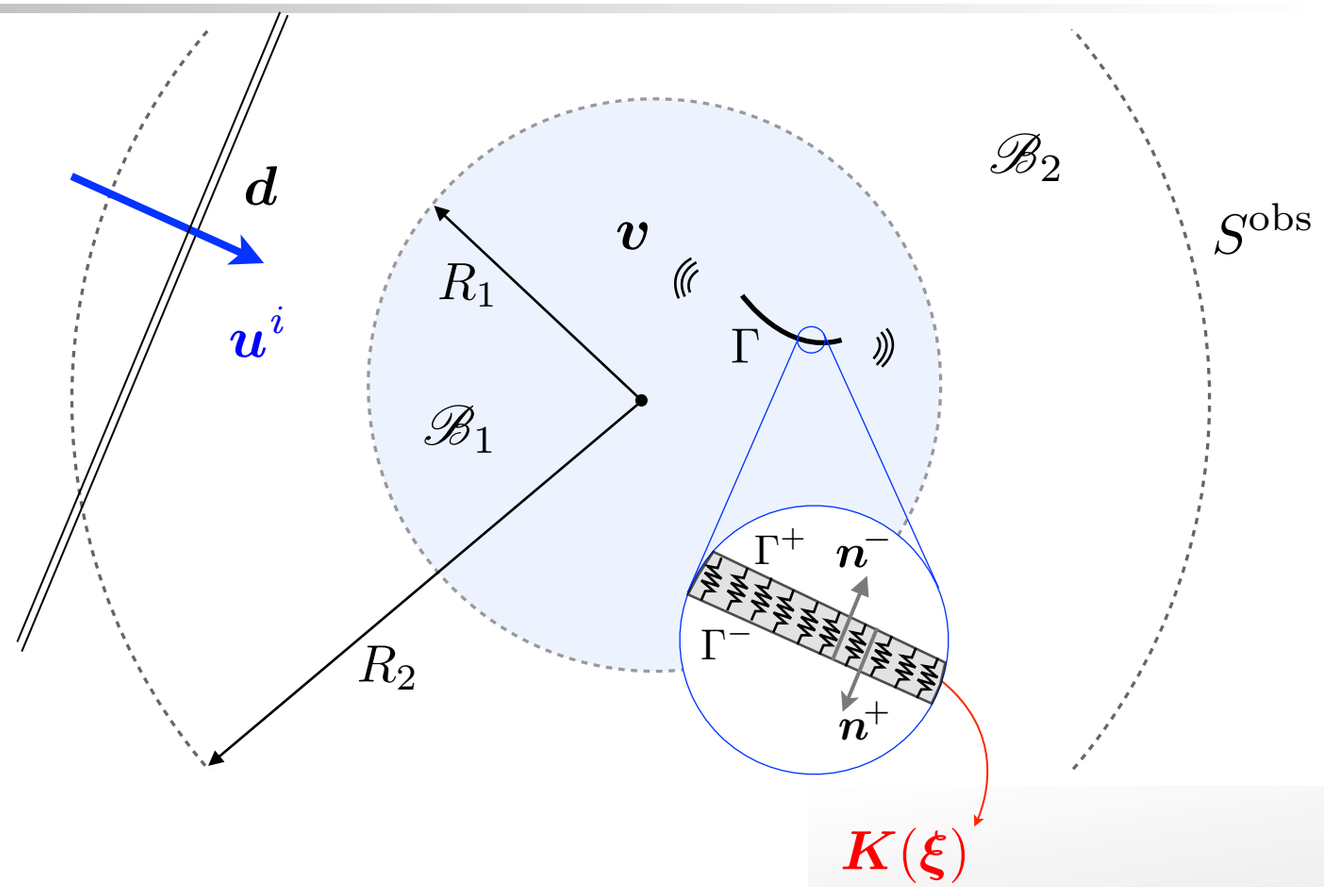
COD

Kupradze - Sommerfeld

$R_2 \to \infty$

dimensional platform $\rho,\ c = \sqrt{\dfrac{\mu}{\rho}},\ R_1$



elastic Herglotz wave function

$$\boldsymbol{u}^i(\boldsymbol{\xi}) = \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_p(\boldsymbol{d})\, e^{\mathrm{i}k_p\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}} + \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_s(\boldsymbol{d})\, e^{\mathrm{i}k_s\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}}$$

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$$Fg = \Phi_L^\infty(\hat{\xi})$$

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$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) + \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) = 0, \quad \text{away from } \Gamma$$

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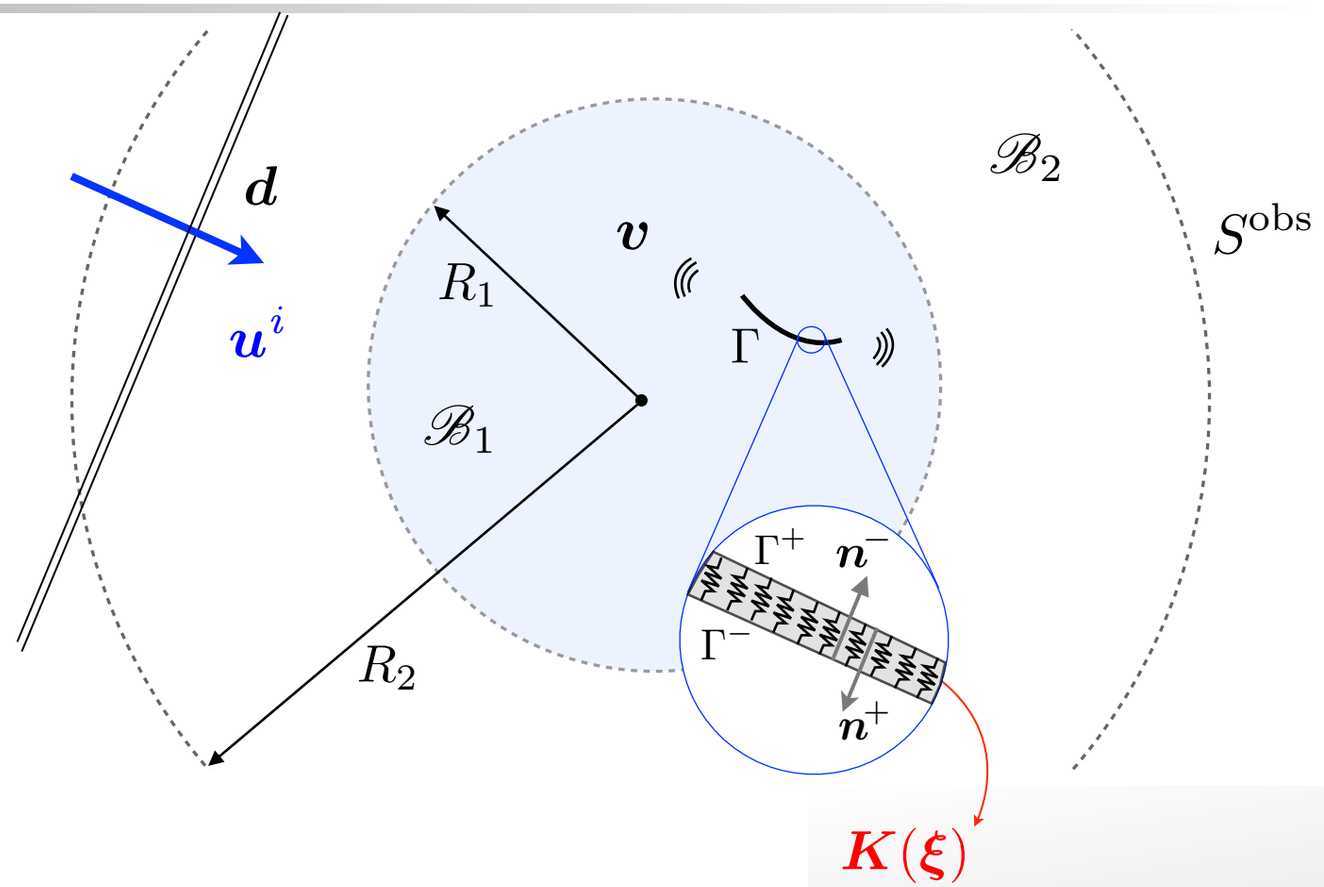
COD

Kupradze - Sommerfeld

$R_2 \to \infty$

dimensional platform $\rho,\ c = \sqrt{\frac{\mu}{\rho}},\ R_1$



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GLSM cost functional

$$J_\alpha^\delta = \alpha\left(|(F_\sharp^\delta \boldsymbol{g}, \boldsymbol{g})| + \delta\,\|\boldsymbol{g}\|^2\right) + \|F^\delta\boldsymbol{g} - \boldsymbol{\Phi}_L^\infty\|^2$$

$\boldsymbol{K}(\boldsymbol{\xi})$

- heterogeneous
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(reciprocity)

$\Im(\boldsymbol{K}) \leqslant 0,$

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Problem

$$Fg = \Phi_L^\infty(\hat{\xi})$$

scattered field $\mathbb{R}^3$

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) + \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) = 0, \quad \text{away from } \Gamma$$

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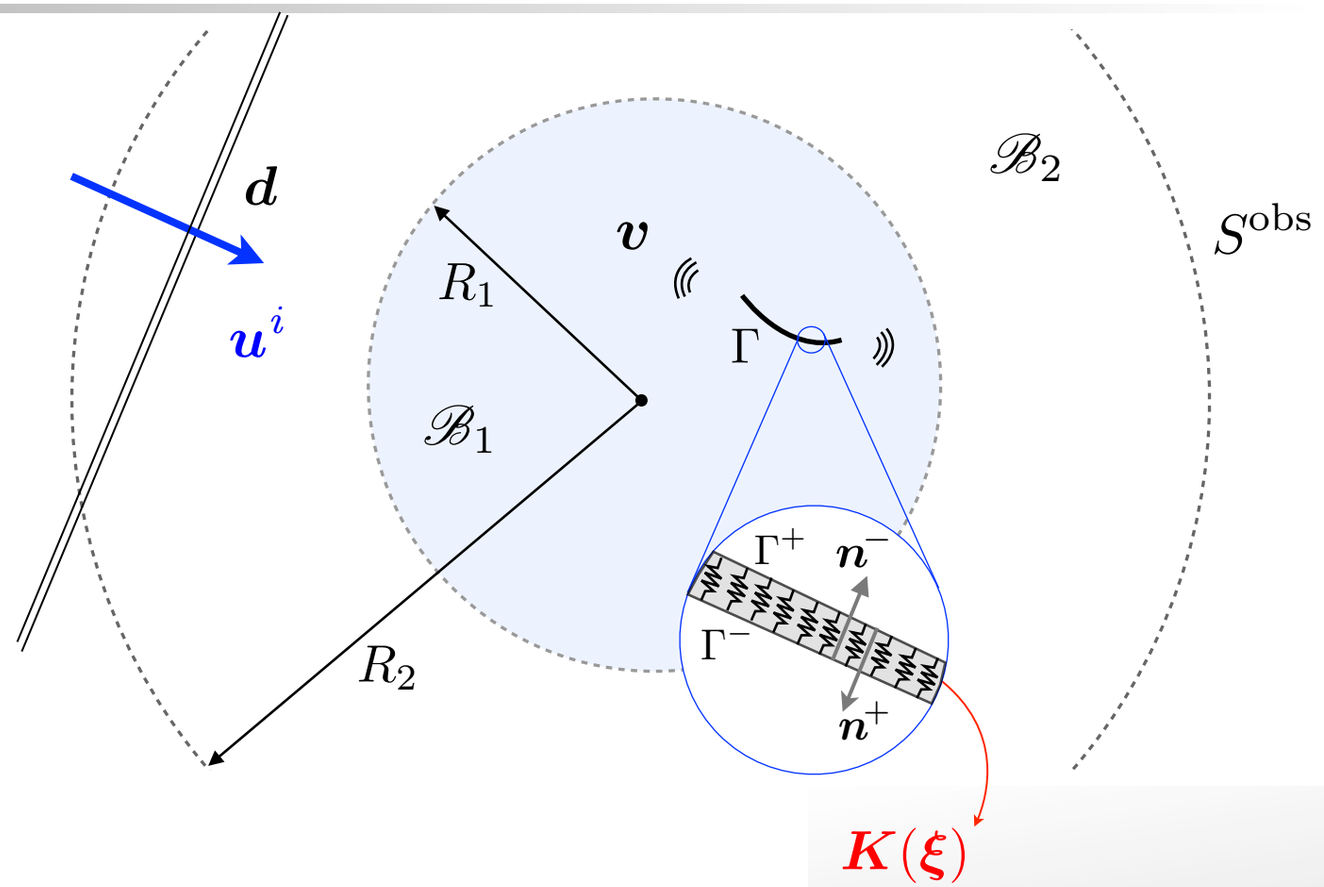
COD

Kupradze - Sommerfeld

$R_2 \to \infty$

dimensional platform $\rho,\ c = \sqrt{\dfrac{\mu}{\rho}},\ R_1$



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$$\boldsymbol{u}^i(\boldsymbol{\xi}) = \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_p(\boldsymbol{d})\, e^{\mathrm{i}k_p\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}} + \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_s(\boldsymbol{d})\, e^{\mathrm{i}k_s\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}}$$

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GLSM cost functional

noisy operator

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$$F_\sharp^\delta = |\Re(F^\delta)| + \Im(F^\delta)$$

Problem

$$Fg = \Phi_L^\infty(\hat{\xi})$$

scattered field $\mathbb{R}^3$

$$\nabla\cdot(\boldsymbol{C}:\nabla\boldsymbol{v})(\boldsymbol{\xi}) + \rho\omega^2\boldsymbol{v}(\boldsymbol{\xi}) = 0, \quad \text{away from } \Gamma$$

