1)

(a) $$\overline{DO} = \frac{1.8 + 2.0 + \cdots 2.2}{10} = 2.03\,mg/l$$

$$s^2 = \frac{(1.8 - 2.03)^2 + (2.0 - 2.03)^2 + \cdots + (2.2 - 2.03)^2}{10-1}$$

$$s^2 = 0.2357$$
$$s = 0.485\ mg/l$$

95% Confidence interval for the true mean is

$$\left(\overline{DO} - t_{0.025,9}\frac{s}{\sqrt{10}},\ \overline{DO} + t_{0.025,9}\frac{s}{\sqrt{10}}\right)$$

$$= \left(2.03 - 2.2622\,\frac{0.485}{\sqrt{10}},\ 2.03 + 2.2622\,\frac{0.485}{\sqrt{10}}\right)$$

$$= (1.683,\ 2.377)\ mg/l.$$

(b) New interval $= (2.377 - 1.683)\,0.9 = 0.6246$

$$\frac{s}{\sqrt{n}} \cdot t_{0.025,9} = \frac{0.6246}{2}$$

$$\boxed{n \simeq 13}$$

additional 2-3 observations are required

(c) 95% Confidence interval for the variance.

$$\left(\frac{(n-1)\, s^2}{\chi^2_{0.975,\, n-1}} , \quad \frac{(n-1)\, s^2}{\chi^2_{0.025,\, n-1}} \right)$$

$$\left(\frac{(10-1)\ 0.2357}{(19.023)} , \quad \frac{(10-1)\ 0.2357}{2.7} \right)$$

$$= (0.1115, \quad 0.7856)$$

(d) 95% confidence interval of $\overline{DO}$ is $(1.683, 2.377)$

- action is required $\overline{DO}$ exceeds 2.5 mg/l.
- The True $\overline{DO}$ is likely to be within the CI with 95% probability, which is less than 2.5 mg/l. Therefore, no clean up is required

(e) 95% CI of DO over the new set of 10 days is (2.5, 3.2). This does not overlap the CI of the 10 days computed in (a). Therefore the two sets of 10 days have different $\overline{DO}$.

2)

(a) $f(x) = \frac{1}{6500} e^{-\left(\frac{x-5500}{6500}\right)} \quad x \geq 5500$

$$\int_{5500}^{\infty} f(x)\,dx = \frac{1}{6500}\int_{5500}^{\infty} e^{-\frac{x-5500}{6500}}\,dx$$

$$u = \frac{x-5500}{6500} \qquad du = \frac{1}{6500}\,dx.$$

$$= \frac{1}{6500} \cdot \int e^{-u} (6500)\,du$$

$$= -e^{-u} = -e^{-\frac{x-5500}{6500}} \Big|_{5500}^{\infty}$$

$$= -\left[\frac{1}{e^{\infty}} - \frac{1}{e^{0}}\right] = -[0-1] = 1$$

$$\boxed{\int_{5500}^{\infty} f(x)\,dx = 1}$$

$f(x)$ is a valid pdf

(b) $\bar{x} = \int x f(x)\, dx$

$$= \int_{550}^{\infty} x \frac{1}{650} e^{-\frac{x-550}{650}}\, dx$$

$$\frac{x-550}{650} = u \Rightarrow du = \frac{dx}{650}.$$

$$\bar{x} = \int \frac{(650u+550)}{\cancel{650}} e^{-u}\, du\,(\cancel{650})$$

$$= 650 \int u e^{-u}\, du + 550 \int e^{-u}\, du$$

$$= 650 \int u e^{-u}\, du + 550\left(-e^{-u}\right)$$

$$\int u e^{-u}\, du = -ue^{-u} - \int -e^{-t}\, du$$

$$= -ue^{-u} - e^{-u}$$

$$= -e^{-u}(u+1)$$

$$= -e^{-\frac{x-550}{650}}\left(\frac{x-550}{650}+1\right)\Big|_{550}^{\infty}$$

$$= -\left[\frac{1}{e^{\infty}}(\infty+1) - \frac{1}{e^{0}}(0+1)\right]$$

$$= -(0-1) = 1$$

$$\bar{X} = (6500)1 + 5500\left(-e^{-\frac{x-5500}{6500}}\right)\Big|_{5500}^{\infty}$$

$$= 6500 + 5500\left(-\frac{1}{e^{\infty}} - \left(-\frac{1}{e^{\frac{5500-5500}{6500}}}\right)\right)$$

