APPLICATIONS OF THEORY OF ELASTICITY IN ROCK MECHANICS

1. INTRODUCTION

2. CLOSED-FORM SOLUTIONS VS. NUMERICAL METHODS

3. TWO-DIMENSIONAL FORMULATION

3.1 Plane Strain Analysis
3.2 Plane Stress Analysis
3.3 Generalized Plane Strain Analysis
3.4 Analysis in Polar or Cylindrical Coordinates

4. EXAMPLES

4.1 Hollow Cylinder under Pressure
4.2 Circular Hole Drilled Under a Triaxial Principal Stress Field at Infinity
4.3 Circular Hole Drilled Under a Triaxial Stress Field-General Solution

5. USING STRESSES IN ROCK ENGINEERING

6. REFERENCES

Recommended Readings:

1) Chapter 4 in *Rock Mechanics and the Design of Structures in Rocks*, by Obert, L. and Duvall, W.I., Wiley, New York, 1967.

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1. INTRODUCTION

The theory of elasticity is used widely in rock mechanics to predict how rock masses respond to loads and excavation (surface and underground). The assumptions inherent to the theory of elasticity are:

- the material is elastic (linear or non-linear) which implies an immediate response during loading and a fully reversible response upon unloading,

- the material behaves as a continuum.

If time is involved (time-deferred response), the theory of viscoelasticity should be used instead.

In classical linear elastic theory, it is assumed that no distinction needs to be made between the Lagrangian and Eulerian descriptions of strain. The six components of stress, the six components of strain and the three components of displacement must satisfy some basic equations called *field equations*, e.g. 3 equations of equilibrium, 6 equations relating strains to displacement and 6 equations relating stresses to strains. Thus, a total of 15 equations are available to solve for 15 unknowns, i.e. 6 stresses, 6 strains, and 3 displacements.

In solving for the 15 unknowns, stresses and displacements must satisfy the boundary conditions which can take different forms: prescribed displacements only, prescribed stresses (surface tractions) only, or mixed stress and displacement boundary conditions. In all cases, the body forces are assumed to be given throughout the continuum.

Most problems in rock mechanics are three-dimensional. Under certain assumptions of plane strain or plane stress, a two-dimensional approximation can be made. In plane stress problems, the geometry of the rock mass is essentially that of a plate with one dimension much smaller than the others. Plane stress is used for instance to model surface problems. In plane strain problems, the geometry of the body is essentially that of a prismatic cylinder with one dimension much larger than the others. These notes are limited to two-dimensional elastostatic problems.

Rocks and rocks masses do not always behave elastically or as continua. Nevertheless, for a wide range of engineering problems, useful solutions may be obtained by treating the rock as a homogeneous, linearly elastic continuum. If necessary, anisotropy and nonlinearity may be taken into account. Despite its limitation, the elastic analysis can be used to evaluate a number of factors of importance in rock engineering. Examination of solutions to elastic stress distribution problems can provide useful qualitative guidelines for the design of engineering structures built in or on rock. For instance, elastic stress analysis provides an assessment of the extent of compression and tension zones, the zones of influence of excavations, the stress intensity and the extent of highly stressed zones. A failure criterion can be superposed on the elastic solution to assess the extent of overstressed rock around the excavations of interest. The

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basic assumption behind this approach is that the existence of the plastic zone does not disturb the stress distribution. A failure criterion can also be included in the solution (elasto-plastic analysis). Elastic analysis can also help in determining the rock deformation associated with the stresses. Deformation, and not stress, may sometimes become the critical factor in rock engineering design.

2. CLOSED-FORM SOLUTIONS VS. NUMERICAL METHODS

Elasticity problems can be solved using *closed-form solutions* or *numerical methods* depending on the complexity of the material of interest and the geometry of the problem being addressed.

Closed-form solutions are usually used when the material is homogeneous, isotropic (or anisotropic), and the boundary of the problem is of simple shape. Closed-form solutions are available in various texts such as Obert and Duvall (1967), Timoshenko and Goodier (1970), Lekhnitskii (1977), Jaeger and Cook (1979), Poulos and Davis (1974), etc... Closed-form solutions can be derived, for instance, to study the response of a rock mass to surface loads, and surface or underground excavation. Various closed-form solutions are also available for the analysis of field and laboratory rock mechanics tests.

Numerical methods are used instead when the material has a complex constitutive behavior (non-linear behavior, elasto-plastic, etc..) and/or the problem geometry is complicated. The most frequently applied numerical procedures in rock engineering and rock mechanics are the finite element method (FEM), boundary element methods (BEM), and the discrete element methods (DEM). Both FEM and BEM are used when the rock mass is modelled as a continuum with several discrete planes of weakness. DEM is used when the rock mass contains a large number of blocks, and its deformation is controlled mostly by the opening of, closing of, and sliding along the discontinuities and to a lesser extent by the block deformation. Hybrid methods are also used in order to preserve the advantages of each method and eliminate their disadvantages. A review of the numerical methods in rock mechanics can be found in Pande et al. (1990).