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COD

Kupradze - Sommerfeld

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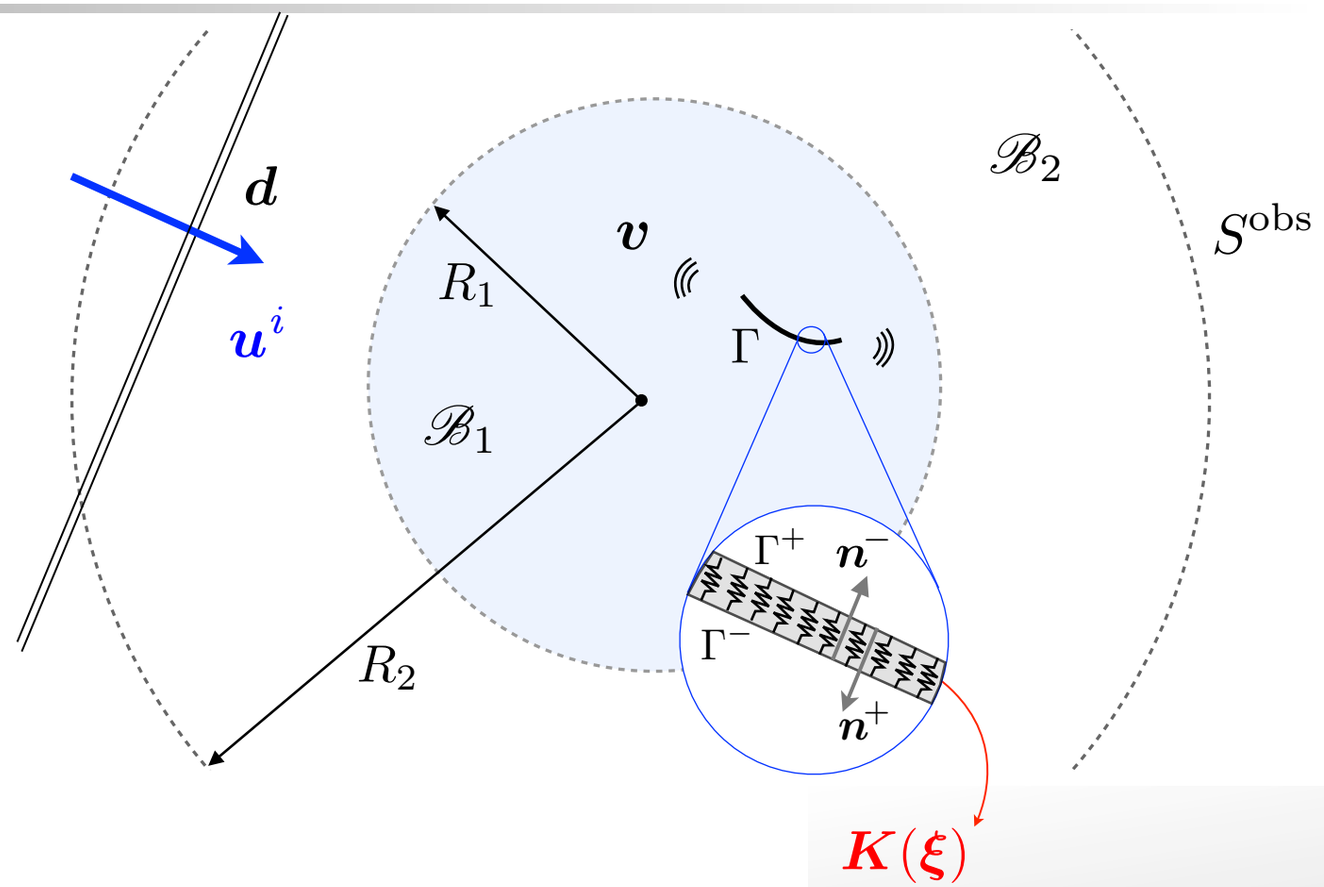
dimensional platform $\rho,\ c = \sqrt{\frac{\mu}{\rho}},\ R_1$



elastic Herglotz wave function

$$\boldsymbol{u}^i(\boldsymbol{\xi}) = \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_p(\boldsymbol{d})\, e^{\mathrm{i}k_p\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}} + \int_{\Omega_{\boldsymbol{d}}} \boldsymbol{g}_s(\boldsymbol{d})\, e^{\mathrm{i}k_s\boldsymbol{d}\cdot\boldsymbol{\xi}}\,\mathrm{d}S_{\boldsymbol{d}}$$

$\boldsymbol{K}(\boldsymbol{\xi})$

- heterogeneous
- complex
- symmetric

(reciprocity)

$\Im(\boldsymbol{K}) \leqslant 0,$

(uniqueness)

GLSM cost functional

noisy operator

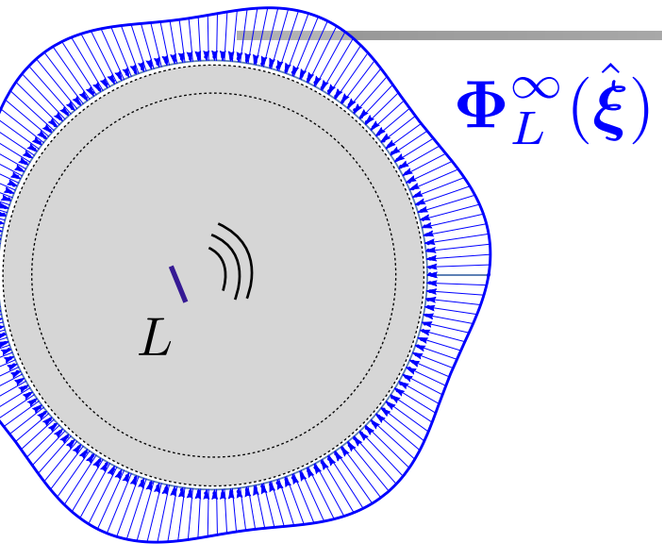
$$J_\alpha^\delta = \alpha\left(|(F_\sharp^\delta \boldsymbol{g}, \boldsymbol{g})| + \delta\,\|\boldsymbol{g}\|^2\right) + \|F^\delta\boldsymbol{g} - \boldsymbol{\Phi}_L^\infty\|^2$$

KNOWN OPTIMIZED MINIMUM $\boldsymbol{g}$

$$F_\sharp^\delta = |\Re(F^\delta)| + \Im(F^\delta)$$

Main Theorem

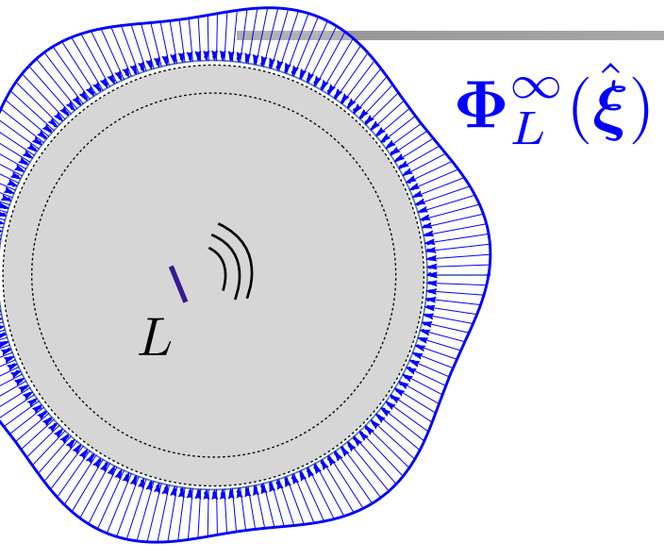
$$Fg = \mathbf{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$$



$$F = \mathscr{H}^* T \mathscr{H}$$

Main Theorem

$$Fg = \mathbf{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$$



dense range

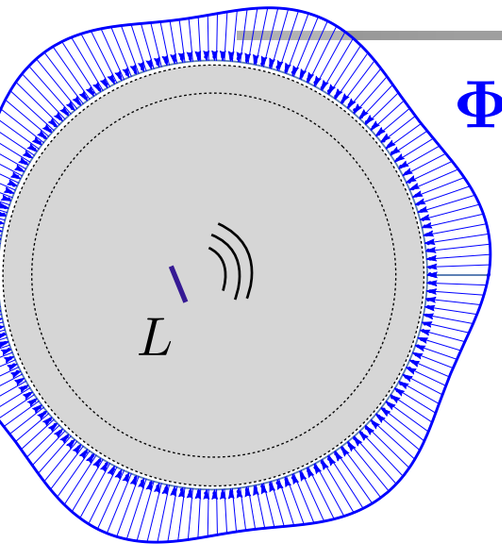
coercive

$$F = \mathscr{H}^* T \mathscr{H}$$

compact

Main Theorem

$$Fg = \mathbf{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$$



dense range

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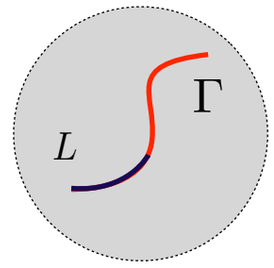
$$F = \mathscr{H}^* T \mathscr{H}$$

compact

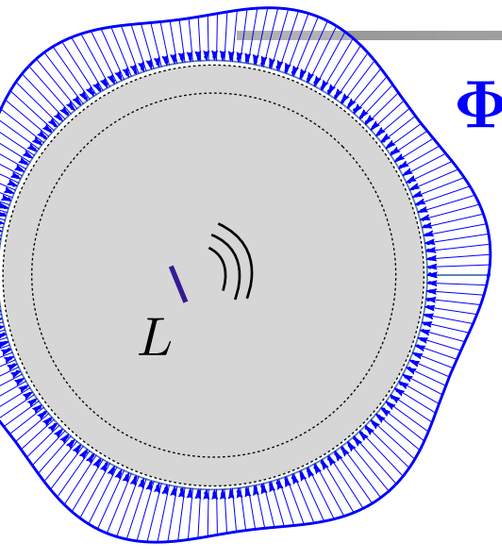
$$L \subset \Gamma \quad \longrightarrow \quad \mathbf{\Phi}_L^\infty(\hat{\boldsymbol{\xi}}) \in Range(\mathscr{H}^*)$$

$$L \not\subset \Gamma \quad \longrightarrow \quad \mathbf{\Phi}_L^\infty(\hat{\boldsymbol{\xi}}) \notin Range(\mathscr{H}^*)$$



Main Theorem

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$$



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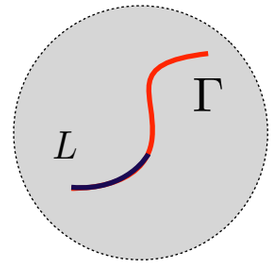
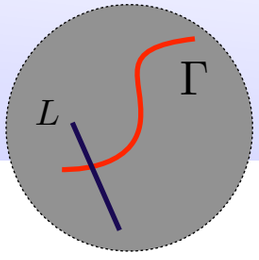


$$\limsup_{\alpha\to 0} \left\|\mathscr{H}\boldsymbol{g}\right\|_{H^{-\frac{1}{2}}(\Gamma)} < \infty \qquad L \subset \Gamma$$

$$\lim_{\alpha\to 0} \left\|\mathscr{H}\boldsymbol{g}\right\|_{H^{-\frac{1}{2}}(\Gamma)} = \infty \qquad L \not\subset \Gamma$$



Main Theorem

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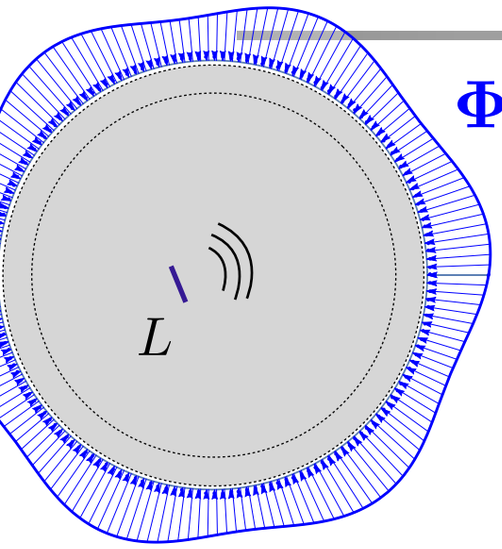

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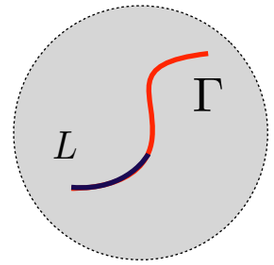

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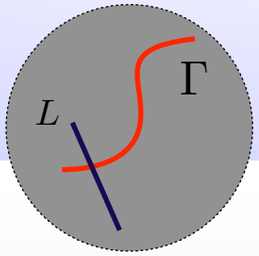


$$J_\alpha \;=\; \left\| F^\delta \boldsymbol{g} - \boldsymbol{\Phi}_L^\infty \right\|^2 + \alpha \left\| \boldsymbol{g} \right\|^2$$

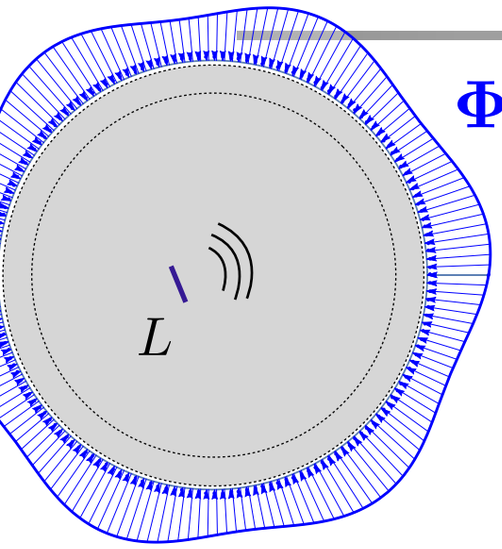
LSM

indicator functionals

$$\mathscr{I}_{\mathrm{LSM}} \;=\; 1 / \left\| \boldsymbol{g} \right\|^2_{L^2(\Omega_{\boldsymbol{d}})}$$

Main Theorem

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^\infty(\hat{\boldsymbol{\xi}})$$



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compact

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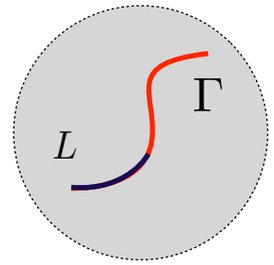

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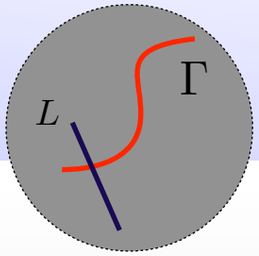

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LSM

indicator functionals

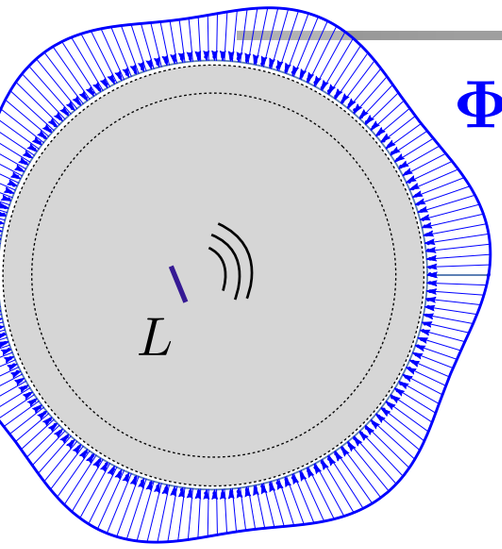
$$\mathscr{I}_{\mathrm{LSM}} \;=\; 1/\, \| \boldsymbol{g} \|^2_{L^2(\Omega_{\boldsymbol{d}})}$$

GLSM

$$\mathscr{I}_{\mathrm{GLSM}} \;=\; 1/\sqrt{\delta \, \| \boldsymbol{g} \|^2 + \left| (F_\sharp^\delta \boldsymbol{g}, \boldsymbol{g}) \right|}$$

Main Theorem

$$F\boldsymbol{g} \;=\; \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}})$$



dense range — coercive

$$F \;=\; \mathscr{H}^{*} T \mathscr{H}$$

compact

$$L \subset \Gamma \quad \longrightarrow \quad \boldsymbol{\Phi}_L^{\infty}(\hat{\boldsymbol{\xi}}) \in Range(\mathscr{H}^{*})$$

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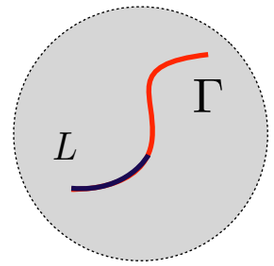

