CVEN 6831 SP TPC: Computational Finite Inelasticity

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office hrs: MW 12-1:30pm

Course Description: The course will cover kinematics, thermodynamics, constitutive modeling, numerical time integration, and finite element implementation of finite deformation (in)elasticity. Kinematics of multiplicative decomposition, and resulting objective stress rates in the current configuration will be covered. Linearization for formulation and finite element implementation of algorithmic (consistent) tangent moduli, and local Newton-Raphson iteration for solution of nonlinear constitutive models will be covered. *The emphasis is on being able to formulate, and numerically implement within a nonlinear finite element program, your own finite strain inelastic constitutive model for a solid material of interest to you.*

For the final project, students will be asked to choose a solid material (soil, rock, concrete, polycrystalline metal, soft/hard biological tissue, foam, polymer, ...) for which to develop a finite strain constitutive model, and then implement that constitutive model using the `UMAT` (or `VUMAT` for high strain rate problems) in `ABAQUS` (`www.simulia.com`) or another finite element/finite difference program of your choosing. Various model types include hyper-elasticity, hyper-viscoelasticity, hyper-elasto-plasticity, hyper-elasto-plasticity-damage, etc.

Proper formulation of finite strain constitutive models (multiplicative decomposition of the deformation gradient and comparison with additive strain decomposition forms, thermodynamic-consistency, ...) will be emphasized, as well as the numerical details of implementing such models using `ABAQUS UMAT, UHYPER, UANISOHYPER_INV, UEL, VUMAT`.

Prerequisites: CVEN 5511 (Intro to FEM), CVEN 5131 (Continuum Mechanics), CVEN 5788 (small strain elasto-plasticity and nonlinear FEM), or equivalents, some knowledge of Matlab and Fortran programming; or instructor consent. Some introductory finite element method (FEM) and continuum mechanics courses are required.

Grading: problem sets 40%, in-class midterm exam 30%, final project 30%.

Primary References: course notes written on board

Some more references: (books on reserve in Engineering library), selected journal articles given in class or on CULearn

1. *T. Belytschko, W.-K. Liu, B. Moran, *Nonlinear Finite Elements for Continua and Structures*, John Wiley, 2000.
2. A.C. Eringen, *Nonlinear Theory of Continuous Media*, McGraw-Hill, 1962.
3. *G.A. Holzapfel, *Nonlinear Solid Mechanics*, John Wiley & Sons, 2000.
4. J. Lubliner, *Plasticity Theory*, Macmillan Pub., 1990.
5. J.E. Marsden, T.J.R. Hughes, *Mathematical Foundations of Elasticity*, Dover, 1994.
6. S. Nemat-Nasser, *Plasticity: a treatise on the finite deformation of heterogeneous inelastic materials*, Cambridge University Press, 2004.
7. R.W. Ogden, *Non-Linear Elastic Deformations*, Dover, 1997.
8. J.C. Simo, "Numerical Analysis and Simulation of Plasticity," *Handbook of Numerical Analysis*, Vol. VI, eds. P.G. Ciarlet and J.L. Lions, Elsevier Science, 1998, pgs.183-499.

9. C. Truesdell, W. Noll, *The Non-Linear Field Theories of Mechanics*, 3rd ed, Springer, 2004.

10. C. Truesdell, R.A. Toupin, "The Classical Field Theories," *Encyclopedia of Physics: Principles of Classical Mechanics and Field Theory*, vol.III, ed.S.Flugge, Springer-Verlag, 1960.

11. *J.C. Simo, T.J.R. Hughes, *Computational Inelasticity*, Springer-Verlag, 1998.

    *recommended purchase, but not required

Course Outline: (tentative)

1. **Review of Nonlinear Continuum Mechanics (2 weeks):** (a) motion (material and spatial descriptions), curvilinear coordinates, base vectors, metric tensors, strain; (b) material time derivative, deformation rate, spin; (c) stress, objectivity, isotropy, balance of momenta (material and spatial descriptions)

2. **Kinematics of Finite Strain Elastoplasticity (2 weeks):** (a) multiplicative decomposition of the deformation gradient $\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p$; (b) elastic and plastic strains in reference, intermediate, and current configurations; (c) velocity gradients, deformation rates, and spins; (d) extensions to include thermal deformation, damage, and viscoelasticity; (e) choice of covariant metric coefficients in intermediate configuration $\bar{G}_{\bar{I}\bar{J}}$ (also contravariant metric), (f) elastic Lie derivative

3. **Thermodynamics (2 weeks):** (a) balance of energy, second law of thermodynamics (intermediate and current configurations), Clausius-Duhem inequality; (b) Helmholtz free energy function, constitutive forms, reduced dissipation inequality; (c) plastic evolution equation forms

4. **Constitutive Equations (3 weeks):** (a) isotropic elasticity and J_2 flow plasticity, map forward to current configuration; (b) #anisotropic elastoplasticity; (c) other elastoplasticity/constitutive models (student interest; journal articles), such as hyperelasticity (volumetric/isochoric split), anisotropic hyperelasticity, thermo-elasto-plasticity, ...

5. **Numerical Integration of Constitutive Equations (3 weeks):** (a) Backward Euler method; (b) semi-implicit schemes; (c) `ABAQUS` user subroutines

6. **Nonlinear Finite Element Solution (4 weeks):** (a) Total Lagrangian formulation; (b) Updated Lagrangian formulation; (c) linearization for solution by Newton-Raphson method; (d) algorithmic tangent moduli; (e) finite element implementation of J_2 flow elastoplasticity model via `UMAT`; (f) finite element implementation of J_2 flow elastoplasticity model via `VUMAT`

   #if we have time

FE software and Project: The commercially-available FE software program `ABAQUS` will be used to learn how to implement finite strain constitutive models via `UMAT` (implicit, requires consistent tangent, pre-defined rotated configuration, Jaumann objective stress rate), `UHYPER` (implicit, requires derivatives of free energy with respect to deformation invariants), or `VUMAT` (explicit, no consistent tangent, pre-defined rotated configuration, Green-McGinnis-Naghdi rate), as well as analyze engineering problems using your implemented constitutive model.