# Solution of Homework # 4

1 . The inverse functions of $y = x^2$ are $x_1 = \sqrt{y}$, $x_2 = -\sqrt{y}$ for $0 < y < 1$ and $x_3 = \sqrt{y}$ for $1 < y < 4$. Now $|J_1| = |J_2| = |J_3| = 1/2\sqrt{y}$ from which we get

$$g(y) = f(\sqrt{y})|J_1| + f(-\sqrt{y})|J_2| = \frac{2(\sqrt{y}+1)}{9} \cdot \frac{1}{2\sqrt{y}} +$$

$$\frac{2(-\sqrt{y}+1)}{9} \cdot \frac{1}{2\sqrt{y}} = 2/9\sqrt{y}, \quad 0 < y < 1$$

and
$$g(y) = f(\sqrt{y})|J_3| = \frac{2(\sqrt{y}+1)}{9} \cdot \frac{1}{2\sqrt{y}}$$

$$= (\sqrt{y}+1)/9\sqrt{y}, \quad 1 < y < 4.$$

2. $\mu_1 - \mu_2 = 0$, $\sigma_{\bar{X}_1-\bar{X}_2} = 50\sqrt{1/32 + 1/50} = 11.319$.

(a) $z_1 = -20/11.319 = -1.77$, $z_2 = 20/11.319 = 1.77$;

$P(|\bar{X}_1 - \bar{X}_2| > 20) = 2P(Z < -1.77) = (2)(0.0384) = 0.0768.$

(b) $z_1 = 5/11.319 = 0.44$, $z_2 = 10/11.319 = 0.88$.

$P(-10 < \bar{X}_1 - \bar{X}_2 < -5) + P(5 < \bar{X}_1 - \bar{X}_2 < 10)$

$= 2P(5 < \bar{X}_1 - \bar{X}_2 < 10) = 2P(0.44 < Z < 0.88)$

$= 2(0.8106 - 0.6700) = 0.2812.$

4. $\bar{x} = 0.475$; $s^2 = 0.0336$; $t = (0.475-0.5)/\ 0.0648 = -0.39$;
$P[\ \bar{x} < 0.5\ ] = P[\ t < -0.39\ ] \cong\ 0.35$; In conclusive.

6. $n_p = 8$, $n_u = 8$, $x_p = 86{,}250.00$, $x_u = 79{,}837.50$, $\sigma_1 = \sigma_2 = 4000$

$$(86{,}250.00 - 79{,}837.00) - (1.96)(4000)\sqrt{\frac{1}{8}+\frac{1}{8}}$$

$$< \mu_p - \mu_u < (86{,}250.00 - 79{,}837.00) + (1.96)(4000)\sqrt{\frac{1}{8}+\frac{1}{8}}$$

$$2492.50 < \mu_p - \mu_u < 10{,}332.50$$

Polishing increases the average indurance limit.

**7.1 (5.5)**

| Yields with fracturing | Yields without |
|---|---|
| $r_{crit}$ = .932, accept normality | $r_{crit}$ =.928, reject normality |

Because one of the groups is non-normal, the rank-sum test is performed.
$W_{rs} = \Sigma R_{without} = 121.5$. The one-sided p-value from the large-sample approximation p= 0.032. Reject equality. The yields from fractured rocks are higher.

**7.2 (5.6)** The test statistic changes very little ($W_{rs} = 123$), indicating that most information contained in the data below detection limit is extracted using ranks. Results are the same (one-sided p-value = 0.039. Reject equality). A t-test could not be used without improperly substituting some contrived values for less-thans which might alter the conclusions.

**8.1 (6.4)** Because of the data below the reporting limit, the sign test is performed on the differences Sept–June. The one-sided p-value = 0.002. Sept atrazine concentrations are significantly larger than June concs before application.

**8.2 (6.5)** For the t-test, t=1.07 with a one-sided p-value of 0.15. The t-test cannot reject equality of means because one large outlier in the data produces violations of the assumptions of normality and equal variance.

9. $H_0$: $\sigma_1 = \sigma_2$ $\quad$ $s_1 = 281.0667$ (1980 models)

$H_1$: $\sigma_1 \neq \sigma_2$ $\quad$ $s_2 = 119.3946$ (1990 models)

$f = 5.54 \quad P = 0.0005$

**Decision: Hydrocarbon emissions are more consistent in the 1990 model cars.**