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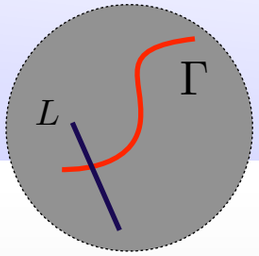


$$J_\alpha \;=\; \| F^{\delta}\boldsymbol{g} - \boldsymbol{\Phi}_L^{\infty} \|^2 + \alpha \, \| \boldsymbol{g} \|^2$$

indicator functionals

LSM

$$\mathscr{I}_{\mathrm{LSM}} \;=\; 1/\, \| \boldsymbol{g} \|^2_{L^2(\Omega_{\boldsymbol{d}})}$$

$$J_\alpha^{\delta} \;=\; \alpha\,(|(F_\sharp^{\delta}\boldsymbol{g},\boldsymbol{g})| + \delta\,\|\boldsymbol{g}\|^2) \;+\; \| F^{\delta}\boldsymbol{g} - \boldsymbol{\Phi}_L^{\infty} \|^2$$

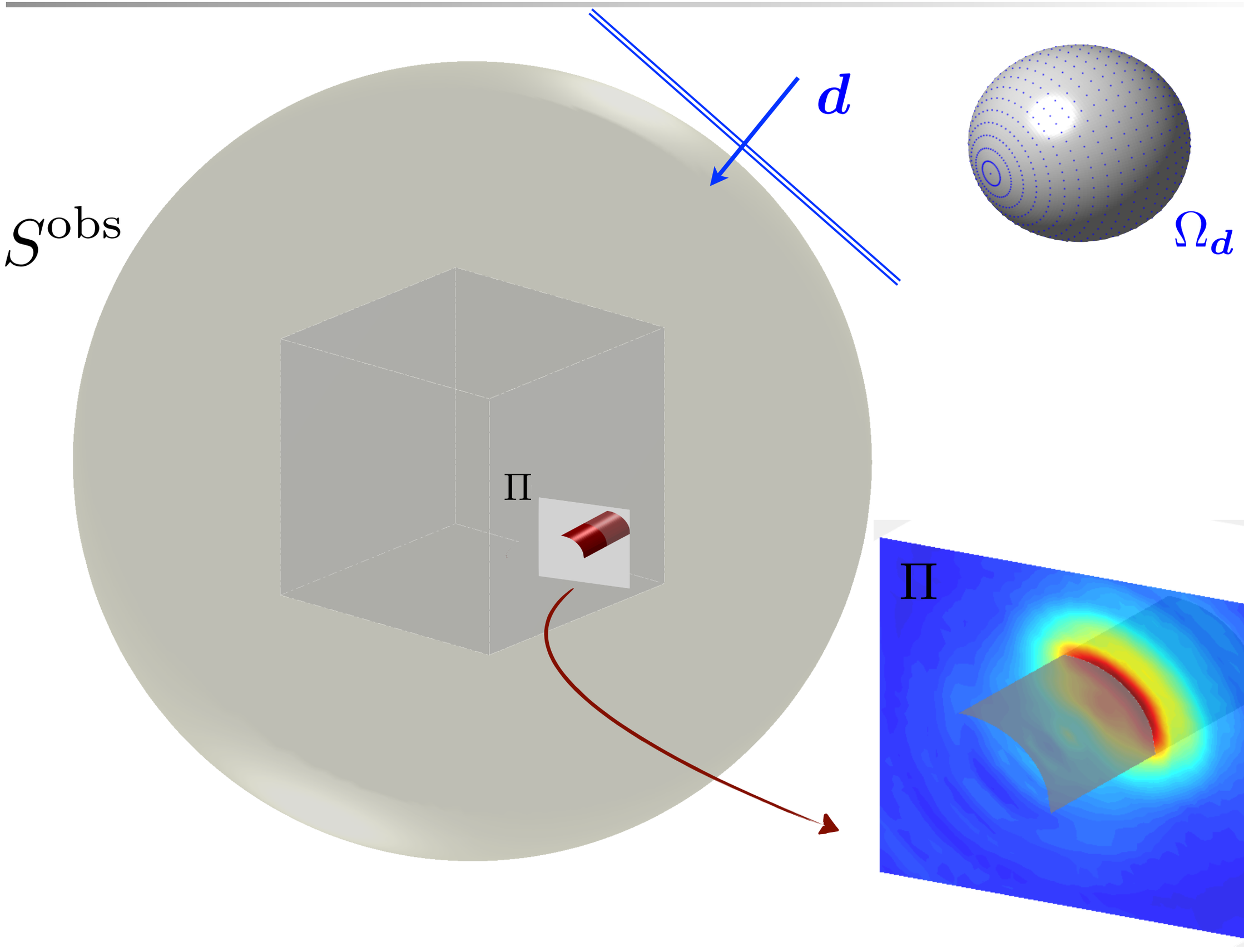
GLSM

$$\mathscr{I}_{\mathrm{GLSM}} \;=\; 1/\sqrt{\delta\,\|\boldsymbol{g}\|^2 + |(F_\sharp^{\delta}\boldsymbol{g},\boldsymbol{g})|}$$

GLSM$^{\mathrm{opt}}$

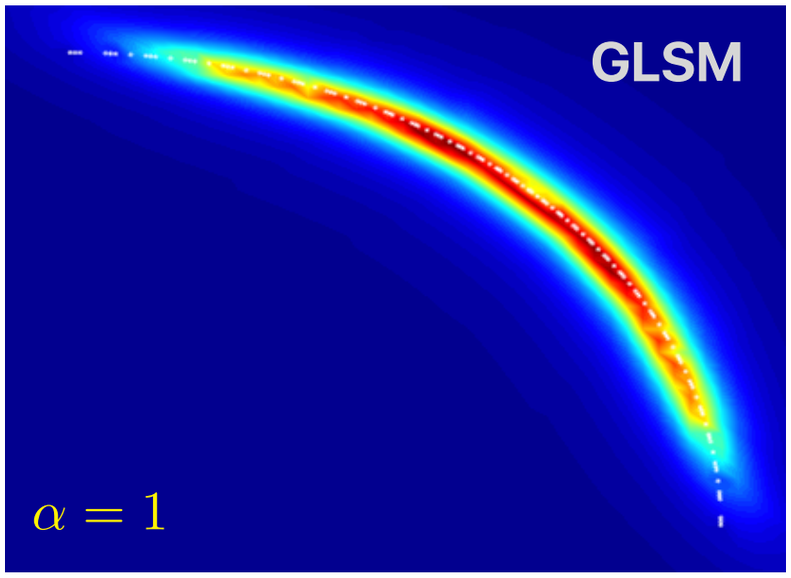
$$\mathscr{I}_{\mathrm{GLSM}} \;=\; 1/\sqrt{\delta\,\|\boldsymbol{g}\|^2 + |(F_\sharp^{\delta}\boldsymbol{g},\boldsymbol{g})|}$$

