Solution of Homework # 4

1. The inverse functions of \( y = x^2 \) are \( x_1 = \sqrt{y}, \ x_2 = -\sqrt{y} \) for \( 0 < y < 1 \) and \( x_1 = \sqrt{y} \) for \( 1 < y < 4 \). Now \( |J_1| = |J_2| = |J_3| = 1/2\sqrt{y} \) from which we get

\[
g(y) = f(\sqrt{y})|J_1| + f(-\sqrt{y})|J_2| = \frac{2(\sqrt{y} + 1)}{9} \cdot \frac{1}{2\sqrt{y}} + \frac{2(-\sqrt{y} + 1)}{9} \cdot \frac{1}{2\sqrt{y}} = 2/9\sqrt{y}, \ 0 < y < 1
\]

and

\[
g(y) = f(\sqrt{y})|J_1| = \frac{2(\sqrt{y} + 1)}{9} \cdot \frac{1}{2\sqrt{y}} = (\sqrt{y} + 1)/9\sqrt{y}, \ 1 < y < 4.
\]

2. \( u_1 - u_2 = 0, \sigma_{\bar{R}_1 - \bar{R}_2} = \frac{50}{\sqrt{1/32} + 1/750} = 11.319. \)

(a) \( z_1 = \frac{-20/11.319}{-1.77} = z_2 = \frac{20/11.319}{1.77}; \)

\[
P(|\bar{R}_1 - \bar{R}_2| > 20) = 2P(Z < -1.77) = (2)(0.0384) = 0.0768.
\]

(b) \( z_1 = \frac{5/11.319}{0.44}, z_2 = \frac{10/11.319}{0.88}. \)

\[
P(-10 < \bar{R}_1 - \bar{R}_2 < -5) + P(5 < \bar{R}_1 - \bar{R}_2 < 10) = 2P(5 < \bar{R}_1 - \bar{R}_2 < 10) = 2P(0.44 < 2 < 0.88) = 2(0.8106 - 0.6700) = 0.2812.
\]

4. \( \bar{x} = 0.475; \ s^2 = 0.0336; \ t = (0.475 - 0.5)/0.0648 = -0.39; \)

\[
P(\bar{x} < 0.5) = P(t < -0.39) \approx 0.35; \text{ in conclusive.}
\]

6. \( n_p = 8, \ n_u = 8, \ x_p = 86,250.00, \ x_u = 79,837.50, \ \sigma_1 = \sigma_2 = 4000 \)

\[
(86,250.00 - 79,837.00) - (1.96)(4000)\sqrt{\frac{1}{8} + \frac{1}{8}}
\]

\[
< \mu_p - \mu_u < (86,250.00 - 79,837.00) + (1.96)(4000)\sqrt{\frac{1}{8} + \frac{1}{8}}
\]

\[
2492.50 < \mu_p - \mu_u < 10,332.50
\]

Polishing increases the average indurance limit.
Yields with fracturing
\[ r_{\text{crit}} = .932, \text{ accept normality} \]
Yields without
\[ r_{\text{crit}} = .928, \text{ reject normality} \]

Because one of the groups is non-normal, the rank-sum test is performed.
\[ W_{rs} = 71, \text{ reject equality}. \] The one-sided \( p \)-value from the large-sample approximation
\[ p = 0.032. \] Reject equality. The yields from fractured rocks are higher.

7.2 (5.6) The test statistic changes very little \( (W_{13} = 123) \), indicating that most information contained in the data below detection limit is extracted using ranks. Results are the same (one-sided \( p \)-value = 0.039. Reject equality). A t-test could not be used without improperly substituting some contrived values for less-thans which might alter the conclusions.

8.1 (6.4) Because of the data below the reporting limit, the sign test is performed on the differences Sept—June. The one-sided \( p \)-value = 0.002. Sept atrazine concentrations are significantly larger than June cones before application.

8.2 (6.5) For the t-test, \( t = 1.07 \) with a one-sided \( p \)-value of 0.15. The t-test cannot reject equality of means because one large outlier in the data produces violations of the assumptions of normality and equal variance.

9. \( H_0: \sigma_1 = \sigma_2 \quad s_1 = 281.0667 \) (1980 models)
\( H_1: \sigma_1 \neq \sigma_2 \quad s_2 = 119.3946 \) (1990 models)
\[ f = 5.54 \quad P = 0.0005 \]

Decision: Hydrocarbon emissions are more consistent in the 1990 model cars.