CVEN 7511: Computational Finite Inelasticity and Multiphase Mechanics

Meeting time, room, and zoom link: MWF 9:05-9:55am, ECCE 1B41, and on Zoom (`https://cuboulder.zoom.us/j/91077991693`). Recorded lectures posted after class at `https://drive.google.com/drive/folders/12O4ykGVZCHnaiLrl0deRiXqIjUoa4N8h`.

Instructor: Professor Richard Regueiro, 303.492.8026, `richard.regueiro@colorado.edu`; hybrid office hours (in-person in ECOT 421): Wednesday, 10-11am, `https://cuboulder.zoom.us/j/92308364754`

Course Description: The course will cover kinematics, thermodynamics, constitutive modeling, numerical time integration, and finite element implementation of large deformation (in)elasticity and multiphase mechanics. Kinematics of the multiplicative decomposition (see Fig.1), finite strain mixture theory, and resulting objective stress rates in the current configuration may be covered. Linearization for formulation and finite element implementation of algorithmic (consistent) tangent moduli, and local Newton-Raphson iteration for solution of nonlinear constitutive models, mixed formulations for nearly-incompressible elasticity, and coupled balance equations may also be covered.

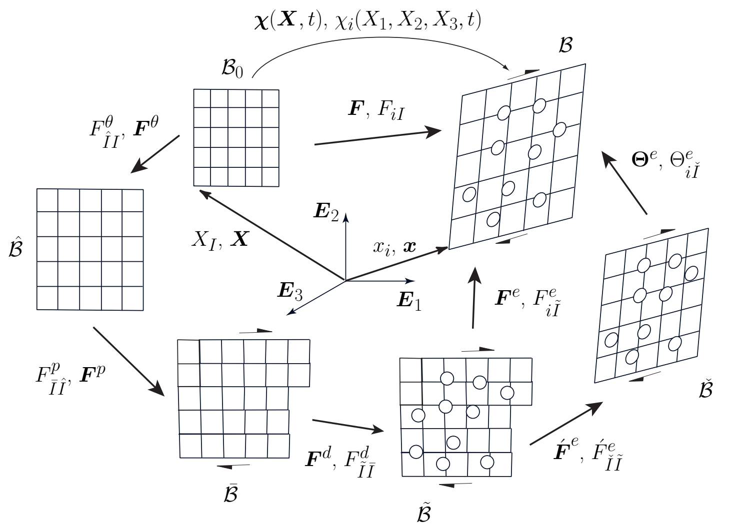

Figure 1. Multiplicative decomposition of the deformation gradient into elastic (e) with isochoric-volumetric split, damage (d), plastic (p), and thermal (θ) parts: $\boldsymbol{F} = \boldsymbol{\Theta}^e \acute{\boldsymbol{F}}^e \boldsymbol{F}^d \boldsymbol{F}^p \boldsymbol{F}^\theta$, $F_{iI} = \Theta^e_{i\check{I}} \acute{F}^e_{\check{I}\tilde{I}} F^d_{\tilde{I}\bar{I}} F^p_{\bar{I}\hat{I}} F^\theta_{\hat{I}I}$.

For the Final Project, students will be asked to choose a solid or multiphase material (soil, rock, concrete, polycrystalline metal, soft/hard biological tissue, foam, polymer, ...) for which to develop a finite strain constitutive model and/or multiphase balance equations, and then implement the model using `UMAT`, `VUMAT`, or `UHYPER` in `ABAQUS` (`www.3ds.com/products-services/simulia/products/abaqus`), full source code in `Python`, *or* another nonlinear finite element/finite difference/meshfree program of your choosing. Various model types include hyper-elasticity, hyper-viscoelasticity, hyper-elasto-plasticity, hyper-elasto-plasticity-damage, etc.