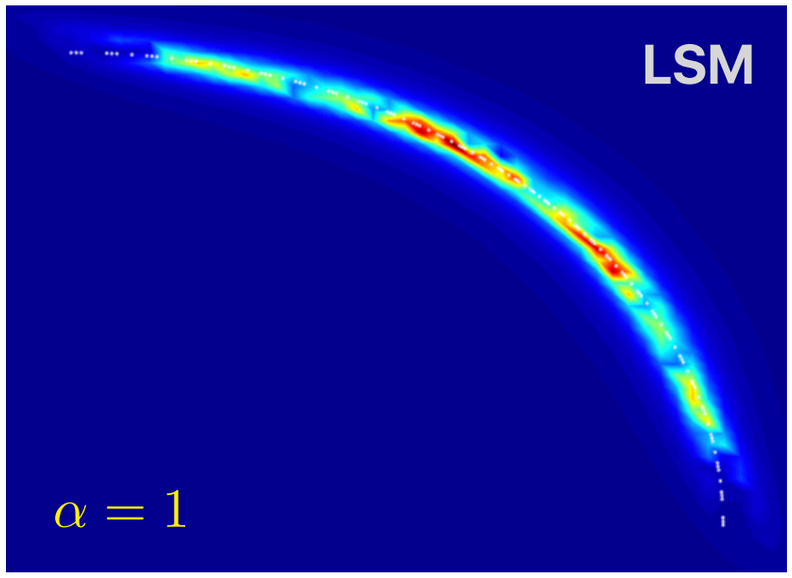
Numerical results

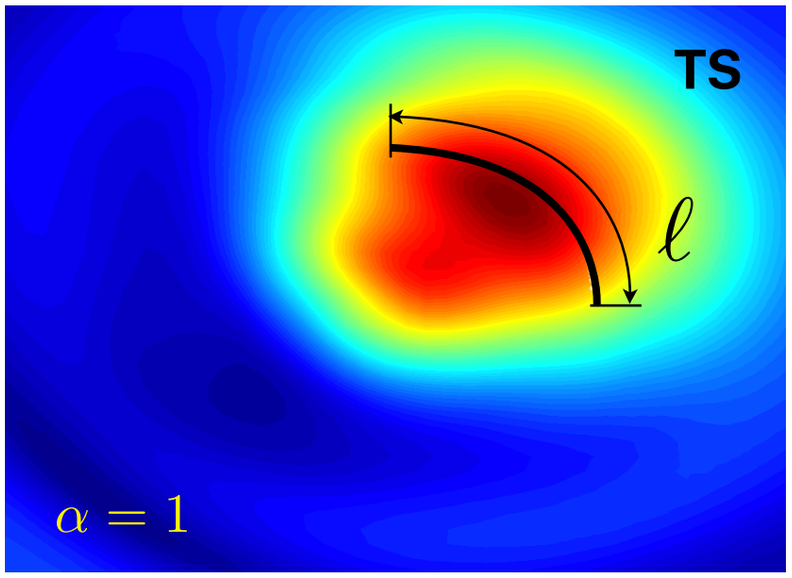


Mid-plane reconstruction

$\alpha = \ell/\lambda$

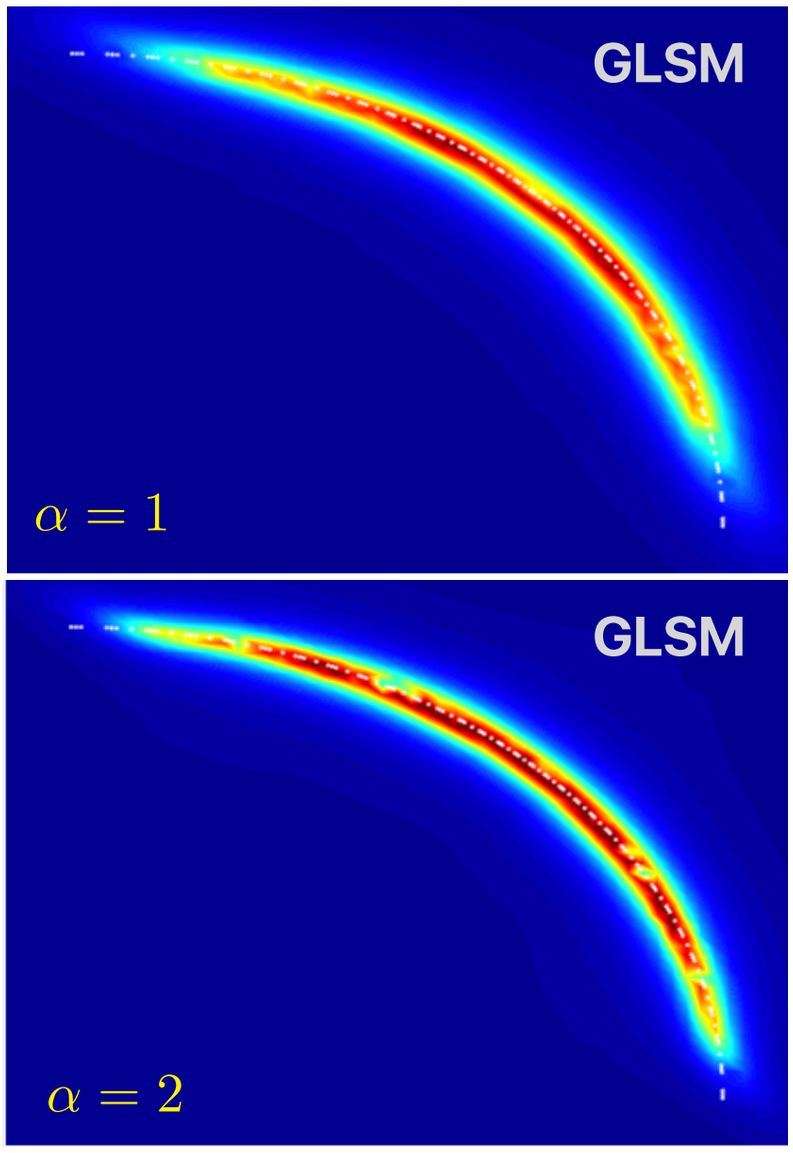


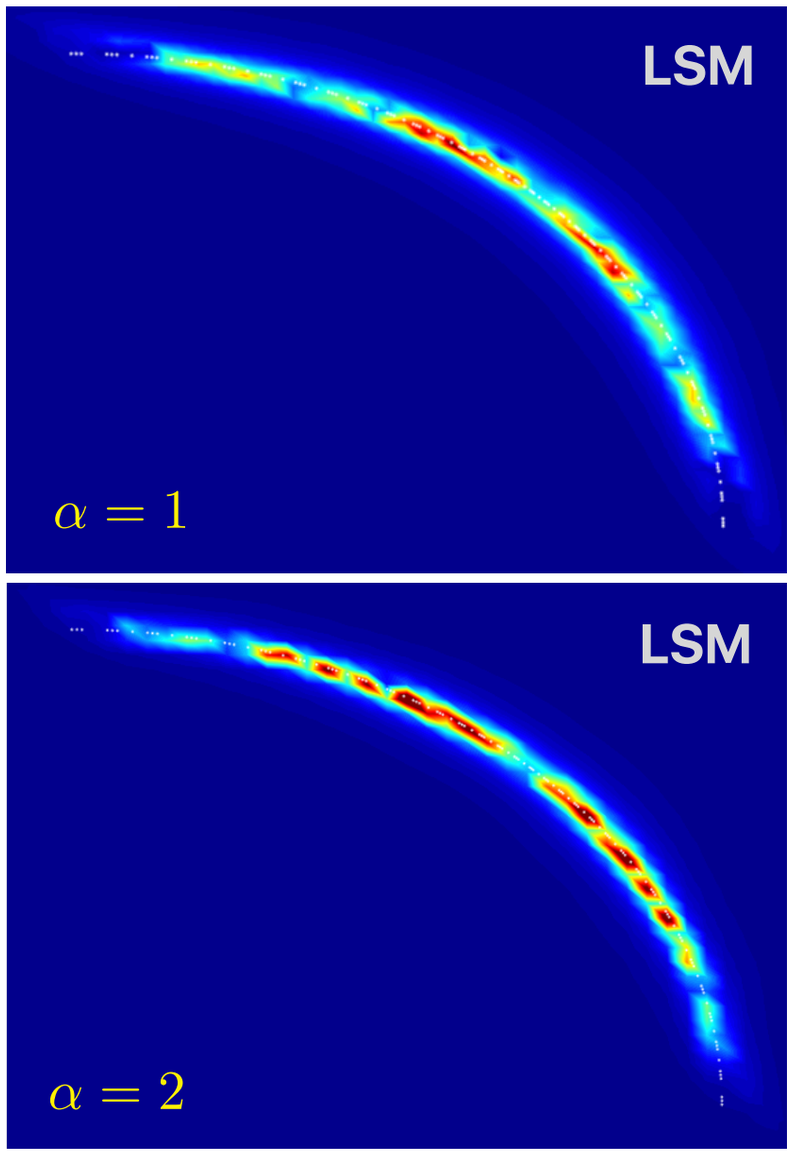


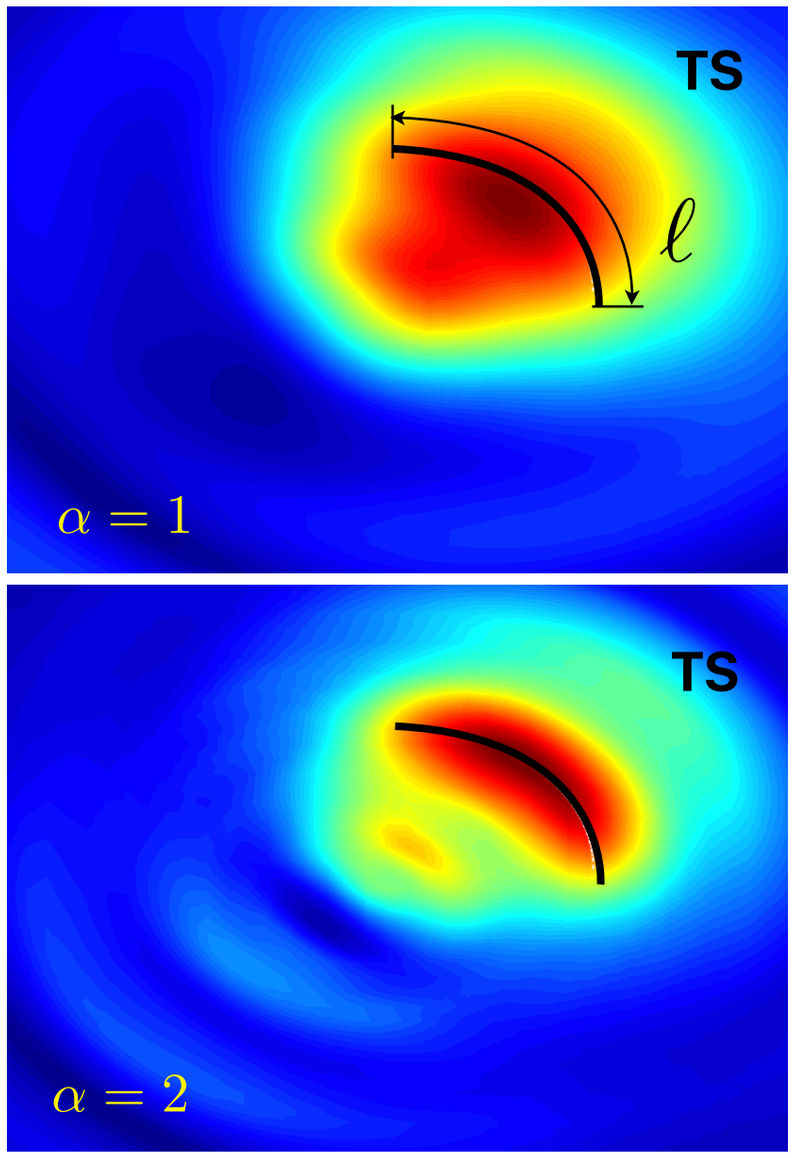


Mid-plane reconstruction

$\alpha = \ell/\lambda$





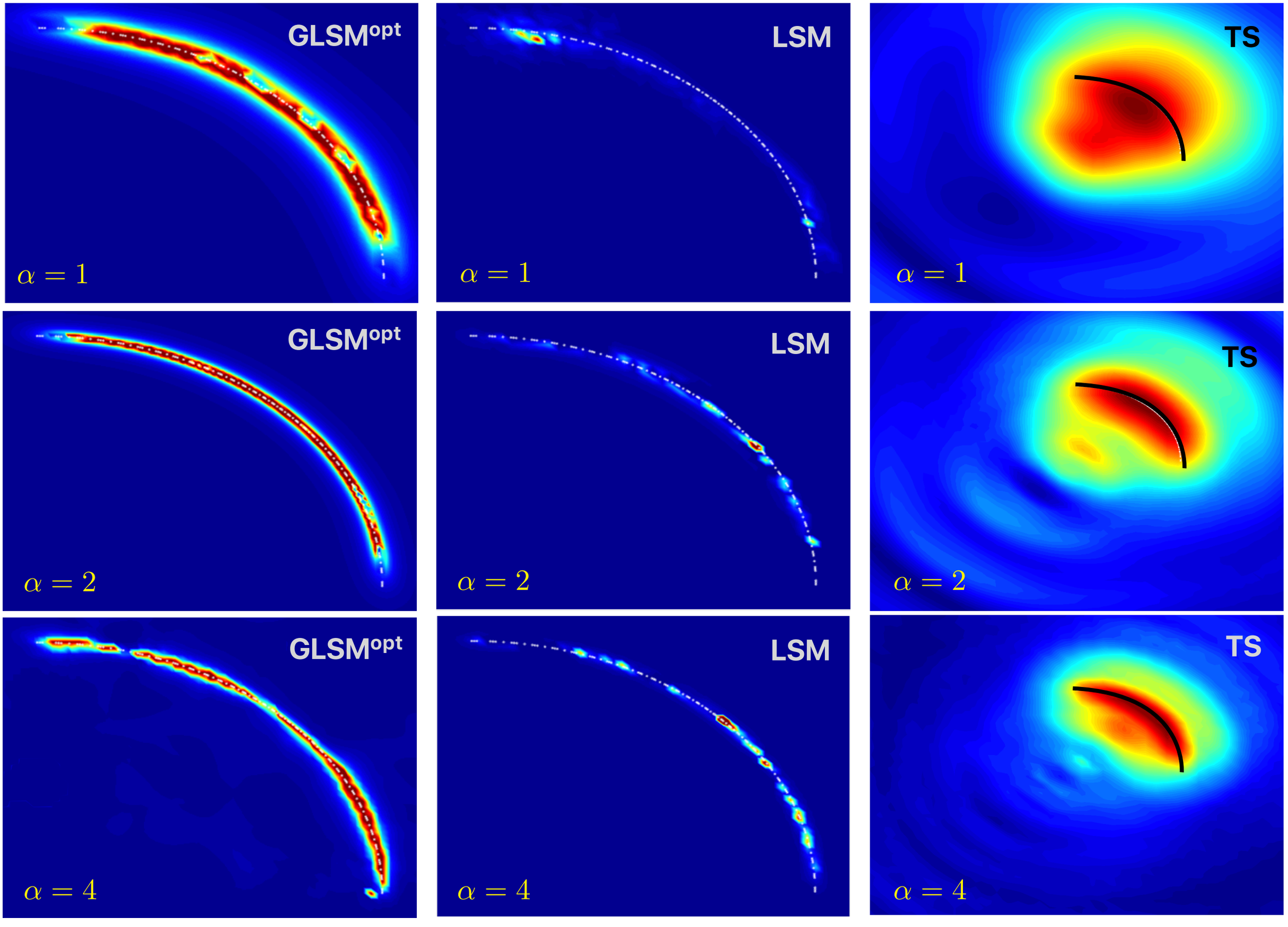


Mid-plane reconstruction

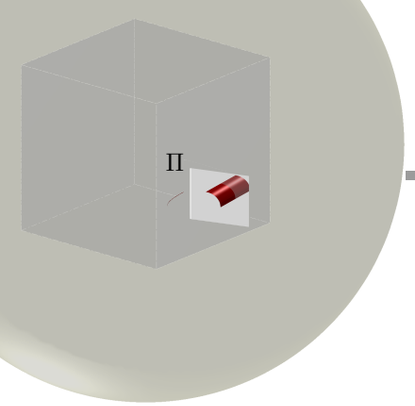
$\alpha = \ell/\lambda$

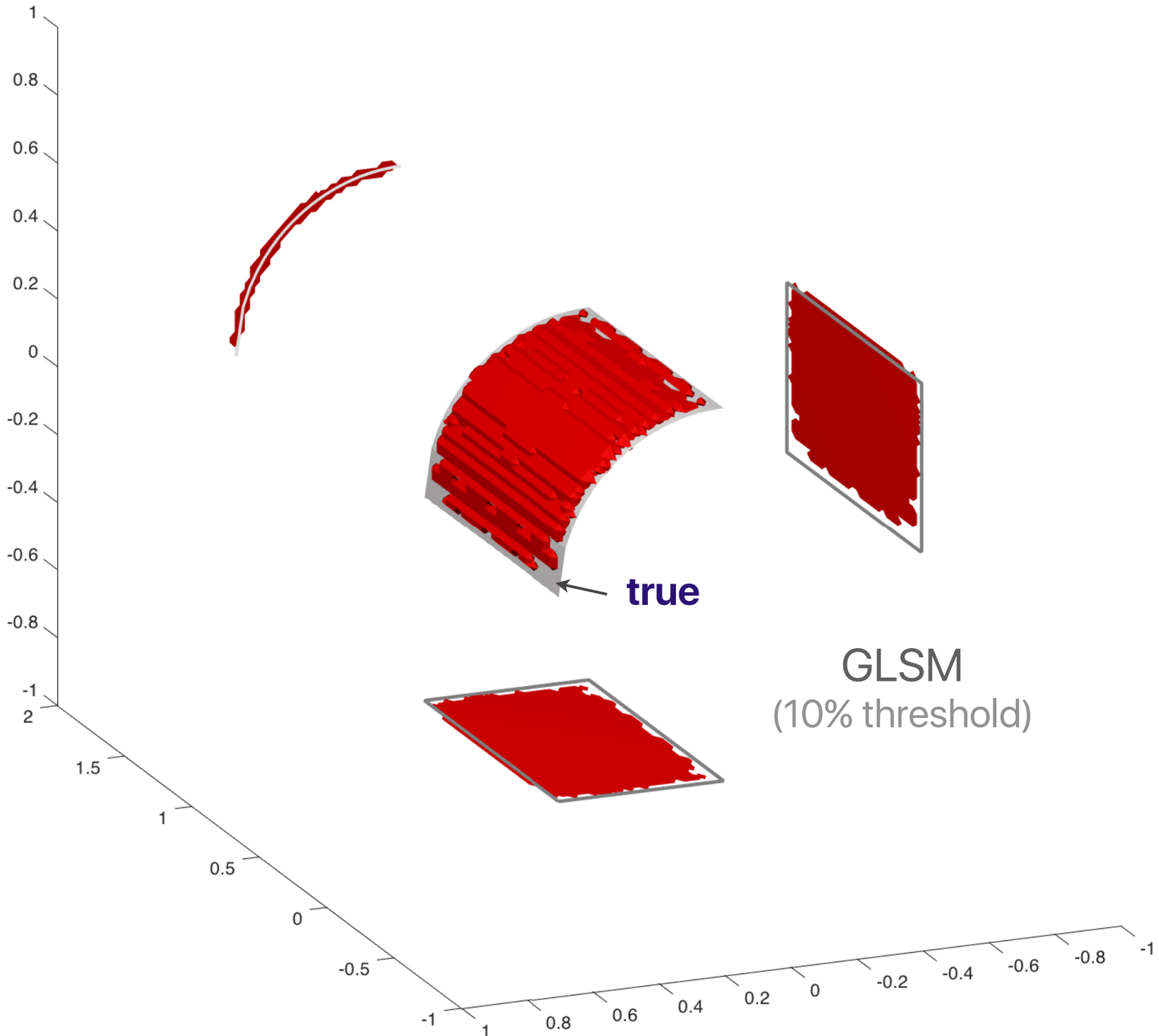
5% noise

$\alpha = \ell/\lambda$



3D Reconstruction





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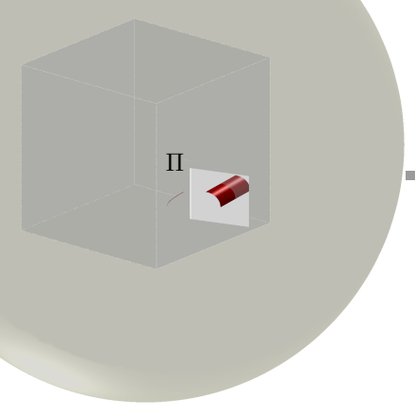
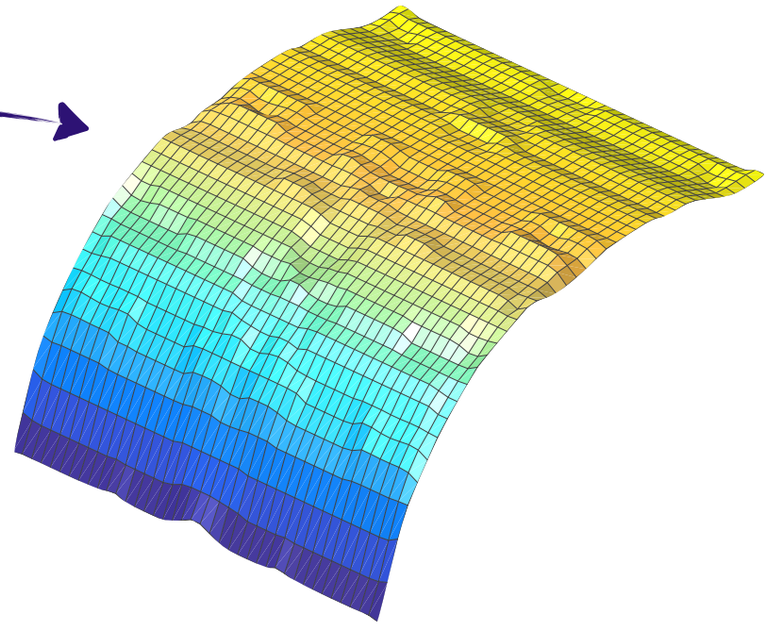
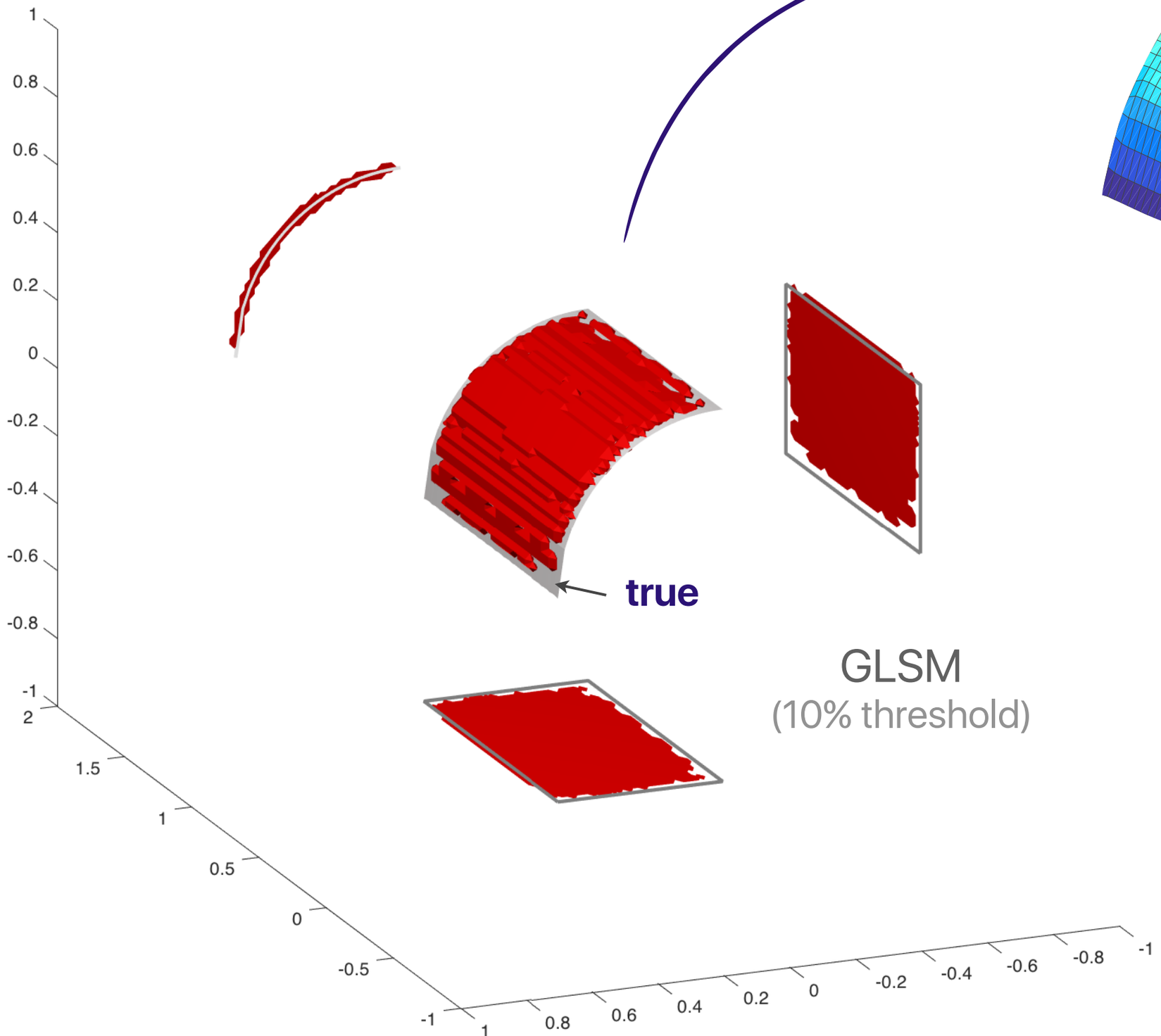