Finite Element Method

- efficient numerical solution procedure,
- complex constitutive behavior can be modelled,
- requires discretization of the complete problem domain,
- arbitrary external boundaries are needed,
- the size of the numerical problem to be solved is related to the volume of the problem domain.

Boundary Element Methods

- no requirement to define arbitrary external boundaries from the problem area,
- size of the numerical problem increases with the size of surface area of excavation,

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- volume of the problem is considered explicitly in the analysis,
- discretization of problem boundaries only,
- low demand on computer storage,
- simplicity of data input,
- calculate stresses, strains and displacements at points of interest,
- limited to rock masses with linear constitutive behavior.

Discrete Element Methods

- analysis of large block movement in geologically complex regions having many joint blocks,
- can be performed by a microcomputer and displayed interactively,
- mostly two-dimensional,
- precise location of joints needs to be input.

BEM, FEM, DEM Coupling

- BEM for far-field rock mass (continuum) and DEM near-field rock mass (discontinuum),
- coupling BEM and FEM eliminates boundaries and uncertainties associated with outer boundary conditions,
- far-field rock mass is a homogeneous continuum modeled with BEM and near-field rock mass is modeled as a continuum with zones (usually small and localized) of complex constitutive behavior.

3. TWO-DIMENSIONAL FORMULATION

The material is assumed to be isotropic an linearly elastic and under plane strain or plane stress. Body forces are assumed to be absent or constant. Stresses and strains are defined in the x,y plane of an x,y,z coordinate system.

3.1 Plane Strain Analysis

As discussed in Section 4.0 in Lecture Notes 3, plane strain in the x,y plane implies that the displacement components u, v and w in the x, y and z directions are such that

$$u = u(x,y) \ ; \quad v = v(x,y) \ ; \quad w = 0 \tag{1}$$

The geometry of the body is essentially that of a prismatic cylinder with one dimension (z direction) much larger than the others. Loading of the body (body and surface forces) is directed normal to the z-axis, and all cross-sections of the body are identically loaded.

In view of equation (1), the strains ϵ_{xx}, ϵ_{yy} and ϵ_{xy} are the only non-vanishing strain components. They satisfy the following compatibility equation

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$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \epsilon_{xy}}{\partial y \partial x} \tag{2}$$

Since ϵ_{zz}, ϵ_{yz} and ϵ_{xz} vanish, according to Hooke's law, the stresses τ_{xz} and τ_{yz} also vanish and $\sigma_z = \nu(\sigma_x + \sigma_y)$. The strains ϵ_{xx}, ϵ_{yy} and ϵ_{xy} are related to the stresses σ_x, σ_y and τ_{xy} as follows

$$\begin{aligned}
\epsilon_{xx} &= \frac{1-\nu^2}{E}\left(\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right) \\
\epsilon_{yy} &= \frac{1-\nu^2}{E}\left(\sigma_y - \frac{\nu}{1-\nu}\sigma_x\right) \\
\epsilon_{xy} &= \frac{(1+\nu)}{E}\tau_{xy}
\end{aligned} \tag{3}$$

The stresses σ_x, σ_y and τ_{xy} satisfy the following equations of equilibrium

$$\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho b_x &= 0 \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \rho b_y &= 0
\end{aligned} \tag{4}$$

where ρ is the density and ρb_x and ρb_y are the components of the body force per unit volume in the x and y directions, respectively. Assume that the body force components can be derived from a potential U(x,y) such that

$$\rho b_x = -\frac{\partial U}{\partial x} \ ; \quad \rho b_y = -\frac{\partial U}{\partial y} \tag{5}$$

and that there is a stress function F(x,y) (also known as the *Airy stress function*) such that

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} + U \ ; \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} + U \ ; \quad \tau_{xy} = -\frac{\partial^2 F}{\partial y \partial x} \tag{6}$$

The expressions of the stress components in (6) satisfy automatically the equilibrium equations (4). Combining equations (2), (3) and (6) gives a differential equation of the type

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4} = \frac{(2\nu-1)}{(1-\nu)}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{7}$$

If the body force components are constant or vanish, the right hand side of equation (7) vanishes and equation (7) becomes the so-called *biharmonic equation* $\nabla^2(\nabla^2 F)=0$. Functions F(x,y) which satisfy such equation are called *biharmonic functions.* Various mathematical expressions for F(x,y) can be found in the mathematical literature and can be applied to solve rock mechanics problems with various boundary conditions.

