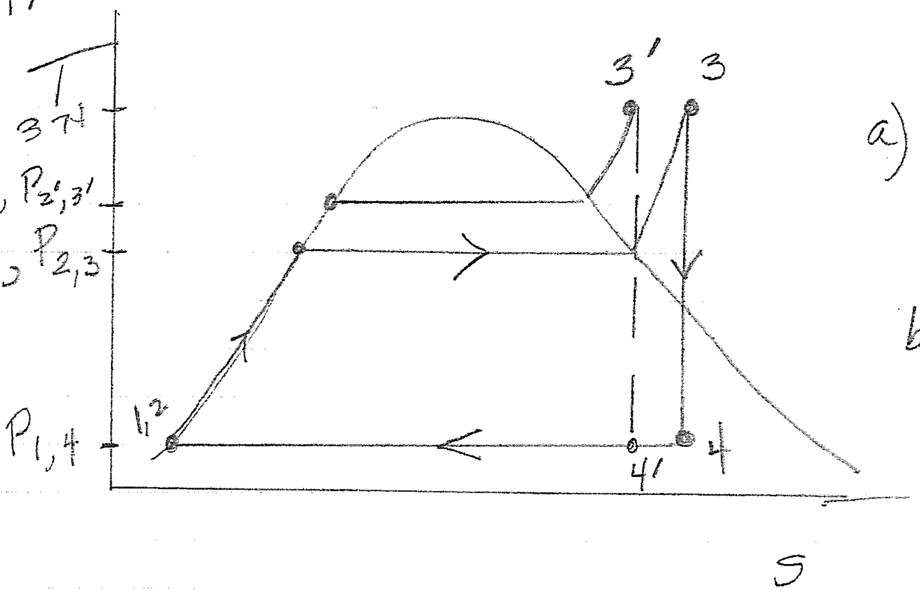
SOLUTIONS, HW 11

1.



a) $-w = v(P_2 - P_1)$
P_2 increases
w_p increases

b) as P increases,
h(T) decreases for superheated vapor
but, X_4 decreases too,
so h_4 decreases also
w_T increases overall

b) example let $P_1 = 50\text{kPa}$, $T_3 = 400°C$, $P_2 = 8\text{MPa}$
$P_2' = 10\text{MPa}$

$h_3 = 3139.4 \text{ kJ/kg}$ $h_3' = 3097.5 \text{ kJ/kg}$

$s_3 = 6.3658 \text{ kJ/kgK}$ $s_3' = 6.2141 \text{ kJ/kgK}$

$X_4 = \frac{6.3658 - 1.0912}{6.5019} = 0.811$ $X_4' = \frac{6.2141 - 1.0912}{6.5019}$

$h_4 = 0.811(2304.7) + 340.54$ $= 0.788$

$= 2210.2 \text{ kJ/kg}$ $h_4' = 0.788(2304.7) + 340.54$

$w_T = 3139.4 - 2210.2$ $= 2156.4$
$= 929.2 \frac{\text{kJ}}{\text{kg}}$ $w_T' = 3097.5 - 2156.4$
$= 941.07 \text{ kJ/kg}$

small increase in w_T
(~1%)

effect will be in same direction for all P incr.

1\. c) q_H decreases (see diagram or check)

$q_H = h_3 - h_2 = 3139.4 - (0.00103(8000-50) + 340.54)$

$= 2790.7$ kJ/kg

$q_H' = 3097.5 - (0.00103(10000-50) + 340.54)$

$= 2746.7$ kJ/kg $< q_H$

d) rejected heat clearly decreases (from diagram)
and h_1 same $h_4' < h_4$

e) $\eta = 1 - \frac{q_L}{q_H}$, $\eta' = 1 - \frac{q_L'}{q_H'}$ where $q_L' < q_L$ and $q_H' < q_H$

$$\eta = 1 - \frac{h_4 - h_1}{2790.7} = 1 - \frac{(2210.2 - 340.54)}{2790.7} = 0.33$$

$$\eta' = 1 - \frac{h_4' - h_1}{2746.7} = 1 - \frac{(2156.4 - 340.54)}{2746.7} = 0.34$$