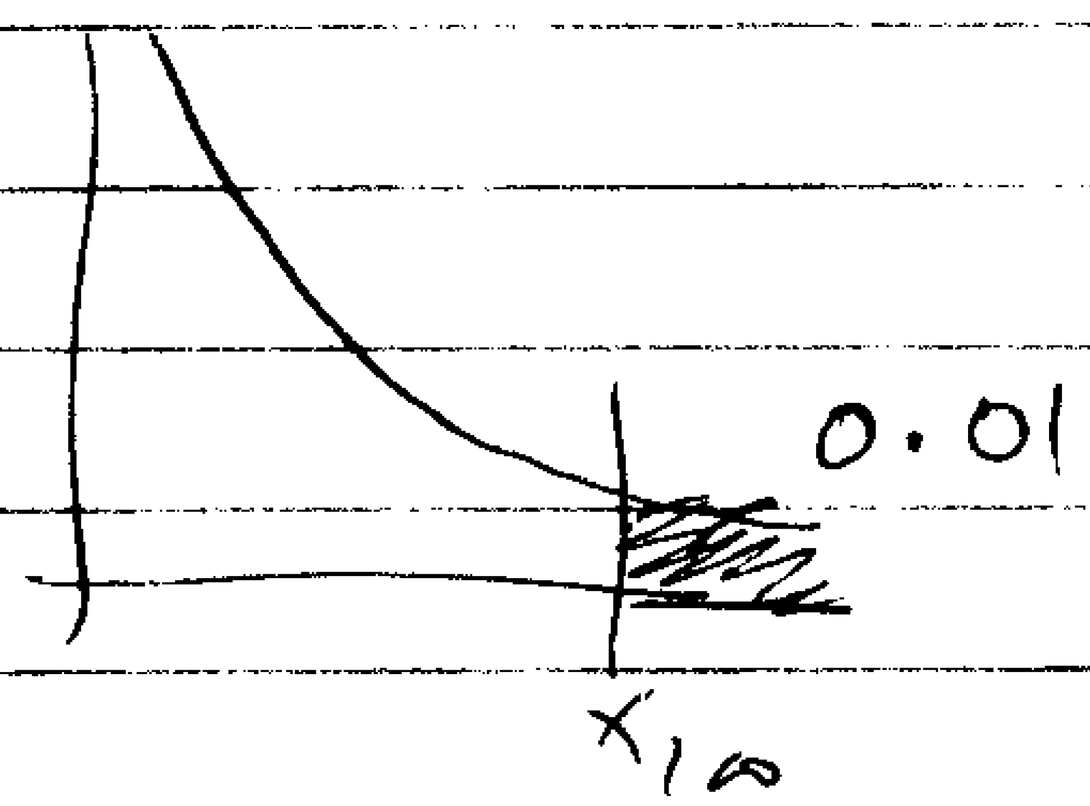
$$= 6500 + 5500(-0+1)$$

$$\boxed{\bar{X} = 12000}$$

(b) variance $= \int (x-\bar{x})^2 f(x)\,dx$

$$\boxed{\sigma_x^2 = \int_{5500}^{\infty} (x-12000)^2 \frac{1}{6500} e^{-\frac{(x-5500)}{6500}}\,dx}$$

(c) 100 yr. flood



$$\int_{5500}^{x_{100}} f(x)\,dx = 0.01$$

(from 2(a))

$$-e^{-\frac{x-5500}{6500}}\Big|_{5500}^{x_{100}} = 0.99$$

$$-\frac{1}{e^{\frac{x_{100}-5500}{6500}}} + \frac{1}{e^{\frac{5500-5500}{6500}}} = 0.99$$

$$-\frac{1}{e^{\frac{\lambda_{100}-5500}{6500}}} + \frac{1}{e^{0}} = 0.99$$

$$0.99 = \frac{1}{e^{\frac{\lambda_{100}-5500}{6500}}}$$

$$e^{\frac{\lambda_{100}-5500}{6500}} = 1/0.01 = 100$$

$$\lambda_{100} = \log(100)\,6500 + 5500$$

$$\boxed{\lambda_{100} = 35434}$$

3) P = Probability of flood in each year $= \frac{1}{T} = 0.1$

$T = 10$ yr (return period)

$\bar{P}$ = probability of no flood in a year $= 1 - P = 0.9$

(a) P(flood first time in 3^{rd} year) $= \bar{P} \cdot \bar{P} \cdot P$

$= (0.9)(0.9)(0.1)$

(Geometric distn) $\boxed{= 0.081}$

(b) P(flooding in the first three years)

= P(atleast one flood in three years)

= $1 - P$(no flood in three years)

$= 1 - \bar{P} \cdot \bar{P} \cdot \bar{P} = 1 - (0.9)^3 = \boxed{0.271}$

(c) P(flooding in 3 of the first 5 years)

$$= \binom{5}{3} P^3 (\bar{P})^{5-3} = \boxed{0.0081}$$

(Binomial distn)

(d) P(only one flood in 3 years)

$$= \binom{3}{1} (0.1)^1 (0.9)^{3-1} = \boxed{0.243}$$

(e) $\nu = 1/8$ (rate of occurrence for Poisson process)

P(Flooding in 10 years) = P(atleast one flood in 10 years) $= 1 - P$(no flood in 10 years)

$$= 1 - P(n=0) = 1 - \frac{(\nu t)^n e^{-\nu t}}{n!} = 1 - \frac{(\nu t)^0 e^{-\frac{1}{8}(10)}}{0!} = \boxed{0.7135}$$

4) (a) T = Time to complete project in days

G = Good weather,

B = Bad weather

$P(PG|G) = 0.9, \quad P(PB|G) = 0.1$

$P(PB|B) = 0.9 \quad P(PG|B) = 0.1$

$P(G) = P(B) = 0.5$

$P(PG) = P(PG|G)P(G) + P(PG|B)P(B)$

$= 0.9 \times 0.5 + 0.1 \times 0.5 = 0.5$

$P(PB) = 0.5$

$$P(G|PG) = \frac{P(PG|G)P(G)}{P(PG)} = \frac{0.9 \times 0.5}{0.5} = 0.9$$

$$P(B|PG) = \frac{P(PG|B)P(B)}{P(PG)} = \frac{0.1 \times 0.5}{0.5} = 0.1$$

$= P(B)PB)$

(b) $P(\text{Delay}) = P(T > 30) = P(\text{Delay}|G)P(G|PG)$

$+ P(\text{Delay}|B)P(B|PB)$

$= 0.1112(.9) + (0.5)(0.1)$

$$\boxed{P(\text{Delay}) = 0.15011}$$