A special case of plane strain solution is when ϵ_z does not vanish and has a constant value ϵ_{zzo}. In that case $\sigma_z=\nu(\sigma_x+\sigma_y) + E\epsilon_{zzo}$ and $-\nu\epsilon_{zzo}$ needs to be added to the right hand side of ϵ_{xx} and ϵ_{yy} in (3). The stress function F(x,y) still satisfies equation (7).

3.2 Plane Stress Analysis

Plane stress in the x,y plane implies that the stress components σ_z, τ_{xz} and τ_{yz} vanish. The geometry of the body is essentially that of a plate with one dimension (z direction) much smaller than the others. Loading of the body is the x,y plane only.

The vanishing character of σ_z, τ_{xz} and τ_{yz} implies that ϵ_{yz} and ϵ_{xz} also vanish if the material is isotropic and linear elastic. The non-vanishing strains ϵ_{xx}, ϵ_{yy}, ϵ_{zz} and ϵ_{xy} are related to the stresses σ_x, σ_y and τ_{xy} as follows

$$\begin{aligned}
\epsilon_{xx} &= \frac{1}{E}(\sigma_x-\nu\sigma_y) \\
\epsilon_{yy} &= \frac{1}{E}(\sigma_y-\nu\sigma_x) \\
\epsilon_{zz} &= -\frac{\nu}{E}(\sigma_x+\sigma_y) \\
\epsilon_{xy} &= \frac{(1+\nu)}{E}\tau_{xy}
\end{aligned} \tag{8}$$

Combining equations (2), (6) and (8) gives a differential equation of the type

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4} = (\nu-1)\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{9}$$

If the body force components are constant or vanish, the right hand side of equation (9) vanishes

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and equation (9) becomes a *biharmonic equation* $\nabla^2(\nabla^2 F)=0$.

Thus, in the absence of body forces or in the case when the body forces are such that the right hand side of equations (7) and (9) vanishes, the stress distribution in the x,y plane is independent of the elastic properties of the material.

Also, comparing equation (3) with (8), a plane strain solution can be obtained from a plane stress solution as follows: replace E by $E/(1-\nu^2)$, and replace ν by $\nu/(1-\nu)$,

3.3 Generalized Plane Strain Analysis

The generalized plane strain analysis is a variant of the plane strain analysis. The geometry of the body is the same as in plane strain, loading is again in the x,y plane and does not vary with the z-axis. The only difference with plane strain is that w does not vanish and is independent of z. The displacement components are such that

$$u = u(x,y) \;; \quad v = v(x,y) \;; \quad w = w(x,y) \tag{10}$$

The strains are related to the stresses as follows

$$\begin{aligned} \epsilon_{xx} &= \frac{1-\nu^2}{E}\left(\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right) \\ \epsilon_{yy} &= \frac{1-\nu^2}{E}\left(\sigma_y - \frac{\nu}{1-\nu}\sigma_x\right) \\ \epsilon_{xy} = \frac{(1+\nu)}{E}\tau_{xy} \;; \quad &\epsilon_{xz} = \frac{(1+\nu)}{E}\tau_{xz} \;; \quad \epsilon_{yz} = \frac{(1+\nu)}{E}\tau_{yz} \end{aligned} \tag{11}$$

The equations of equilibrium reduce to

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho b_x &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \rho b_y &= 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \rho b_z &= 0 \end{aligned} \tag{12}$$

where ρ is the density and ρb_x, ρb_y and ρb_z are the components of the body force per unit volume in the x, y and z directions, respectively.

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The strain components satisfy two compatibility equations

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2}+\frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \epsilon_{xy}}{\partial y \partial x}$$
$$\frac{\partial \epsilon_{xz}}{\partial y}-\frac{\partial \epsilon_{yz}}{\partial x} = 0 \tag{13}$$

Two potential functions U(x,y) and V(x,y) are introduced such that

$$\rho b_x = -\frac{\partial U}{\partial x} \; ; \quad \rho b_y = -\frac{\partial U}{\partial y} \; ; \quad \rho b_z = \frac{\partial V}{\partial y}-\frac{\partial V}{\partial x} \tag{14}$$

There are now two stress functions F(x,y) and G(x,y) such that

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}+U \; ; \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}+U \; ; \quad \tau_{xy} = -\frac{\partial^2 F}{\partial y \partial x}$$
$$\tau_{xz} = \frac{\partial G}{\partial y}+V \; ; \quad \tau_{yz} = -\frac{\partial G}{\partial x}-V \tag{15}$$

which satisfy the equations of equilibrium in (12). Combining equations (11), (13) and (15) gives two differential equations that F(x,y) and G(x,y) must satisfy, e.g.