η ~ 3% increase in η in this case
again, trend is same regardless of size of P increase

f) $x_4' < x_4$, so % moisture @ turbine
outlet increases

2. Rankine cycle with R-134a

$P_b = 1600\,kPa$, $P_c = 400\,kPa$

$T_3 = 80°C$, $X_4 = 0.98$ (not isentropic turbine)

	P	T	h	s
1	400	8.91	63.94	0.24761
2	1600	8.91	64.9	0.24761
3	1600	80	305.07	0.9875
4	400	8.91	251.73	0.91331

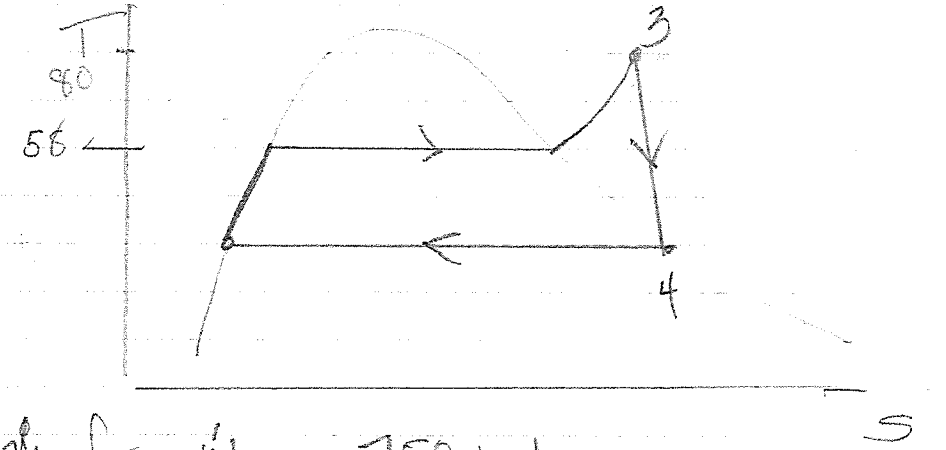
$h_1 = h_f$ @ 400 kPa

$h_2 = h_1 + v(P_2 - P_1) = 63.94 + 0.0007907(1600-400)$
$= 64.9\ kJ/kg$

$h_3 = 305.07$ (table 8)

$h_4 = 0.98(191.62) + 63.94 = 251.73\ kJ/kg$

$s_4 = 0.98(0.67929) + 0.24761 = 0.91331$



a) $\dot m$ for $\dot W_{net} = 750\,kW$

$\dot W_{net} = 750\,kW = \dot m(w_T + w_p)$

$= \dot m(q_H + q_L)$

$750\,kW = \dot m((h_3 - h_2) + (h_1 - h_4))$

2. a) $$\dot{m} = \frac{750\text{ kW}}{(305.07-64.9)+(63.94-251.73)}$$

$$\dot{m} = \frac{750\text{ kW}}{52.38\text{ kJ/kg}} = \boxed{14.3\text{ kg/s}}$$

b) $$\eta = 1 - \frac{q_L}{q_H} = 1 - \frac{(251.73-63.94)}{(305.07-64.9)}$$

$$\eta = \boxed{0.22}$$

3. Rankine

	P	T	h	s
1	50 kPa	81.32	340.54	1.0912
2	5 MPa	81.32	345.64	1.0912
3	5 MPa	263.94	2794.2	5.9737
4	50 kPa	81.32	2071.25	5.9737