Proper formulation of finite strain constitutive models (multiplicative decomposition of the deformation gradient and comparison with additive strain decomposition forms, thermodynamic-consistency, ...) and multiphase mechanics balance equations will be emphasized, as well as the numerical details of implementing such models using `ABAQUS UMAT, UHYPER, UANISOHYPER_INV, VUMAT`, and also model implementation in full `Python` codes.

Course Objective: *To formulate and numerically implement within a nonlinear finite element (FE) program (or other numerical method of your choosing), your own finite strain inelastic constitutive model for a solid or multiphase material of interest, as well as possibly the nonlinear coupled physics governing equations themselves.*

Recommended prerequisites: CVEN 5511 (intro to linear FEM), CVEN 5131 (nonlinear continuum mechanics), CVEN 6511 (small strain elasto-plasticity, mixture theory, and nonlinear FEM), **or equivalents**, some knowledge of `Python` and `Fortran` or `C/C++` programming; or instructor consent. *Introductory finite element method (FEM) and continuum mechanics courses are required.*

Grading: problem sets 50%, final project 45%, class participation 5%.
*The *class participation* grade is based on in-class attendance. You may miss no more than 2 class periods without a legitimate reason (e.g., sickness, travel to conference, etc.), and must participate in in-class discussion in a meaningful manner when appropriate.

Primary References:
-G.A. Holzapfel, *Nonlinear Solid Mechanics*, John Wiley & Sons, 2000 (**required**) `https://libcat.colorado.edu/Record/b3190074`
-course notes written on board; journal articles and book chapters provided as pdf; other learning modules (such as combined code and notes) provided for self-learning.

Secondary References: (books in Engineering library or online as e-book), selected journal articles and book chapters provided as pdf on `https://canvas.colorado.edu/`

- *T. Belytschko, W.-K. Liu, B. Moran, *Nonlinear Finite Elements for Continua and Structures*, John Wiley, 2000. `https://libcat.colorado.edu/Record/in00000089213`
- **R. de Boer, *Trends in Continuum Mechanics of Porous Media*, Springer, 2005. `https://libcat.colorado.edu/Record/b5495120`
- *J. Bonet, R.D. Wood, *Nonlinear Continuum Mechanics for Finite Element Analysis*, Cambridge University Press, 2008. `https://libcat.colorado.edu/Record/in00000038727`
- *R.I. Borja, *Plasticity Modeling & Computation*, Springer-Verlag, 2013. `https://libcat.colorado.edu/Record/b7425079`
- **R.M. Brannon, *Curvilinear Analysis in a Euclidean Space*, 2004. `https://my.mech.utah.edu/~brannon/public/curvilinear.pdf`
- O. Coussy, *Poromechanics*, Wiley, 2004. `https://libcat.colorado.edu/Record/b12203408`
- *R. de Borst, M.A. Crisfield, J.J.C. Remmers, C.V. Verhoosel, *Non-linear Finite Element Analysis of Solids and Structures*, Wiley, 2012. `https://libcat.colorado.edu/Record/b8500804`
- M.A. Crisfield, *Non-linear Finite Element Analysis of Solids and Structures*, Vols. 1 (1991) and 2 (1997), Wiley. `https://libcat.colorado.edu/Record/b7609123`
- **A.C. Eringen, *Nonlinear Theory of Continuous Media*, McGraw-Hill, 1962. `https://libcat.colorado.edu/Record/b1247322`
- *T.J.R. Hughes, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*, Prentice-Hall, 1987. `https://libcat.colorado.edu/Record/b7459745`
- J. Lubliner, *Plasticity Theory*, Macmillan Pub., 1990. `https://libcat.colorado.edu/Record/b10848188`
- **J.E. Marsden, T.J.R. Hughes, *Mathematical Foundations of Elasticity*, Dover, 1994. `https://libcat.colorado.edu/Record/b1058388`
- *L.E. Malvern, *Introduction to the mechanics of a continuous medium*, Prentice-Hall, 1969. `https://libcat.colorado.edu/Record/b1105592`