scattered
interpolant
(Matlab)

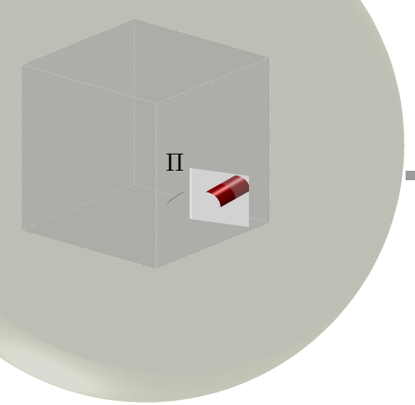




true

GLSM
(10% threshold)

3D Reconstruction



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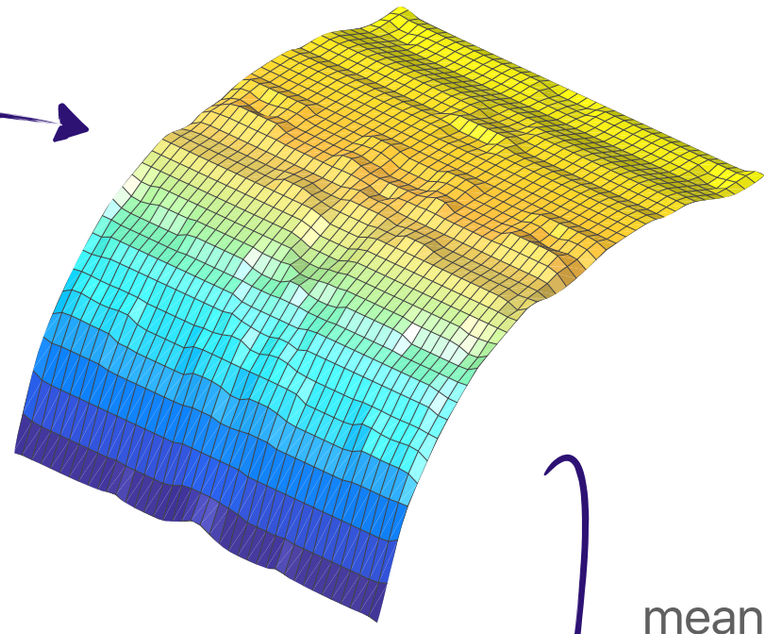
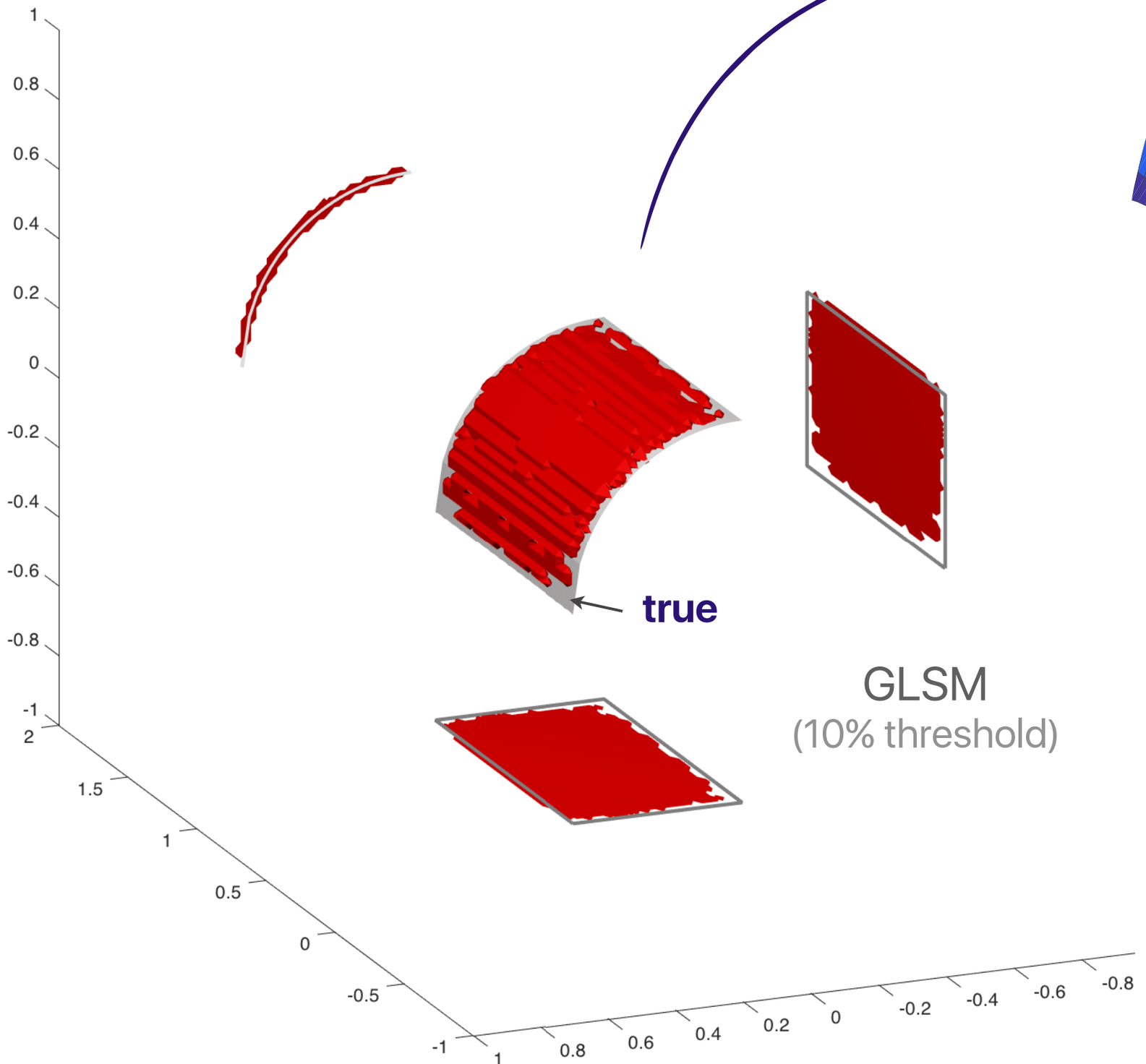


mean filter
(Matlab)



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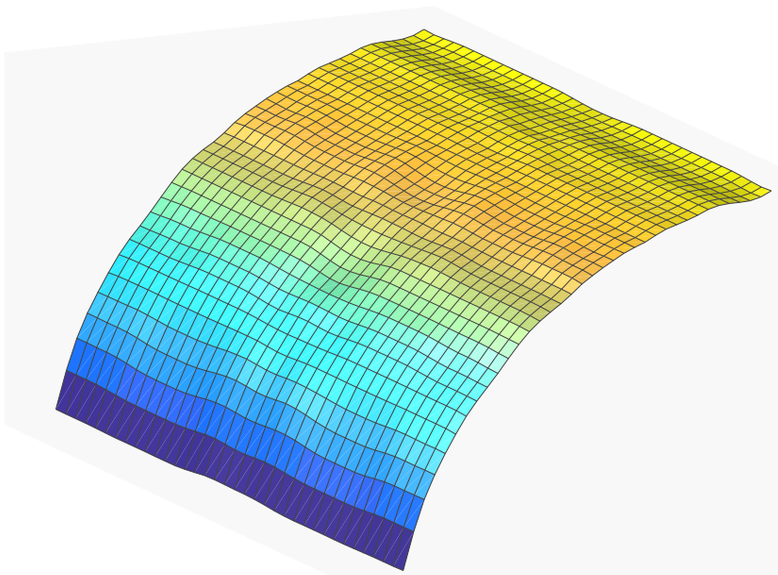
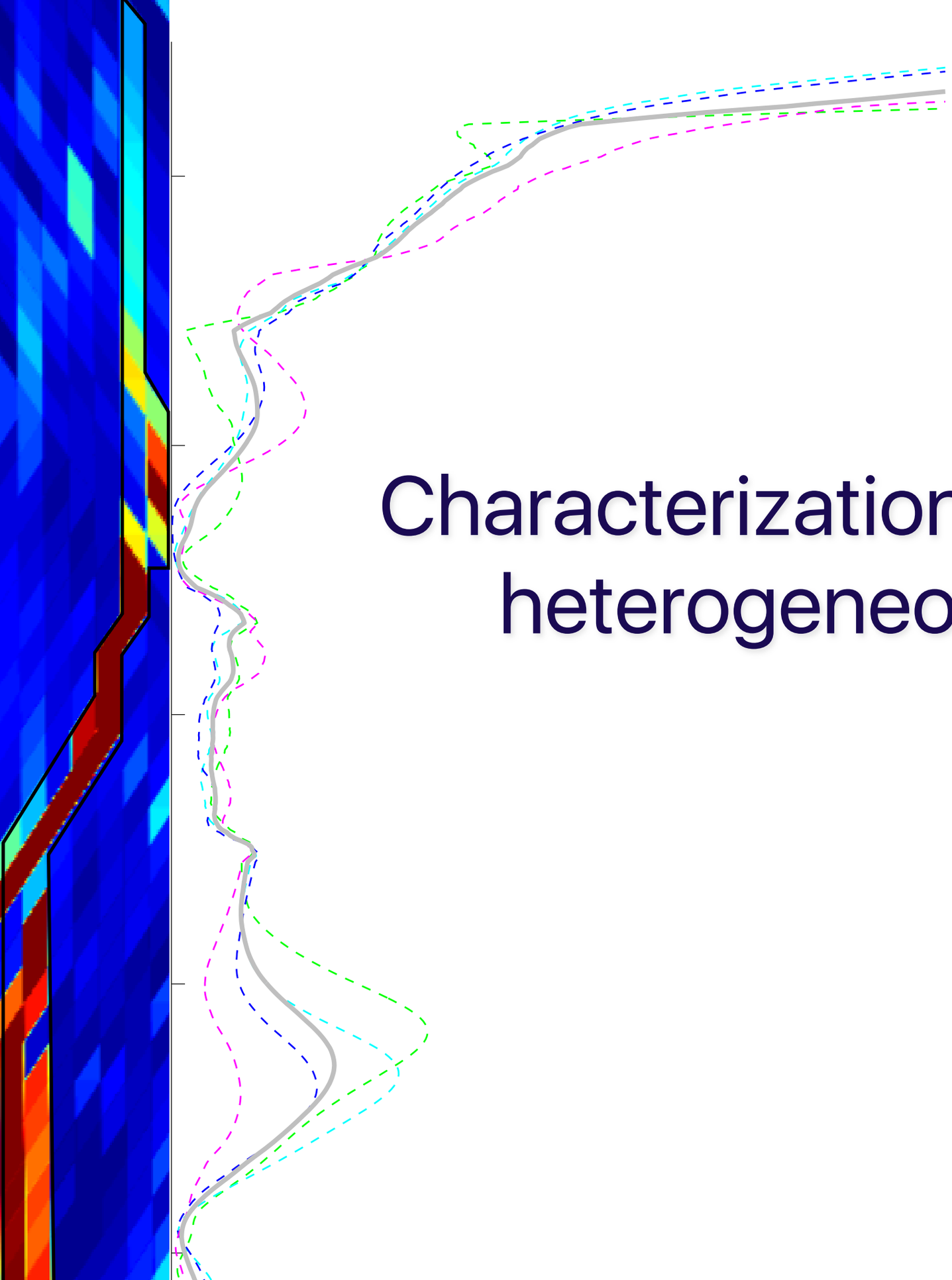
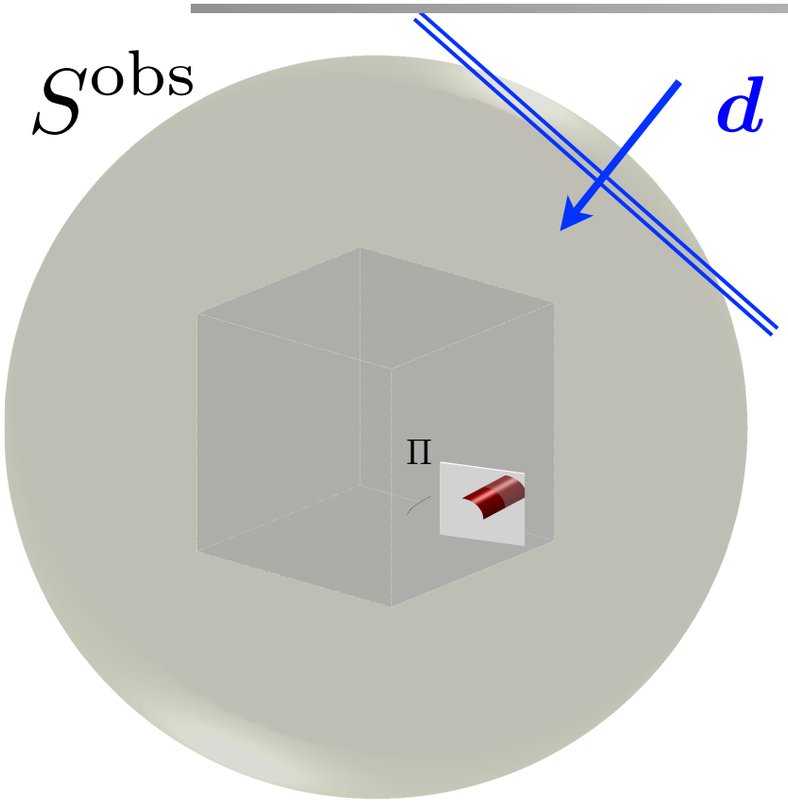