$$h_2 = h_1 + v(P_2 - P_1) = 340.54 + 0.00103(5000-50)$$

$$-w_p = 5.1\text{ kJ/kg}$$

$$h_2 = 345.64\text{ kJ/kg}$$

$$x_4 = \frac{5.9737-1.0912}{6.5019} = 0.75$$

$$h_4 = 0.75(2304.7) + 340.54 = 2071.25\text{ kJ/kg}$$

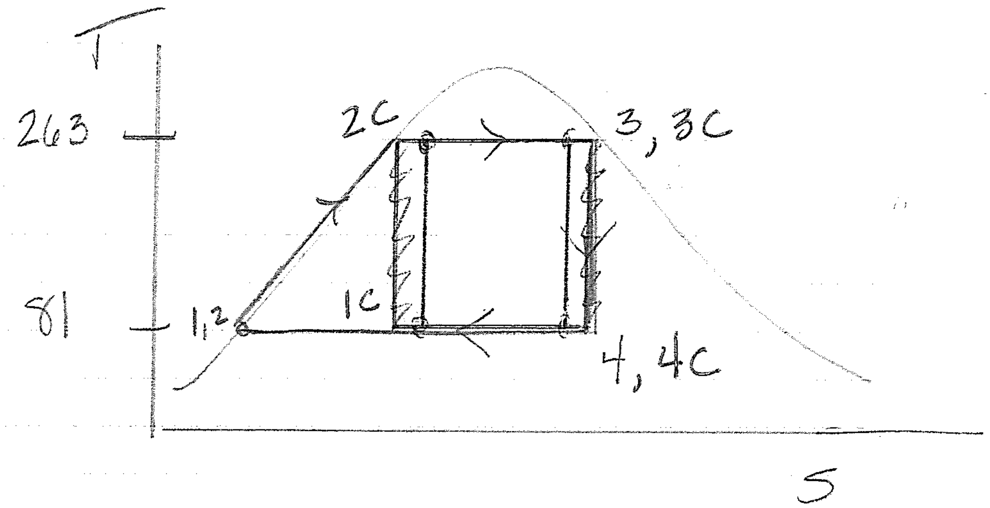
3. CARNOT

	P kPa	T	h	s
1	50	81.32	988.2	2.9207
2	5000	263.94	1154.5	2.9207
3	5000	263.94	2794.2	5.9737
4	50	81.32	2071.25	5.9737

$$x_1 = \frac{2.9207 - 1.0912}{6.5019} = 0.281$$

$$h_1 = 0.281(2304.7) + 340.54 = 988.2 \text{ kJ/kg}$$

a)



b) $$\eta_R = 1 - \frac{q_L}{q_H} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{(2071.25 - 340.54)}{(2794.2 - 345.64)}$$

$$\eta_R = 0.293 \qquad w_{net} = \eta q_{in} = 0.293(2794.2 - 345.64) = 717.4 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_c = 1 - \frac{(2071.25 - 988.2)}{(2794.2 - 1154.5)} = 0.34$$

$$\underline{\text{OR}} = 1 - \frac{(273 + 81.32)}{(273 + 263.94)} = 0.34 \checkmark$$

$$w_{c,net} = 0.34(q_{in}) = 0.34\, T(s_3 - s_2) = 0.34(536.94\text{k})(5.9737 - 2.9207$$
$$= 557.4 \text{ kJ/kg}$$

c) $$S_{gen,R} = -\frac{q_H}{T_H} - \frac{q_L}{T_L} = -\frac{(2794.2 - 345.64)}{(273 + 263.94)} - \frac{(340.54 - 2071.25)}{(273 + 81.32)}$$

3 c) $s_{gen,R} = -4.560 - (-4.885)$ kJ/kgk

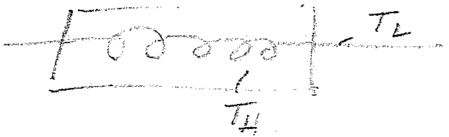
$= 0.325$ kJ/kgk

$$s_{gen,c} = 0 = -\frac{(2794.2 - 1154.5)}{273+263.94} - \frac{(988.2 - 2071.25)}{(273+81.32)}$$

$= -3.054 - (-3.055)$ close enough

$\simeq 0$

4.



both R-134A and cooled space are reservoirs ($\Delta T = 0$)

