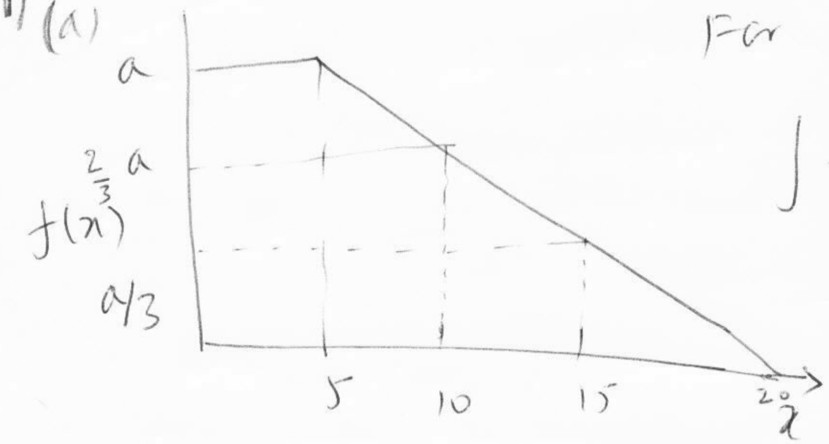
1) (a)



For $f(x)$ to be a PDF

$\int f(x)\,dx = 1$

$\Rightarrow$ area under the curve $= 1$

$\Rightarrow 5 \times a + \frac{1}{2} \times 15 \times a = 1$

$$\boxed{a = 0.08}$$

(b) O = Event of overflow at B

overflow occurs when $X + Y > 40$

~~$P(O) = P(X+Y>40 \mid Y=25) + P(X+Y>40 \mid Y=35)$~~

$P(O) = P((X+Y>40) \cap Y=25) + P((X+Y>40) \cap Y=35)$

$= P(X+Y>40 \mid Y=25)\,P(Y=25) + P(X+Y>40 \mid Y=35)\,P(Y=35)$

$P(X+Y>40 \mid Y=25) = P(X>15)$

$P(X+Y>40 \mid Y=35) = P(X>5)$

$P(x>15) = \frac{1}{2} \times 5 \times \frac{a}{3} = \frac{5}{6}(0.08)$

$P(x>5) = \frac{1}{2} \times 15 \times a = \frac{15}{2}(0.08)$

$P(O) = \frac{5}{6}(0.08)\,0.7 + \frac{15}{2}(0.08)(0.3) = 0.227$

$\boxed{P(O) = 0.227}$

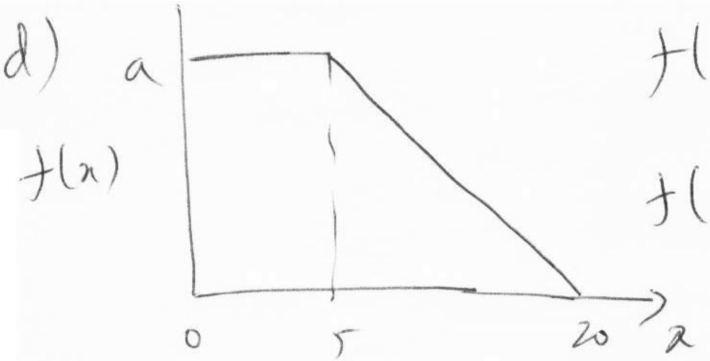
(c) $P(Y=25 \mid \bar{O}) = \dfrac{P(Y=25 \cap \bar{O})}{P(\bar{O})}$

$= \dfrac{P(\bar{O} \mid Y=25)\, P(Y=25)}{P(\bar{O})}$

$P(\bar{O} \mid Y=25) = P(x \leq 15) = 1 - \frac{1}{2}(5)\left(\frac{0.08}{3}\right)$

$\boxed{P(Y=25 \mid \bar{O}) = \dfrac{\left(1 - \frac{1}{2}(5)\frac{0.08}{3}\right) 0.7}{1 - 0.227} = 0.845}$

(d)



