Solution for Homework # 2

1. $P(F) = 0.25,\ P(H) = 0.75,\ P(S) = 0.10,\ P(W) = 0.90$

   C = The structure will collapse.

   $P(C|S) = 1.0,\ P(\overline{C}|WH) = 1.0,\ P(C|WF) = 0.5$

   $P(\overline{C}|WF) = 0.5$

   $P(F|C) = P(SF|C) + P(WF|C)$

   $$= \frac{P(C|SF)\cdot P(SF)}{P(C)} + \frac{P(C|WF)\cdot P(WF)}{P(C)}$$

   Now,

   $$P(C) = P(C|SH)\cdot P(SH) + P(C|SF)\cdot P(SF) + P(C|WH)\cdot P(WH) + P(C|WF)\cdot P(WF)$$
   $$= 1.0\text{x}0.1\text{x}0.75 + 1.0\text{x}0.1\text{x}0.25 + 0 + 0.5\text{x}0.9\text{x}0.25$$
   $$= 0.2125$$

   So,

   $$P(F|C) = \frac{1.0\text{x}0.10\text{x}0.25}{0.2125} + \frac{0.5\text{x}0.9\text{x}0.25}{0.2125} = 0.646$$

2. Let,

   F_S and F_H represent that the county will be flooded in a year by snow melting and by hurricanes respectively.
   H_0, H_1 and H_2 represent that the county will be hit by none, one and two hurricanes respectively in a year.

   $P(F_S) = 0.10$

   $P(H_1) = 0.3,\ P(H_2) = 0.05,\ P(H_0) = 0.65$

   $P(F_H|H_0) = 0.0$

   $P(F_H|H_1) = 0.25,\quad P(\overline{F}_H|H_1) = 0.75$

   $P(F_H|H_2) = 1-0.75^2 = 1-0.5625 = 0.4375$

   Using theorem of total probability,

   $$P(F_H) = P(F_H|H_0)P(H_0) + P(F_H|H_1)P(H_1) + P(F_H|H_2)P(H_2)$$
   $$= 0 + 0.25\text{x}0.30 + 0.4375\text{x}0.05 = 0.096875$$

   P(Flood in the county in a year)

   $$= P(F_S \cup F_H) = P(F_S) + P(F_H) - P(F_S F_H)$$
   $$= 0.10 + 0.096875 - P(F_S)\cdot P(F_H)$$
   $$= 0.196875 - 0.10 \text{ x } 0.096875$$
   $$= 0.187$$

3. **This is a parallel system of two series subsystems.**

   (a) $P = 1 - (1 - 0.7)(0.7))(1 - (0.8)(0.8)(0.8)) = 0.7511.$

   (b) $$P = \frac{(A' \cap C \cap D \cap E}{P(\text{SystemWorks})} = \frac{(0.3)(0.8)(0.8)(0.8)}{0.75112} = 0.2045$$

4. (a) $P(A \cap B \cap C) = P(C \mid A \cap B) = (0.20)(0.225) = 0.045.$

   (b) $P(A \cap B') = P(B'|A)P(A) = (0.25)(0.3) = 0.075;$

   $P(A' \cap B') = P(B'|A')P(A') = (0.80)()(.70) = 0.56.$

   $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C)$

   $= P(C|A \cap B')P(A \cap B') + P(C|A' \cap B')P(A' \cap B')$

   $= (0.80)(0.075) + (0.90)(0.56)$

   $= 0.564.$

   (c) $P(A' \cap B) = P(B|A')P(A') = (0.20)(0.7) = 0.14$

   From (a) $P(A \cap B \cap C) = 0.045$

   $P(A' \cap B \cap C) = P(C|A' \cap B)P(A' \cap B) = (0.15)(0.14) = 0.021.$

   $P(A \cap B' \cap C) = P(C|A \cap B')P(A \cap B') = (0.80)(0.075) = 0.060.$

   $P(A' \cap B' \cap C) = P(C|A' \cap B')P(A' \cap B') = (0.90)(0.56) = 0.504$

   $P(C) = 0.045 + 0.021 + 0.060 + 0.504 = 0.630.$

   (d) $P(A|B' \cap C) = P(A \cap B' \cap C)/P(B' \cap C) = (0.06)/(0.564) = 0.1064.$

5. Referring to the sample space in Exercise 3 and making use of the fact that $P(H) = 2/3$ and $P(T) = 1/3$, we have

   $P(W = -3) = P(TTT) = (1/3)(1/3)(1/3) = 1/27,$

   $P(W = -1) = P(HTT) + P(THT) + P(TTH) = 3(2/3)(1/3)^2 = 2/9,$

   $P(W = 1) = P(HHT) + P(HTH) + P(THH) = 3(2/3)^2(1/3) = 4/9,$

   $P(W = 3) = P(HHH) = (2/3)(2/3)(2/3) = 8/27.$

   The probability distribution for W is then

w	-3	-1	1	3
$P(W = w)$	1/27	2/9	4/9	8/27

6. (a) $$1 = k\int_{30}^{50}\int_{30}^{50}(x^2 + y^2)\,dx\,dy$$

   $$= k \cdot 10^4 \int_3^5\int_3^5 (x^2 + y^2)\,dx\,dy$$

   $$= k \cdot 10^4 \int_3^5 \left(\frac{98}{3} + 2y^2\right)dy = \frac{392k}{3} \cdot 10^4$$

   $$k = \frac{3}{392} \cdot 10^{-4}$$

   (b) Find $P(30 \le X \le 40,\ 40 \le Y \le 50)$

   $$\frac{3}{392} \cdot 10^{-4} \cdot 10^4 \int_3^4\int_4^5 (x^2 + y^2)\,dy\,dx$$

   $$= \frac{3}{392}\int_3^4\left(x^2 + \frac{61}{3}\right)dx = \frac{3}{392}\left[\frac{98}{3}\right] = \frac{98}{392} = \frac{49}{196}$$

   (c) $P[30 \le X \le 40,\ 30 \le Y \le 40)$

   $$\frac{3}{392} \cdot 10^{-4} \cdot 10^4 \int_3^4\int_3^4 (x^2 + y^2)\,dy\,dx$$

   $$= \frac{3}{392}\int_3^4\left(x^2 + \frac{37}{3}\right)dx = \frac{3}{392}\left[\frac{74}{3}\right] = \frac{74}{392} = \frac{37}{196}$$