$$\Delta S_{R134a} = \frac{Q}{T_{R134}}$$

$$\Delta S_{LTR} = \frac{Q}{T_{LTR}}$$

$T_{R134} = T_s @ 160\text{kPa}$

$= -15.6°C$

$= 257.4k$

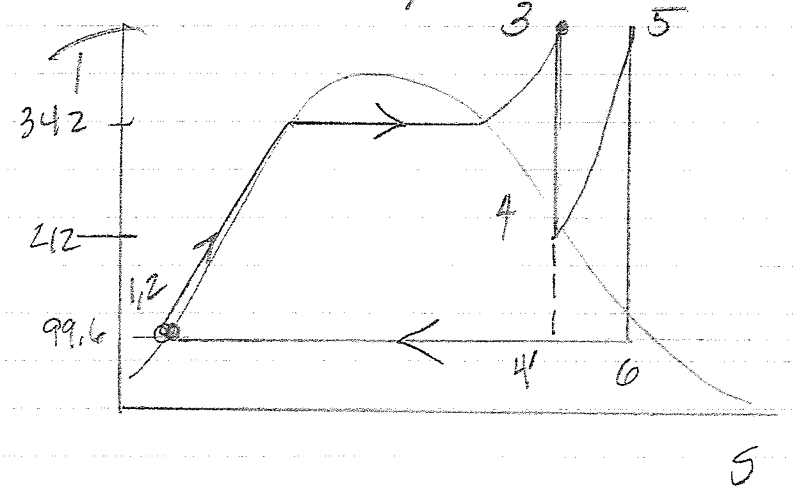
$T_{LTR} = 268k$

$$\Delta S_{R134a} = \frac{170 \text{ kJ}}{257.4} = 0.6605 \frac{\text{kJ}}{\text{k}}$$

$$\Delta S_{LTR} = \frac{-170 \text{ kJ}}{268} = -0.6343 \frac{\text{kJ}}{\text{k}}$$

$$\Delta S_{sys} = 0.6605 - 0.6343 = 0.0262 \frac{\text{kJ}}{\text{k}}$$

5. Rankine cycle, steam, reheat



$P_1 = 100$ kPa $\quad T_3 = T_5 = 450°$

$P_2 = 15$ MPa

$P_4 = 2$ MPa

$\dot{m} = 1.74$ kg/s

	T	P (kPa)	h	s
1	99.61	100	417.51	1.3028
2	99.61	15000	433.05	1.3028
3	450	15000	3157.9	6.1434
4	212.38	2000	2703.3	6.1434
5	450	2000	3358.3	7.282
6	99.61	100	2646.1	7.282

$h_1 = h_f$ @ 100 kPa

$$h_2 = h_1 + v(P_2 - P_1) = 417.51 + 0.001043(15000-100) = 433.05$$

$$-w_p = 0.001043(15000-100) = 15.54 \text{ kJ/kg}$$

$$s_f < s_4 < s_g \text{ @ } 2 \text{ MPa} \Rightarrow \text{mixture}$$

$$x_4 = \frac{6.1434 - 2.4467}{3.8923} = 0.95$$

$$h_4 = 0.95(1889.8) + 908.47 = 2703.3 \text{ kJ/kg}$$

interpolate @ 2 MPa

$$h_5 = \frac{3248.4 + 3468.2}{2} = 3358.3 \text{ kJ/kg}$$

$$s_5 = \frac{7.1292 + 7.4337}{2} = 7.282 \text{ kJ/kgK}$$

$$x_6 = \frac{7.282 - 1.3028}{6.0562} = 0.987$$

$$h_6 = 0.987(2257.5) + 417.51 = 2646.1 \text{ kJ/kg}$$

a) $-\dot{W}_p = 1.74 \text{ kg/s}\left(15.54 \ \frac{\text{kJ}}{\text{kg}}\right) = 27 \text{ kW}$

b) $\dot{W}_t = \dot{m}\left[(h_3 - h_4) + (h_5 - h_6)\right] =$

$= 1.74 \text{ kg/s}\left[(3157.9 - 2703.3) + (3358.3 - 2646.1)\right] = 2{,}030 \text{ kW}$

$\dot{W}_{net} = 2030 - 27 = 2{,}003 \text{ kW}$

5. c) $\dot{Q}_{45} = \dot{m}(h_5 - h_4) = 1.74 \frac{kg}{s}(3358.3 - 2703.3) = 1,140 \text{ kW}$

d) $\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_T - \dot{W}_P}{\dot{Q}_{45} + \dot{Q}_{23}} = \frac{(2030 - 27)}{1140 + 4741} = 0.341$

$\dot{Q}_{23} = \dot{m}(h_3 - h_2) = 1.74 \frac{kg}{s}(3157.9 - 433.05) = 4741 \text{ kW}$

e) Rankine, no reheat

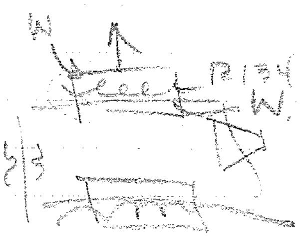
$$\eta = 1 - \frac{q_L}{q_H} = 1 - \frac{h_{4'} - h_1}{h_3 - h_2}$$

$$x_{4'} = \frac{6.1434 - 1.3028}{6.0562} = 0.799$$

$$h_{4'} = 0.799(2257.5) + 417.51 = 2221.9 \text{ kJ/kg}$$

$$\eta = 1 - \frac{(2221.9 - 417.5)}{(3157.9 - 433.05)} = 0.338$$

(0.9% lower η without reheat)



