Solution for Homework #2

1. \[ P(F) = 0.25, P(H) = 0.75, P(S) = 0.10, P(W) = 0.90 \]
   \[ C \text{ - The structure will collapse.} \]
   \[ P(C|S) = 1.0, P(C|WH) = 1.0, P(C|WF) = 0.5 \]
   \[ P(C|WF) = 0.5 \]
   \[ P(F|C) = P(S|F) + P(W|F) \]
   \[ = P(C|S)P(S|F) + P(C|WF)P(W|F) \]
   \[ = P(C|S)P(S|F) + P(C|WF)P(W|F) \]
   \[ = 1.0 \times 0.1 + 0.5 \times 0.9 = 0.646 \]
   \[ \text{Now,} \]
   \[ P(C) = P(C|S)P(S) + P(C|WH)P(WH) + P(C|WF)P(WF) \]
   \[ = 1.0 \times 0.1 \times 0.75 + 1.0 \times 0.1 \times 0.25 + 0.5 \times 0.9 \times 0.25 \]
   \[ = 0.2125 \]
   \[ \text{So,} \]
   \[ P(F|C) = \frac{0.646}{0.2125} \]
   \[ = 0.866 \]

2. Let, \( F_s \) and \( F_h \) represent that the county will be flooded in a year by snow melting and by hurricanes respectively. \( h_0, h_1, \) and \( h_2 \) represent that the county will be hit by none, one, and two hurricanes respectively in a year.

\[ P(F_s) = 1.0 \]
\[ P(H_0) = 0.05, P(H_0) = 0.65 \]
\[ P(F_{h1}|h_1) = 0.75 \]
\[ P(F_{h2}|h_2) = 1.0 \]
\[ \text{Using theorems of total probability,} \]
\[ P(F^\text{flood in the county in a year}) = P(F_s) + P(F_{h1})P(H_0) + P(F_{h2}) \]
\[ = 0.2125 \times 0.75 + 0.05 \times 0.5 \]
\[ = 0.646 \]

3. This is a parallel system of two series subsystems.
   \[ (a) P = 1 - (1 - 0.7)(0.7)(1 - 0.8)(0.8)(0.8) = 0.7511 \]
   \[ P = \frac{(0.7)(0.8)(0.8)(0.8)}{0.7511} = 0.2045 \]

4. \[ (a) P(A \cap B \cap C) = P(A)P(B)P(C) = 0.20(0.25)(0.80) = 0.045 \]
   \[ (b) P(A \cap B') = P(B'|A)P(A) = 0.25(0.25)(0.3) = 0.075 \]
   \[ P(A' \cap B') = P(B'|A)P(A') = 0.80(0.15) = 0.12 \]
   \[ P(B' \cap C) = P(C|B')P(A')P(A') = 0.56 \]
   \[ P(A \cap B' \cap C) = P(C|A \cap B')P(A) = 0.80(0.70) = 0.56 \]
   \[ P(C|A \cap B') = P(A \cap B')P(A)P(A') = 0.56 \]
   \[ P(C|A \cap B') = P(A \cap B')P(A)P(A') = 0.56 \]

5. Referring to the sample space in Exercise 3 and making use of the fact that \( P(H) = 2/3 \) and \( P(T) = 1/3 \), we have
   \[ P(\text{W}) = 1 = P(\text{HTT}) + P(\text{THT}) + P(\text{THH}) = \frac{2}{27} + \frac{2}{27} + \frac{8}{27} = \frac{12}{27} \]
   \[ P(\text{W}) = 3 = P(\text{HHH}) = \frac{8}{27} \]
   \[ \text{The probability distribution for W is then} \]
   \[ \begin{array}{c|cccc}
   \hline
   \text{W} & -3 & -1 & 1 & 3 \\
   \hline
   P(\text{W = w}) & \frac{1}{27} & \frac{2}{27} & \frac{4}{27} & \frac{8}{27} \\
   \hline
   \end{array} \]

6. \[ (a) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x^2 + y^2) dy \]
   \[ = k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{1}{2} + \frac{1}{2} \right) dx \]
   \[ = k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{3}{2} + \frac{3}{2} \right) dx \]
   \[ = k \int_{-\infty}^{\infty} \left( \frac{3}{2} \right) dx \]
   \[ = k \left( \frac{3}{2} \right) 2 \]
   \[ = \frac{3}{2} \cdot 10^{-4} \]

\[ (b) \int_{0}^{3} \int_{0}^{3} (x^2 + y^2) dy \]
   \[ = k \int_{0}^{3} \int_{0}^{3} \left( \frac{3}{2} + \frac{3}{2} \right) dx \]
   \[ = k \int_{0}^{3} \left( \frac{3}{2} \right) dx \]
   \[ = k \left( \frac{3}{2} \right) 3 \]
   \[ = \frac{3}{2} \cdot 10^{-4} \]

\[ (c) \int_{0}^{3} \int_{0}^{3} (x^2 + y^2) dy \]
   \[ = k \int_{0}^{3} \int_{0}^{3} \left( \frac{3}{2} + \frac{3}{2} \right) dx \]
   \[ = k \int_{0}^{3} \left( \frac{3}{2} \right) dx \]
   \[ = k \left( \frac{3}{2} \right) 3 \]
   \[ = \frac{3}{2} \cdot 10^{-4} \]