$f(x) = a \qquad 0 < x \leq 5$

$f(x) = \frac{20-x}{15} \cdot a \qquad 5 < x \leq 20.$

$$E(x) = \int x \, f(x) \, dx$$

$$= \int_0^5 x \cdot a \cdot dx + \int_5^{20} \left(\frac{20-x}{15}\right) \cdot a \cdot dx \cdot x$$

$$= a\frac{x^2}{2}\Big|_0^5 + \frac{a}{15}\left(20x - \frac{x^2}{2}\right)\Big|_5^{20}$$

$$= a\left(\frac{25}{2}\right) + \frac{a}{15}\left[\left(400 - \frac{400}{2}\right) - \left(100 - \frac{25}{2}\right)\right]$$

$$\boxed{E(x) = 1.6}$$

2) Probability of 10-year flood in any year $= \frac{1}{T} = \frac{1}{10} = 0.1 = \boxed{\begin{array}{l} p = 0.1 \\ \bar{p} = 1 - p = 0.9 \end{array}}$

(a) P(flood first time in 3^{rd} year)

$$= \bar{p}\,\bar{p}\,p = (0.9)(0.9)(0.1)$$

$$\boxed{= 0.081}$$

(b) P(flooding in first 3 years) = P(atleast one flooding in 3 years)

$$= 1 - P(\text{no flooding in 3 years})$$

$$= 1 - \binom{3}{0} p^{0} (1-p)^{3-0}$$

$$\boxed{= 0.271}$$

(c) P(flooding in 3 of first 5 years)

$$= \binom{5}{3} p^{3} (1-p)^{5-3}$$

$$= \binom{5}{3} (0.1)^{3} (1-p)^{5-3}$$

$$\boxed{= 0.0081}$$

(d) Flood risk over a 25-year period

= 1 - P(no flood in 25 years)

$$\boxed{= 1 - (1-p)^{25} = 0.93}$$

3(a)
Parametric:
----
1. A functional form is first assumed (i.e. Normal, Gamma, Lognormal etc.)

2. The assumed function is then fitted based on the 'entire' data

3. The goodness-of-fit of the fitted function has to be evaluated

4. Limited variety (e.g., only unimodal) of functional forms possible.

5. Outliers have significant impact on the fit.

Nonparametric:
----
1. No assumption of the underlying functional form.

2. Estimation is "local" - hence, outliers have no effect on the estimates

3 Can fit a variety of functional forms exhibited by the data (e.g., bimodal)

3(b) Suppose that you have the histogram then the Monte Carlo proceeds as follows:
1. Randomly select a bin
2. Randomly select one of the data point within the selected bin
3. Repeat

3(c) Monte Carlo have the following utilities..

1. Generate Synthetic data that have the same statistical properties of the historical data. These can be used in decision models.

2. Provide uncertainty estimates (or confidence intervals) of model parameters

3. Help in testing hypotesis of differences between groups.

4) (a) X = No of damaging hurricanes in 20 years

$\nu t = \frac{2}{50}(20) = 0.8$

ν = rate of occurrence = $\left(\frac{2}{50}\right)$

$\boxed{\text{Poisson distribution}}$

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-0.8}(0.8)^0}{0!} + \frac{e^{-0.8}(0.8)^1}{1!} + \frac{e^{-0.8}(0.8)^2}{2!}$$

$$\boxed{P(X<3) = 0.953}$$

(b) $\bar{E}$ = event tower is (NOT) destroyed by the hurricanes during its life.

$$P(\bar{E}) = P(\bar{E} \cap X=0) + P(\bar{E} \cap X=1) + P(\bar{E} \cap X=2) + P(\bar{E} \cap X=3) + \cdots + P(\bar{E} \cap X=5)$$

$$= P(\bar{E} \mid X=0)\, P(X=0) + \cdots + P(\bar{E} / X=5)\, P(X=5)$$

$P(\bar{E}|x=0) = 1-0 = 1, \quad P(\bar{E}|x=1) = 1-0.2 = 0.8$

$P(\bar{E}|x=2) = 1-0.8 = 0.2, \quad P(\bar{E}|x=3) = 1-1 = 0$

$P(\bar{E}|x=4) = P(\bar{E}|x=5) = 0$

$$P(\bar{E}) = 1\frac{e^{-0.8}\,0.8^{0}}{0!} + (0.8)\frac{e^{-0.8}(0.8)^{1}}{1!} + (0.2)\frac{e^{-0.8}(0.8)^{2}}{2!}$$

$$+ 0 + 0 + 0$$

$$\boxed{P(\bar{E}) = 0.766}$$

5)