6. Refrigerator

$T_L = -30°C = 243 \text{ K}$

$\dot{Q}_H = \dot{m}_W C_p (T_2 - T_1) = 0.25 \frac{kg}{s}\left(4.184 \frac{kJ}{kg/°C}\right)(26 - 18) K$

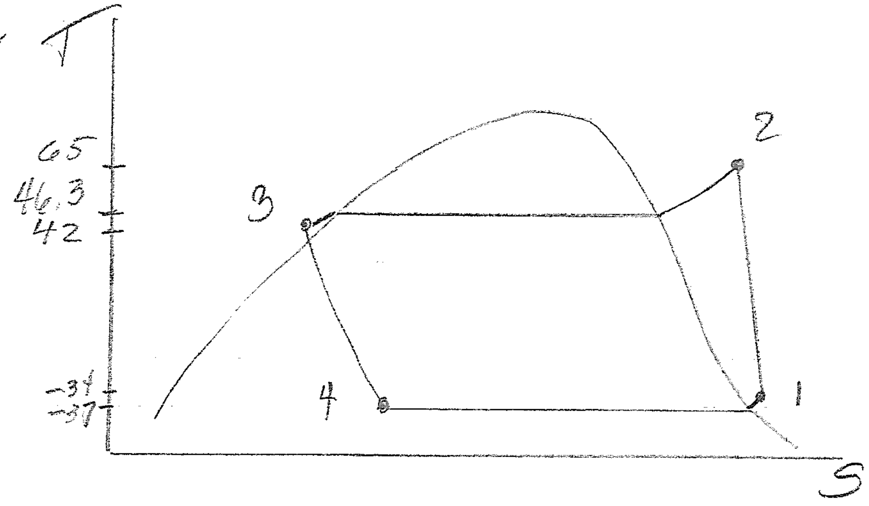
$\dot{Q}_H = 8.37 \text{ kW}$

$P_{2,3} = 1.2 \text{ MPa} \quad T_2 = 65°C, \; T_3 = 42°C$

$P_{1,4} = 60 \text{ kPa} \quad T_1 = -34°C$

$\dot{Q}_C = 0.45 \text{ kW}$

6.



	P (kPa)	T	h	s
1	60	-34	230.05	
2	1200	65	295.2	
3	1200	42	111.26	
4	60	-36.95	111.26	

0.174

$$\frac{-34-(-36.95)}{-20-(-36.95)} = \frac{h_1 - 227.79}{240.76-227.79}$$

$$h_1 = 230.05$$

$$h_2 = \frac{289.64+300.61}{2} = 295.2 \text{ kJ/kg}$$

$$h_3 \simeq h_f @ 42 = 111.26 \text{ kJ/kg}$$

a) $$x_4 = \frac{111.26 - 3.841}{223.95} = 0.48$$

b) $$\dot{m}_R = \frac{-\dot{Q}_H}{(h_3-h_2)} = \frac{-8.37\text{kW}}{(111.26-295.2)} = 0.046 \frac{\text{kg}}{\text{s}}$$

$$\dot{Q}_L = \dot{m}_R(h_1-h_4) = 0.046\frac{\text{kg}}{\text{s}}(230.05-111.26)$$

$$\dot{Q}_L = 5.40 \text{ kW}$$

6. c) $-\dot{W} = \dot{m}(h_2 - h_1) - \dot{Q}_e = 0.046\left[295.2 - 230.05\right] - 0.45$