reconstructed fracture surface

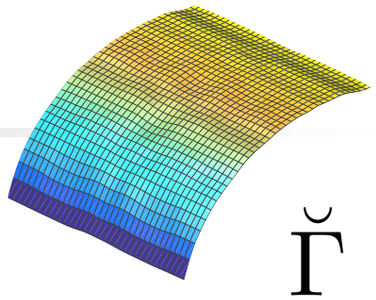


Characterization of the fracture heterogeneous interface

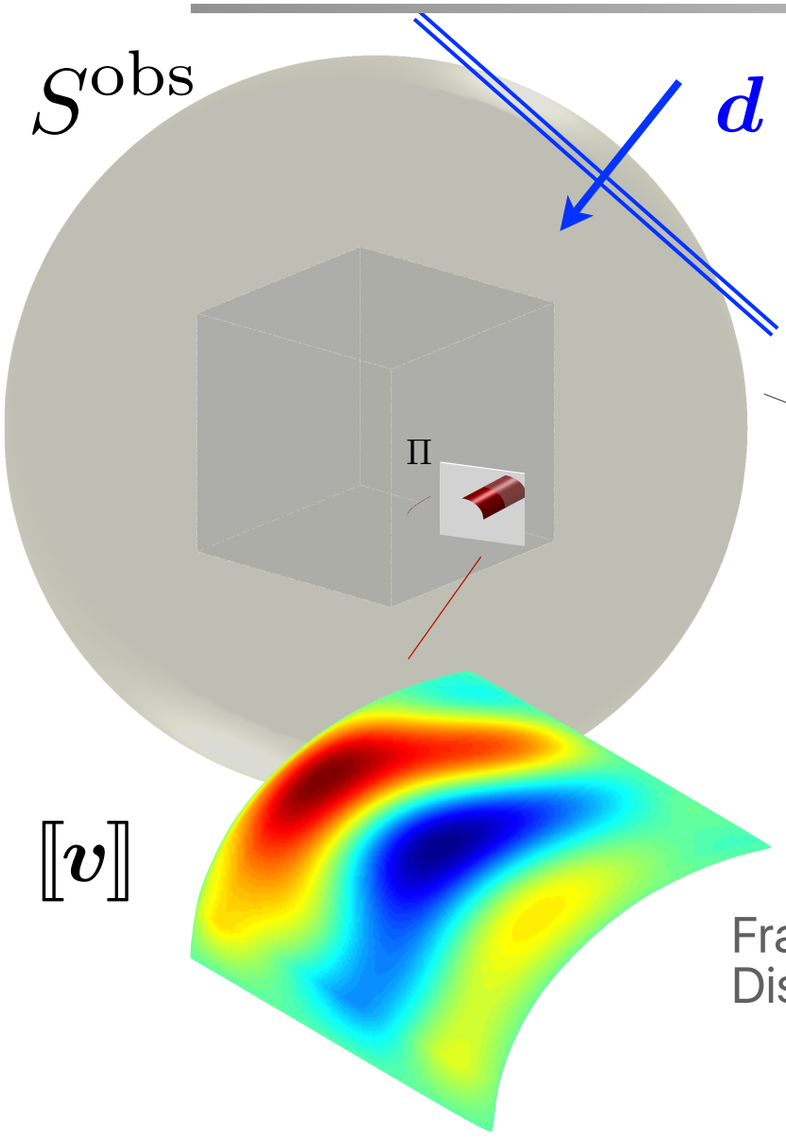
FOD Reconstruction



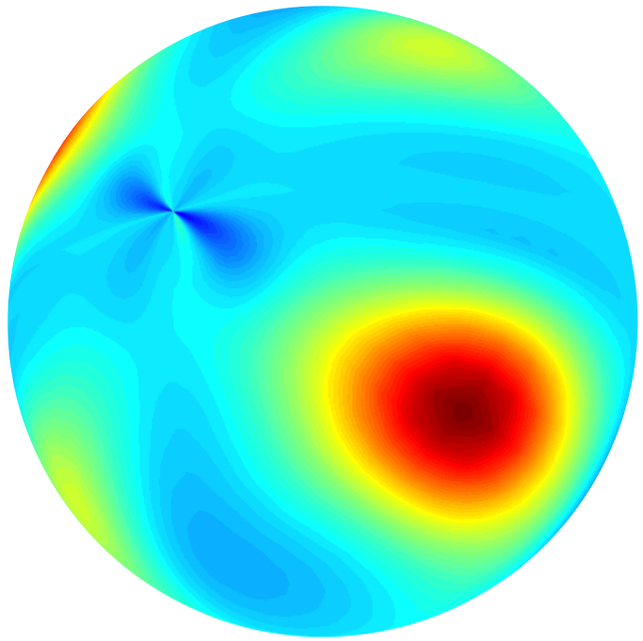
recovered
fracture surface



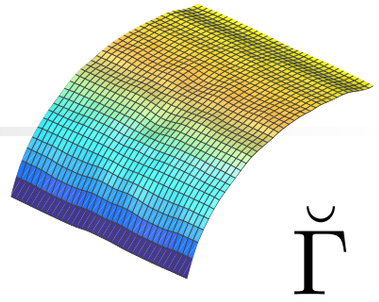
FOD Reconstruction



Fracture Opening
Displacement profile

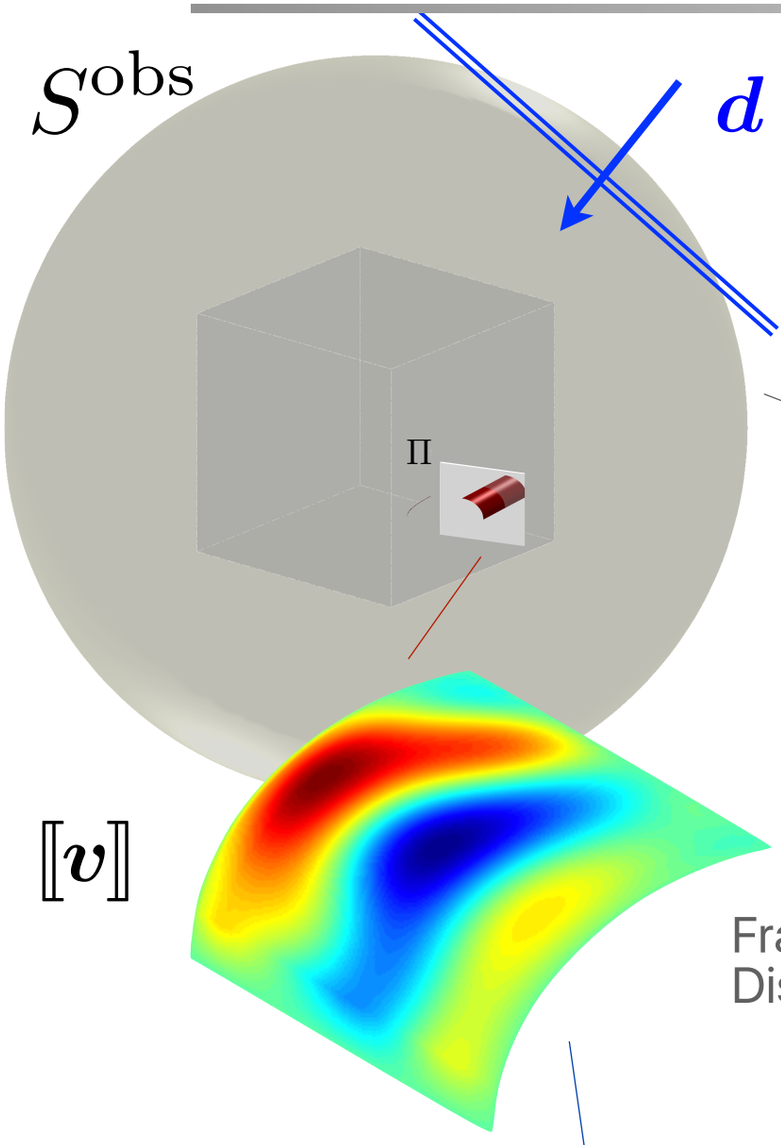


far-field pattern
of the scattered wavefield

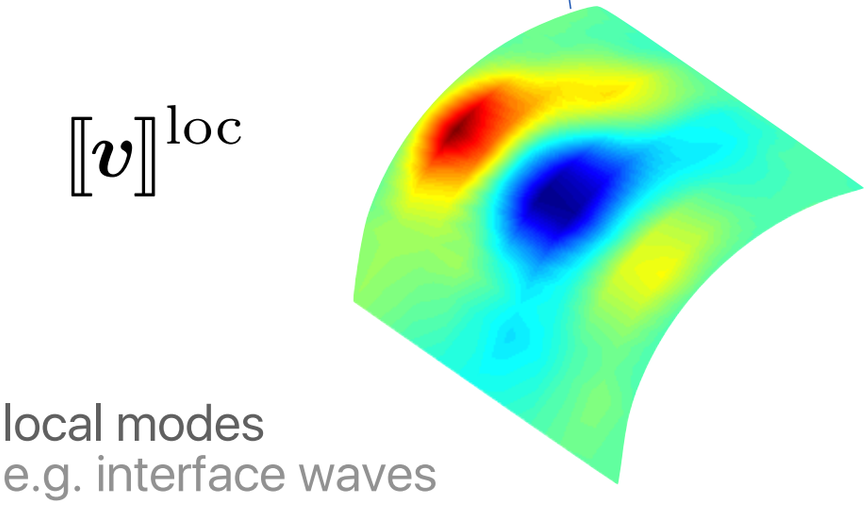


recovered
fracture surface

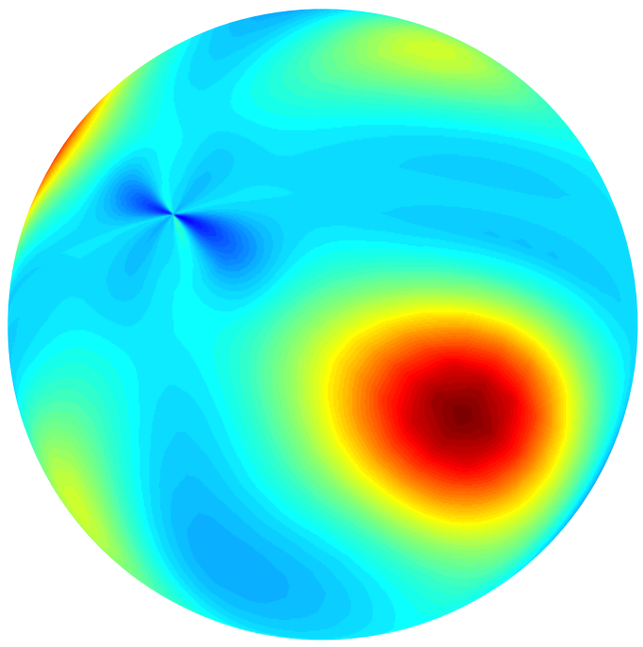
FOD Reconstruction



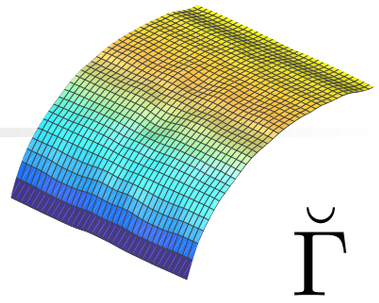
Fracture Opening
Displacement profile



local modes
e.g. interface waves

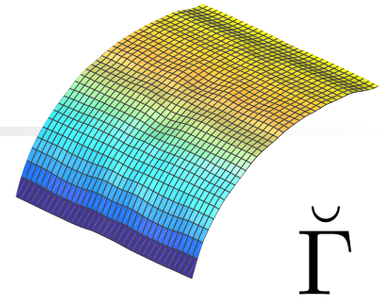


far-field pattern
of the scattered wavefield



recovered
fracture surface

FOD Reconstruction



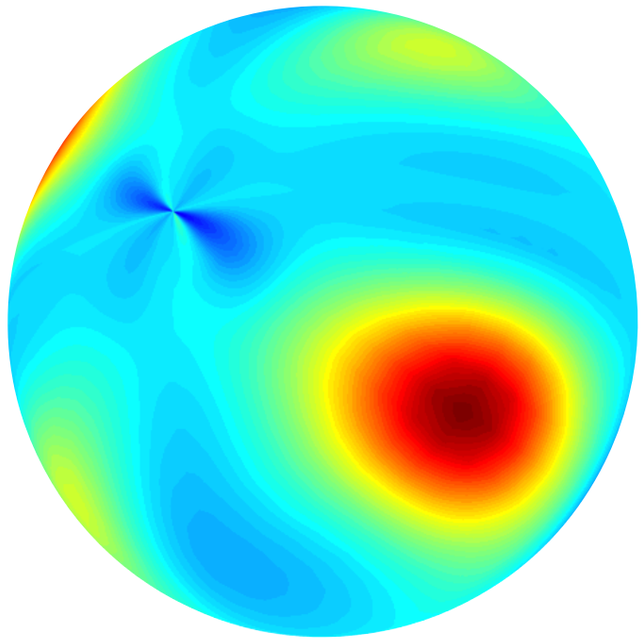
recovered
fracture surface

$\breve{\Gamma}$

S^{obs}

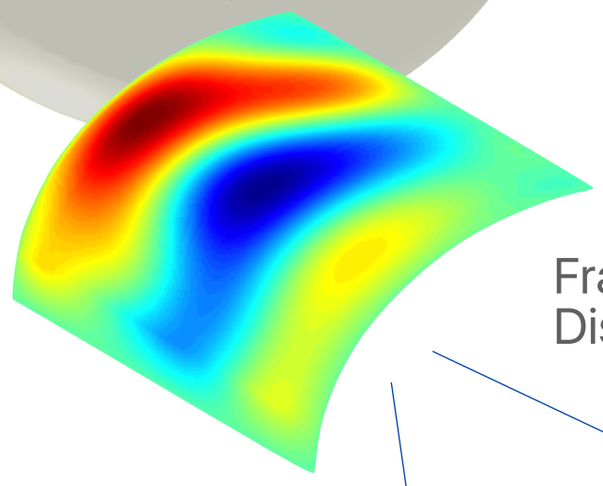
$\boldsymbol{d}$

Π



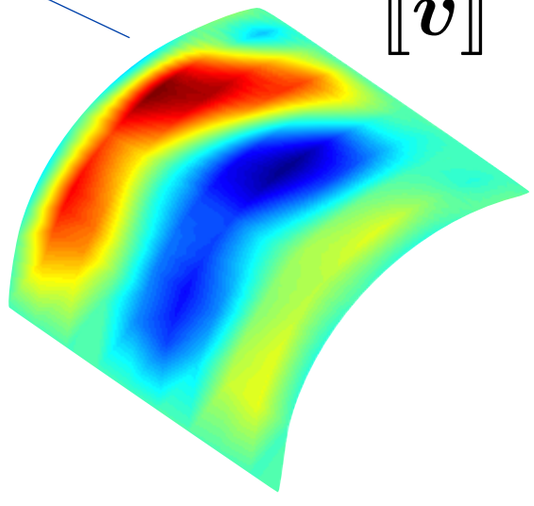
far-field pattern
of the scattered wavefield