- S. Nemat-Nasser, *Plasticity: a treatise on the finite deformation of heterogeneous inelastic materials*, Cambridge University Press, 2004. `https://libcat.colorado.edu/Record/b3715615`
- R.W. Ogden, *Non-Linear Elastic Deformations*, Dover, 1997. `https://libcat.colorado.edu/Record/b8858650`
- N.S. Ottosen, M. Ristinmaa, *The Mechanics of Constitutive Modeling*, Elsevier, 2005. `https://libcat.colorado.edu/Record/b9607672`
- *J.C. Simo, "Numerical Analysis and Simulation of Plasticity," *Handbook of Numerical Analysis*, Vol. VI, eds. P.G. Ciarlet and J.L. Lions, Elsevier Science, 1998, pgs.183-499. `https://libcat.colorado.edu/Record/b10328465`
- *J.C. Simo, T.J.R. Hughes, *Computational Inelasticity*, Springer-Verlag, 1998. `https://libcat.colorado.edu/Record/b7982161`
- **E.A. de Souza Neto, D. Peric, D.R.J. Owen, *Computational Methods for Plasticity: Theory and Applications*, Wiley, 2008. `https://libcat.colorado.edu/Record/b12208558`
- C. Truesdell, W. Noll, *The Non-Linear Field Theories of Mechanics*, 3rd ed, Springer, 2004. `https://libcat.colorado.edu/Record/b3628864`
- *C. Truesdell, R.A. Toupin, "The Classical Field Theories," *Encyclopedia of Physics: Principles of Classical Mechanics and Field Theory*, vol.III, ed. S.Flugge, Springer-Verlag, 1960. `https://libcat.colorado.edu/Record/b12839038`

  **recommended (but it depends on reader preference based upon writing and presentation style of the authors)*
  ***selected chapters provided as pdf*

Course Outline: (tentative; subject to change by the Instructor)

1. **Review of Nonlinear Continuum Mechanics (5 weeks):** (a) motion (material and spatial descriptions), curvilinear coordinates, base vectors, metric tensors, strain; (b) material time derivative, deformation rate, spin; (c) stress, objectivity, isotropy; (d) balance of momenta (material and spatial descriptions), thermodynamics: balance of energy, second law of thermodynamics, Clausius-Duhem inequality, Helmholtz free energy function, constitutive forms; (e) hyperelasticity, isochoric-volumetric split; (f) # viscoelasticity.

2. **Nonlinear Finite Element Implementation and Solution of Hyperelasticity (4 weeks):** (a) Total Lagrangian Finite Element (FE) formulation (strong form, weak form) with finite strain hyperelasticity; (b) #Updated Lagrangian and/or Material Point Method (MPM) formulation and implementation; (c) linearization of weak form for solution by Newton-Raphson method, and algorithmic tangent moduli; (d) FE matrix form (1D uniaxial strain, 3D), `Python` code, and `ABAQUS` User Material subroutines (`UHYPER, UMAT`); (e) Total Lagrangian mixed 3D FE formulation and implementation of nearly-incompressible hyperelasticity via isochoric-volumetric split. (f) #nearly-incompressible hyper-visco-elasticity.

   #time permitting

Figure 2. (left) theory of porous media assumes control space is that of the solid skeleton $\mathcal{B} = \mathcal{B}^{\text{skel}}$ (image is lung parenchyma, Lande and Mitzner *J. Appl. Physiol. 2006*); (middle) foam (`https://www.foambymail.com/`); (right) sand liquefaction (`www.cti.co.jp`).