$-\dot{W} = 2.55\,kW$

$COP = \frac{5.4}{2.55} = 2.1 =$ $\quad COP = \frac{1}{\frac{\text{[illegible]}}{\dot{Q}_L} - 1}$

d) $COP_{CR} = \frac{1}{\frac{T_H}{T_L} - 1}$ $\quad T_H = 273 + 46.3$

$T_L = 273 - 30$

$COP_{CR} = \frac{1}{\frac{319}{243} - 1} = 3.2$

$\dot{Q}_{L\,max} = COP_{CR}\,\dot{W} = 3.2(2.55) = 8.15\,k$

7. heat pump

$T_H = 25°C \qquad T_{1,4} = 20°C$

$P_{2,3} = 1.4\,MPa$

$-w_c = 20\,kJ/kg = h_2 - h_1$

$\dot{Q}_L = 2.7\,kW$

$h_1 = h_g @ 20°C = 261.59\,kJ/kg$

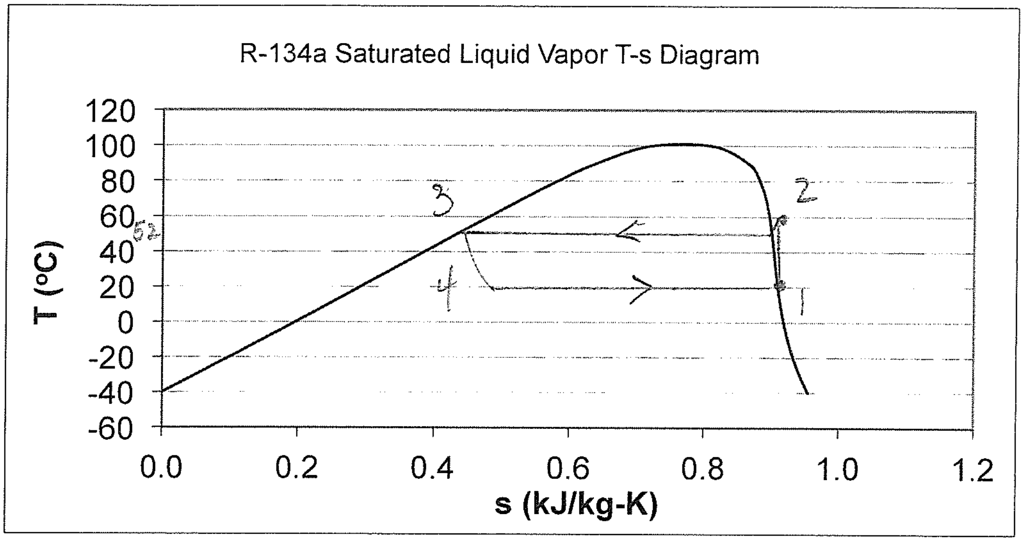
$h_2 = 20\frac{kJ}{kg} + 261.59 = 281.59\,kJ/kg$

$h_3 = h_f @ 1.4\,mPA = 127.22\,kJ/kg = h_4$

$\dot{m} = \frac{2.7\,kW}{(h_1 - h_4)} = \frac{2.7}{(261.59 - 127.22)} = 0.02\,kg/s$

b. The rate of cooling
c. The COP
d. The theoretical maximum cooling rate for the same power input to the compressor.

7. (3 points, 1 per part) A heat pump operates on the ideal vapor-compression refrigeration cycle with R-134a as the working fluid. This heat pump is used to keep a space at 25 °C by absorbing heat at a rate of 2.7 kw from geothermal water flowing through the evaporator heat exchanger. The evaporator operates at 20 °C, and the condenser pressure is 1.4 MPa. The compressor receives work at a rate of 20 kJ/kg.
   a. Show the process on a T-s diagram



   b. Determine the rate of heat transfer to the heated space.
   c. Determine the COP

8. (3 points) A water heater is operated using a heat pump that heats the water by removing heat from the room air and transferring it to the water. The heat pump has a COP of 3.4 and consumes 6 kw power. Determine if the heat pump can be used to cool the room for "free" by absorbing heat from the air in the room. The rate of heat gain in the house in summer is less than 45,000 kJ/hr.

9. (4 points, 1 per part) A Carnot refrigeration cycle uses R-134a as the working fluid. The maximum and minimum temperatures in the cycle are 30 °C and -20 °C, respectively. The quality of the R-134a at the inlet of the evaporator heat exchanger is 0.15 and 0.80 at the outlet.
   a. Show the process on the T-s diagram.
   b. Calculate the coefficient of performance.
   c. Find the condenser and evaporator pressures.
   d. Find the net work required.