(a) $M_A = 30P_1 - 20P_2$

$$\mu_{MA} = E(30P_1) - E(20P_2)$$

$$= (30)\,E(P_1) - (20)\,E(P_2)$$

$$= 30\,(50) - 20\,(20)$$

$$\boxed{\mu_{MA} = 1100}$$

$$\sigma_{MA}^2 = V(30P_1 - 20P_2) = 30^2\,V(P_1) + 20^2\,V(P_2)$$

$$\sigma_{MA}^2 = 30^2 \cdot 5^2 + 20^2 \cdot 3^2$$

$$\boxed{\sigma_{MA} = \sqrt{(30^2)\,5^2 + (20)^2\,3^2} = 161.6}$$

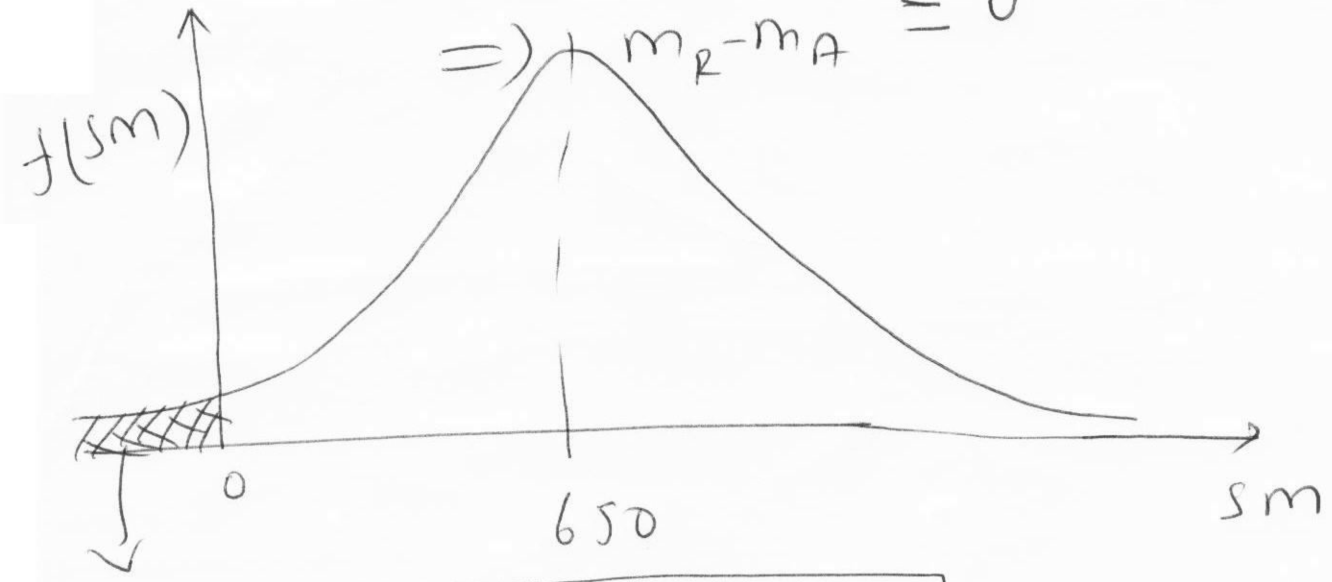
(b) $SM = M_R - M_A$

$$\mu_{SM} = E(M_R) - E(M_A)$$
$$= 1750 - 1100 = 650$$

$$\sigma_{SM} = \sqrt{V(M_R) + V(M_A)} = \sqrt{150^2 + 161.6^2}$$
$$= 220.45$$

Pole fails if $M_R \le M_A$ (resistance $\le$ load)

$$\Rightarrow M_R - M_A \le 0$$



$$\boxed{\text{probability of failure} \quad \text{pnorm}(0, 650, 220.45) = 0.00159}$$