Two options for the next 6 weeks (will depend on student interest, and both may be provided through self-learning modules and limited in-class discussion):

(1)

3. **Finite Strain Elasto-Plasticity (6 weeks):** (a) multiplicative decomposition of the deformation gradient $\boldsymbol{F} = \boldsymbol{F}^e \cdot \boldsymbol{F}^p$; (b) strains in reference, intermediate, and current configurations; (c) velocity gradients, deformation rates, and spins; (d) #extensions to include thermal deformation and damage; (e) elastic Lie derivative; (f) stress, objectivity, isotropy; (g) thermodynamics and constitutive model forms, reduced dissipation inequality, evolution equation forms; (h) yield and plastic potential functions for various materials (metals, geomaterials, ...); (i) Backward Euler time integration method, exponential map for isotropic materials; semi-implicit schemes; `Python` code, and `ABAQUS` User Material subroutine (`UMAT`); (j) # elastic and plastic anisotropy.

(2)

3. **Nonlinear Multiphase Continuum Mechanics (3 weeks):** (a) concept of volume fraction and mixture theory; (b) motion (material and spatial descriptions) and kinematics, material time derivative, deformation rate, spin; (c) balance of mass (material and spatial descriptions); (d) balance of linear momentum (material and spatial descriptions); (e) thermodynamics (first and second laws, constitutive equation forms, including Darcy's law and effective stress principle at finite strain).

4. **Nonlinear Finite Element Implementation and Solution of Finite Strain Poromechanics (3 weeks):** (a) Total Lagrangian formulation and nonlinear FE implementation of bi-phasic solid-fluid mixture with elastic solid skeleton, coupled strong and weak forms; (b) time integration for transient analysis (generalized trapezoidal rule, and # Newmark's method); (c) linearization for solution by Newton-Raphson method; (d) algorithmic tangent moduli; (e) numerical examples in `Python` and `ABAQUS`.

FE software and Final Project: The commercially-available FE software program `ABAQUS` will be used to learn how to implement finite strain constitutive models via `UMAT` (implicit, requires consistent tangent, pre-defined rotated configuration, Jaumann objective stress rate), `UHYPER` (implicit, requires derivatives of free energy with respect to deformation invariants), `VUMAT` (explicit, no consistent tangent, pre-defined rotated configuration, Green-McGinnis-Naghdi objective stress rate), as well as analyze engineering problems of interest to you using your implemented constitutive model in `ABAQUS` (with finite strain mixture theory) *or another non-linear finite element, finite difference, or meshfree program of your choosing.* `Python` codes will be used to demonstrate full source code finite element implementation of finite strain constitutive models, mixed formulation, and/or coupled nonlinear finite strain bi-phasic mixture theory.

Create your own 3DS account: (for accessing online users manual and the learning edition software) `https://www.3ds.com/edu/education/students/solutions/abaqus-le`

Abaqus online users manual (you need to enable your 3ds account first): `https://help.3ds.com/2023/English/DSSIMULIA_Established/SIMULIA_Established_FrontmatterMap/sim-r-DSDocAbaqus.htm?contextscope=all`

Honor Code:
Refer to the webpage:
`https://www.colorado.edu/sccr/students/honor-code-and-student-code-conduct`
If you violate the honor code, you will receive a failing grade of "F" for the course, regardless of the degree of academic dishonestly.

Special considerations: If you have a disability and require special accommodations, please provide Dr. Regueiro with a letter from Disability Services outlining your needs. Refer to the webpage `https://www.colorado.edu/disabilityservices`. If you have a conflict as a result of religious observances, notify Dr. Regueiro at least 2 weeks in advance of the assignment due date: `https://www.colorado.edu/oiec/religious-accommodations`

Review the required syllabus statements: `https://www.colorado.edu/academicaffairs/policies-customs-guidelines/required-syllabus-statements`

Bechtel Computing Laboratory: Refer to instructions posted on the door of ECCE 157 or 161, if you do not currently have Buff OneCard swipe access to the Bechtel Lab in ECCE 157, 161. Instructions for remotely accessing Bechtel computers for use of `ABAQUS` and `Jupyter Notebook (Python)` are provided here `https://oit.colorado.edu/services/learning-spaces-technology/computing-labs/remote-access`, and posted on canvas for using `Splashtop Business`.