$q - w = h_2 - h_1$

0

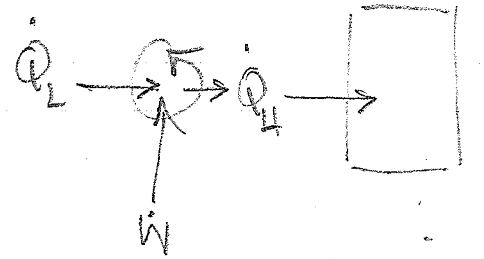
0.45 kw $\quad -w = h_2 - h_1 - 0.45$

7, $\dot{Q}_H = \dot{m}(h_3 - h_2) = 0.02\frac{kg}{s}(127.22 - 281.59)$

b) $\dot{Q}_H = -3.1$ kW

c) $COP = \frac{3.1}{\dot{m}(20\frac{kJ}{kg})} = \frac{3.1 \text{ kW}}{0.02(20) \text{ kW}} = 7.8$

8,



$COP = 3.4 = \frac{\dot{Q}_H}{\dot{W}}$

$\dot{W} = 6$ kW

$\dot{Q}_L = \frac{45000}{3600}$ kW $= 12.5$ kW

required

$COP_R = \frac{12.5 \text{ kW}}{6 \text{ kW}} = 2.08 < 3.4$

So, yes, heatpump can cool the room

9. Carnot refrig

$T_H = 30°C = 303K \quad T_L = -20°C = 253K$

$X_4 = 0.15, \ X_1 = 0.8$

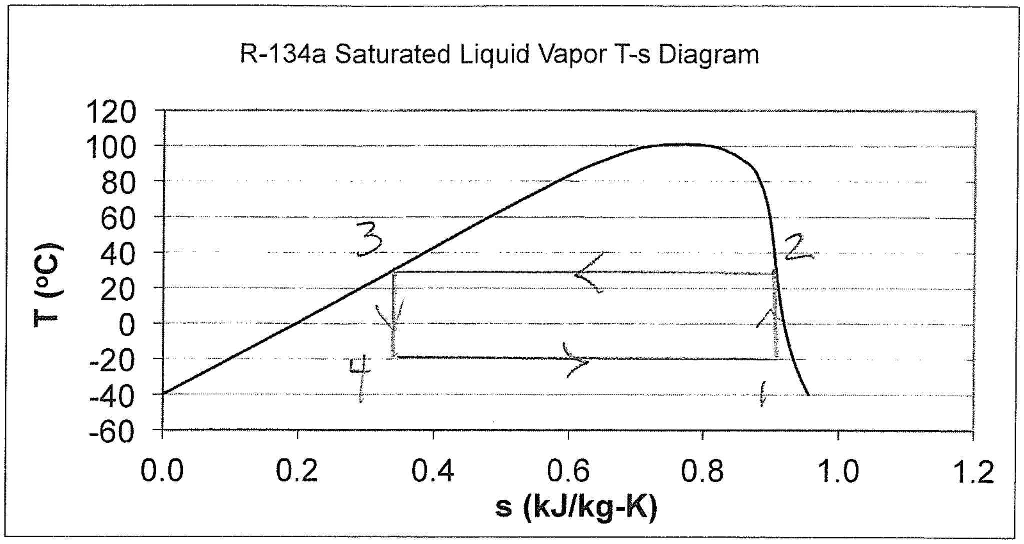
a) see diagram

b) $COP = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\frac{303}{253} - 1} = 5.06$

c) $P_{2,3} = P_{sat}$ @ 30°C = 770.64 kPa (condenser)

$P_{1,4} = P_{sat}$ @ −20°C = 132.82 kPa (evaporator)

13


R-134a Saturated Liquid Vapor T-s Diagram
T (°C)
120
100
80
60
40
20
0
-20
-40
-60
0.0
0.2
0.4
0.6
0.8
1.0
1.2
s (kJ/kg-K)
3
2
4
1

9 d) $h_1 = 0.8(h_{fg}) + h_f$ @ $-20°c$ =

$= 0.8(212.91) + 25.49 = 195.82$ kJ/kg

$h_4 = 0.15(212.91) + 25.49 = 57.43$ kJ/kg

$q_L = (195.82 - 57.43) = 138.4$ kJ/kg

$w = \frac{q_L}{COP} = \frac{138.4}{5.06} = 27.4 \frac{kJ}{kg}$