$[\![\boldsymbol{v}]\!]$

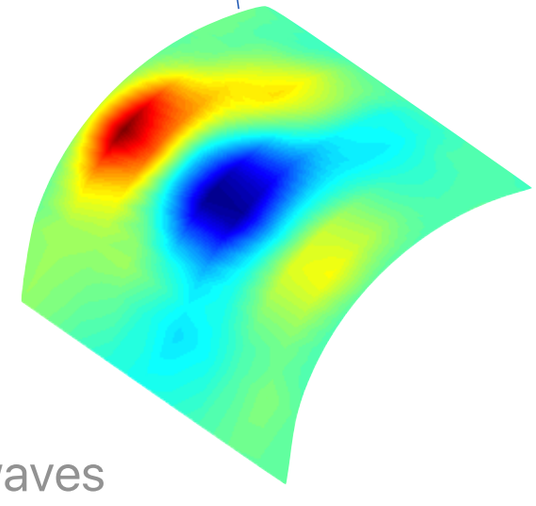


Fracture Opening
Displacement profile

$[\![\breve{\boldsymbol{v}}]\!]$

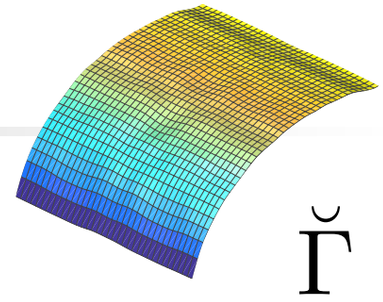


$[\![\boldsymbol{v}]\!]^{\mathrm{loc}}$



local modes
e.g. interface waves

FOD Reconstruction



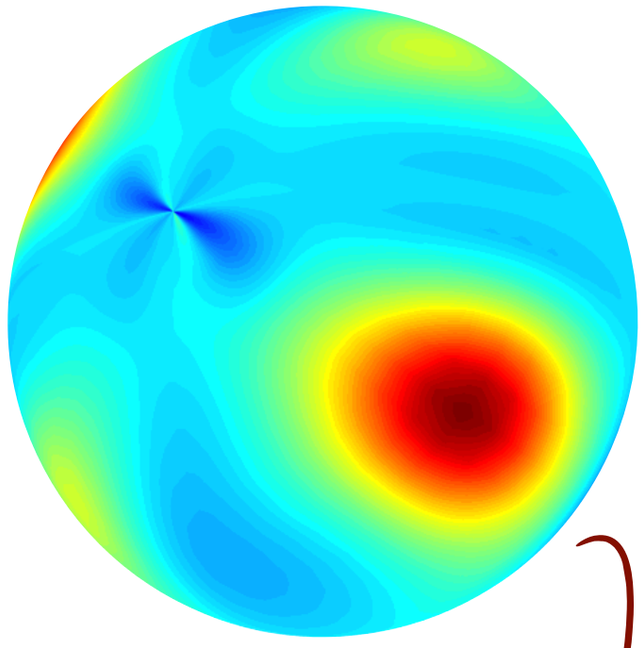
recovered
fracture surface

$\breve{\Gamma}$

S^{obs}

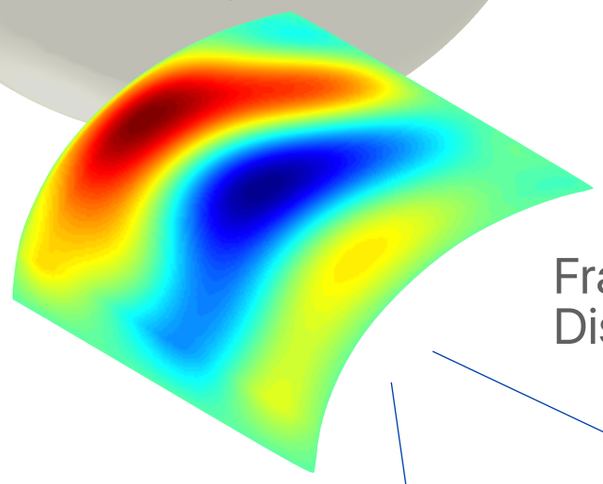
$\boldsymbol{d}$

Π



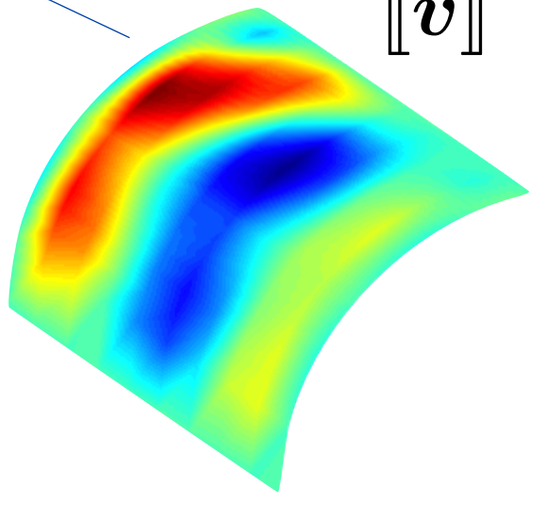
far-field pattern
of the scattered wavefield

$[\![\boldsymbol{v}]\!]$

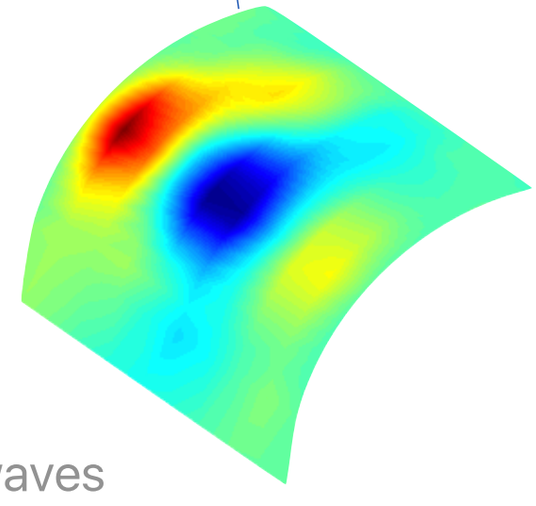


Fracture Opening
Displacement profile

$[\![\breve{\boldsymbol{v}}]\!]$

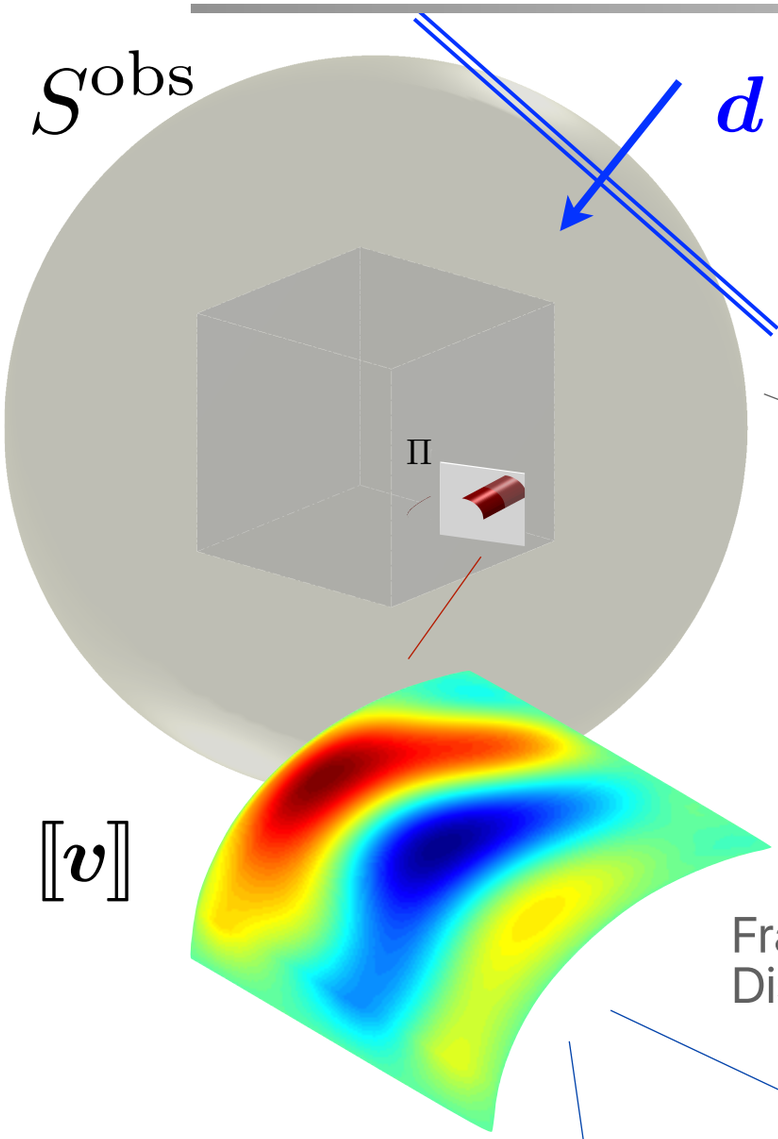


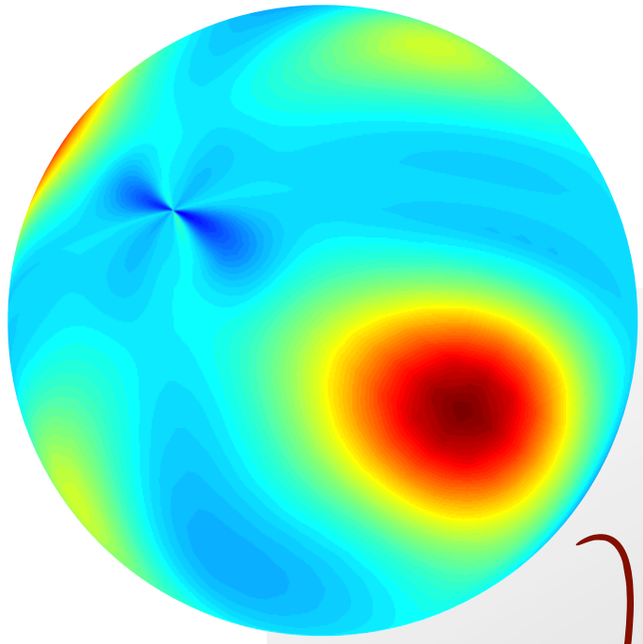
$[\![\boldsymbol{v}]\!]^{\text{loc}}$

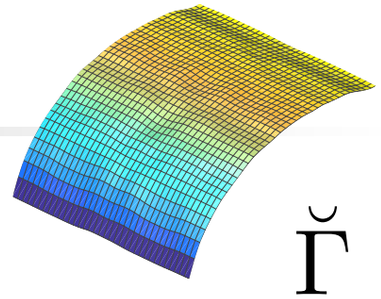


local modes
e.g. interface waves

FOD Reconstruction







recovered
fracture surface

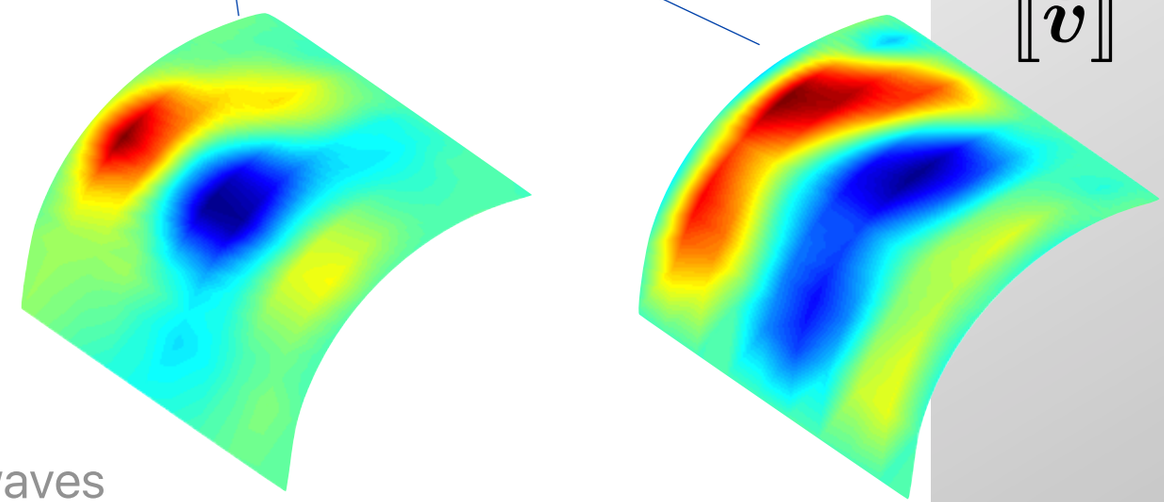
$\breve{\Gamma}$

far-field pattern
of the scattered wavefield

$$\boldsymbol{v}^{\infty} = \int_{\breve{\Gamma}} (\llbracket \breve{\boldsymbol{v}} \rrbracket \otimes \breve{\boldsymbol{n}}) : \boldsymbol{\Sigma}^{\infty} \, \mathrm{d}S_{\breve{\Gamma}}$$

$\llbracket \boldsymbol{v} \rrbracket$

Fracture Opening
Displacement profile

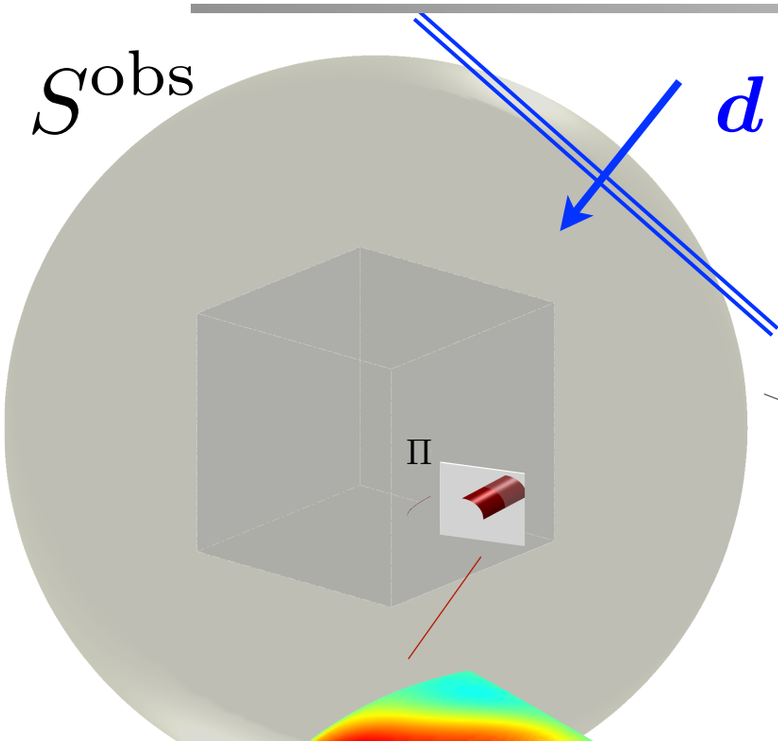


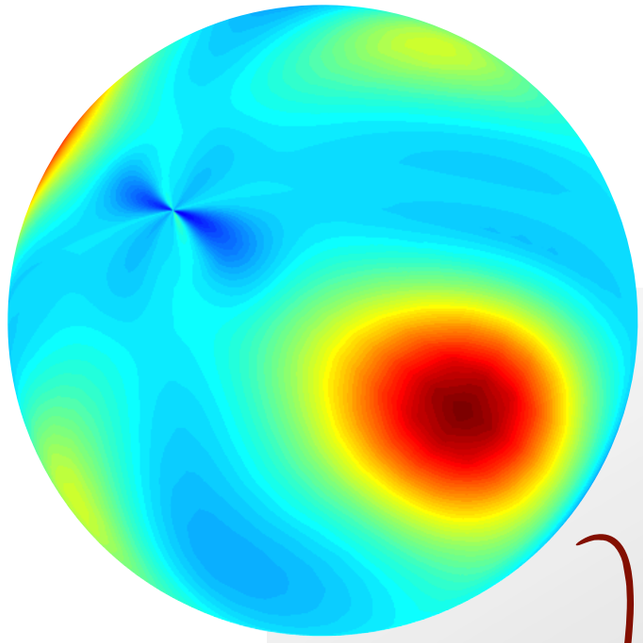
$\llbracket \breve{\boldsymbol{v}} \rrbracket$

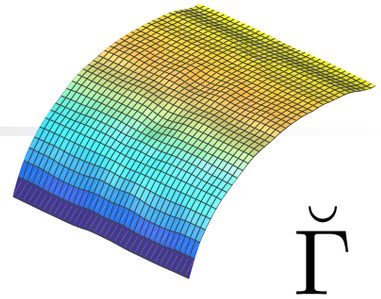
$\llbracket \boldsymbol{v} \rrbracket^{\mathrm{loc}}$

local modes
e.g. interface waves

FOD Reconstruction





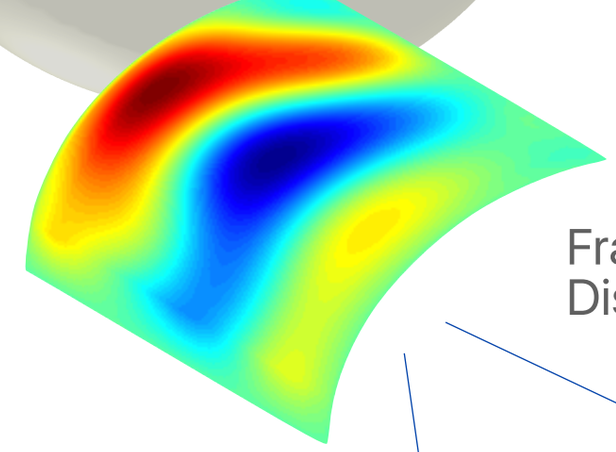


recovered
fracture surface

$\breve{\Gamma}$

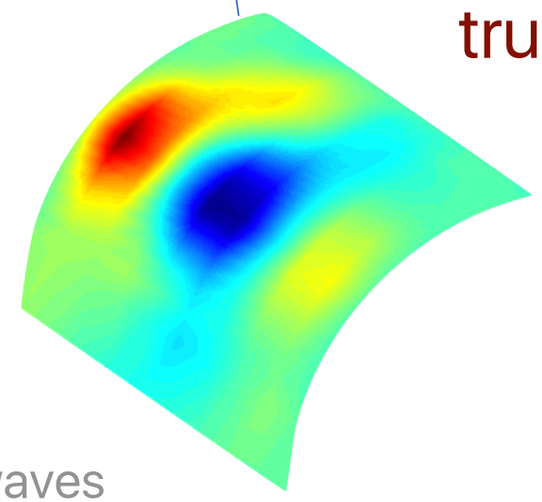
far-field pattern
of the scattered wavefield

$$\boldsymbol{v}^{\infty} = \int_{\breve{\Gamma}} (\llbracket \breve{\boldsymbol{v}} \rrbracket \otimes \breve{\boldsymbol{n}}) : \boldsymbol{\Sigma}^{\infty} \, \mathrm{d}S_{\breve{\Gamma}}$$



$\llbracket \boldsymbol{v} \rrbracket$

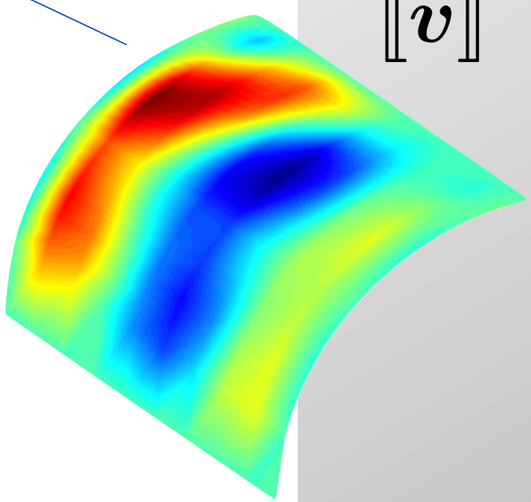
Fracture Opening
Displacement profile



$\llbracket \boldsymbol{v} \rrbracket^{\mathrm{loc}}$

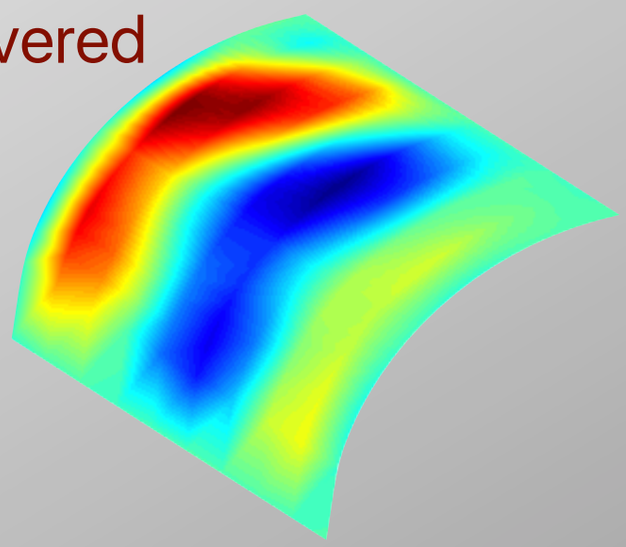
local modes
e.g. interface waves

true

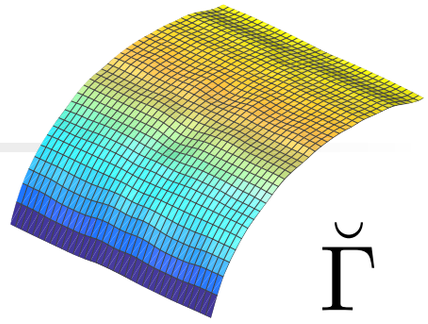


$\llbracket \breve{\boldsymbol{v}} \rrbracket$

recovered



Reconstruction of K



recovered
fracture surface

$\breve{\Gamma}$

TBIE representation

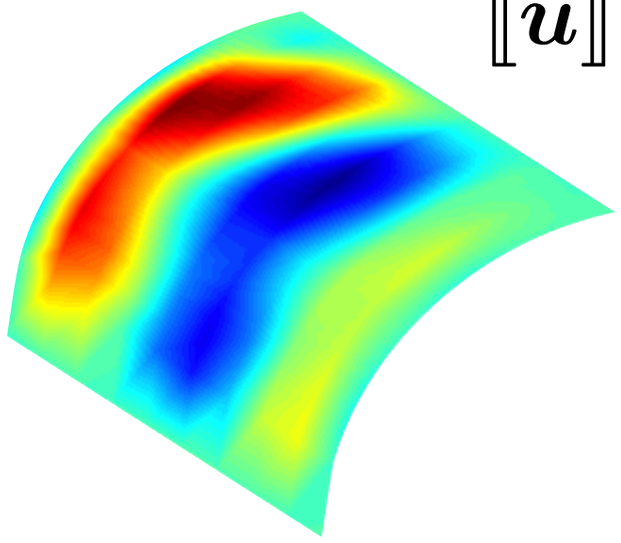
$$\boldsymbol{t}^i(\boldsymbol{\xi}) - \breve{\boldsymbol{K}} \cdot [\![\tilde{\boldsymbol{u}}]\!](\boldsymbol{\xi}) = \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \int_{\breve{\Gamma}} \boldsymbol{\Sigma} : \boldsymbol{D}_{\boldsymbol{x}} [\![\tilde{\boldsymbol{u}}]\!](\boldsymbol{x}) \, \mathrm{d}S_x - \rho\omega^2 \breve{\boldsymbol{n}} \cdot \boldsymbol{C} : \int_{\breve{\Gamma}} \boldsymbol{U} \cdot \left([\![\tilde{\boldsymbol{u}}]\!] \otimes \breve{\boldsymbol{n}} \right)(\boldsymbol{x}) \, \mathrm{d}S_x$$

free field
traction

FEM of BEM simulation

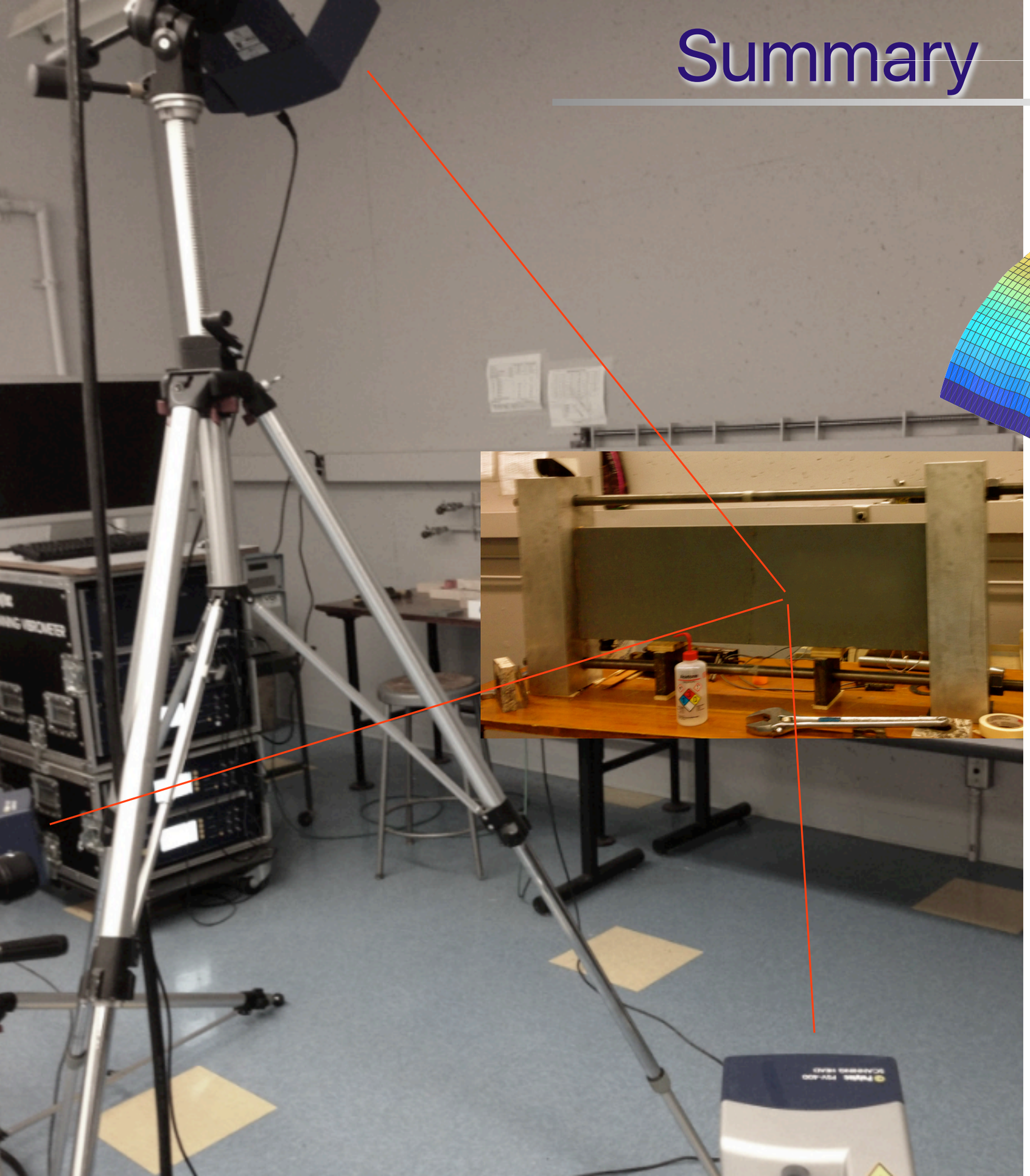
Regularization

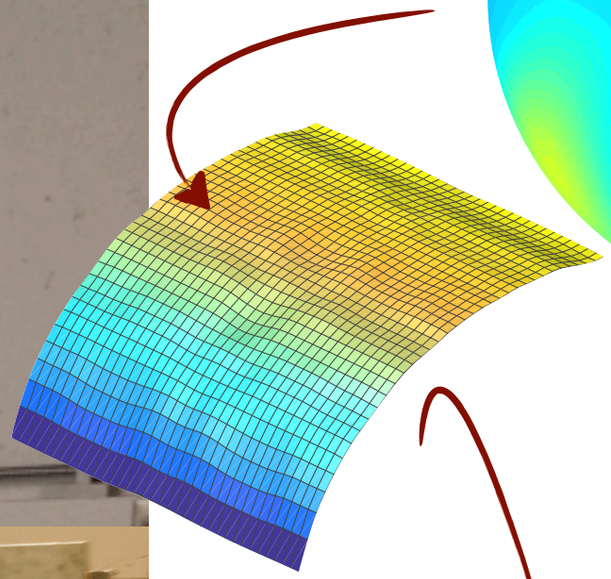
Reconstructed FOD
(propagating modes)

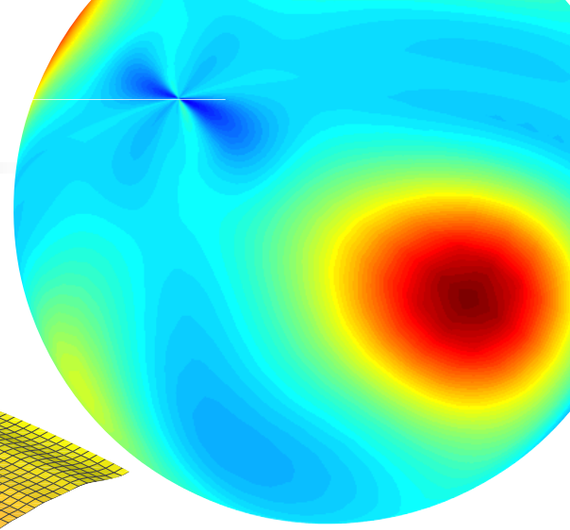


Solve for $\breve{\boldsymbol{K}}(\boldsymbol{\xi})$

Summary







$$\breve{K}(\xi)$$

- low sensitivity to noise
- flexibility in sensing configuration
- no restriction on frequency regime of illumination
- active and non-iterative
- point-by-point characterization of the fracture interfacial condition