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4} = \frac{(2\nu-1)}{(1-\nu)}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)$$
$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2} = -\left(\frac{\partial V}{\partial x}+\frac{\partial V}{\partial y}\right) \tag{16}$$

If the body force components are constant or vanish, the right hand side of the two differential equations in (16) vanish. In that case F(x,y) satisfies a *biharmonic equation* (plane problem), $\nabla^2(\nabla^2 F)=0$, and G(x,y) satisfies a *Laplace equation* (antiplane problem), $\nabla^2 G=0$.

3.4 Analysis in Polar or Cylindrical Coordinates

For 2D problems dealing with circular excavations (boreholes, tunnel, etc..) under plane strain or plane stress, it is more convenient to conduct the analysis using polar coordinates (r, θ) or

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cylindrical coordinates (r, θ, z).

Displacements

At any point P with coordinates x=rcosθ, y=rsinθ, the displacement components are u and v in the x,y coordinate system and u_r and v_θ in the r,θ coordinate system. The two sets of displacement components are related using equations derived from the coordinate transformation law for first-order Cartesian tensor, e.g.

$$
\begin{aligned}
u_r &= u.\cos\theta + v.\sin\theta \\
v_\theta &= -u.\sin\theta + v.\cos\theta
\end{aligned}
\tag{17}
$$

Strains

As shown in Lecture Notes 3, the strain components in the x,y and r,θ coordinate systems are related as follows

$$
\begin{aligned}
\epsilon_{rr} &= \epsilon_{xx}\cos^2\theta + \epsilon_{yy}\sin^2\theta + \epsilon_{xy}\sin 2\theta \\
\epsilon_{\theta\theta} &= \epsilon_{xx}\sin^2\theta + \epsilon_{yy}\cos^2\theta - \epsilon_{xy}\sin 2\theta \\
\epsilon_{r\theta} &= (\epsilon_{yy} - \epsilon_{xx})\sin\theta\cos\theta + \epsilon_{xy}\cos 2\theta
\end{aligned}
\tag{18}
$$

Stresses

As shown in Lecture Notes 3, the stress components in the x,y and r,θ coordinate systems are related as follows

$$
\begin{aligned}
\sigma_r &= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\sin 2\theta \\
\sigma_\theta &= \sigma_x\sin^2\theta + \sigma_y\cos^2 - \tau_{xy}\sin 2\theta \\
\tau_{r\theta} &= (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}\cos 2\theta
\end{aligned}
\tag{19}
$$

Strain-Displacement Relations

Using the engineering mechanics strain convention, the strain and displacement components in the r,θ coordinate system are related as follows

$$
\epsilon_{rr} = \frac{\partial u_r}{\partial r} \; ; \quad \epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \; ; \quad 2\epsilon_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}
\tag{20}
$$

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When using the rock mechanics strain convention, u_r and v_θ in (20) must be replaced by $-u_r$ and $-v_\theta$, respectively

Equations of Equilibrium

The three equations of equilibrium reduce to

$$\begin{aligned} &\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+R = 0 \\ &\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}+\Theta = 0 \end{aligned} \tag{21}$$

where R and Θ are components of body force per unit volume in the r and θ directions, respectively.

Stress Function

If the body force components are constant or vanish, a stress function $\Phi(r,\theta)$ can be introduced that satisfies the biharmonic equation $\nabla^2(\nabla^2\Phi)=0$ with

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{22}$$

In the absence of body forces, the stress components σ_r, σ_θ and $\tau_{r\theta}$ are related to $\Phi(r,\theta)$ as follows

$$\sigma_r = \frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} \quad ; \quad \sigma_\theta = \frac{\partial^2\Phi}{\partial r^2} \quad ; \quad \tau_{r\theta} = -\frac{1}{r}\frac{\partial^2\Phi}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial\Phi}{\partial\theta} \tag{23}$$

The expression of $\Phi(r,\theta)$ can take different forms depending on the problem being analyzed. Several examples are presented below. As shown in Chapter 4 in Obert and Duvall (1967), a general expression of $\Phi(r,\theta)$ is as follows

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$$
\begin{aligned}
\Phi(r,\theta) \; = \; & A_o r^2 + B_o r^2 \ln r + C_o \ln r + D_o \\
& + (E_o r^2 + F_o r^2 \ln r + G_o \ln r + H_o)\theta \\
& + (A_1 r + B_1 r^{-1} + C_1 r^3 + D_1 r \ln r)\sin\theta \\
& + (E_1 r + F_1 r^{-1} + G_1 r^3 + H_1 r \ln r)\cos\theta \\
& + (J_1 r + K_1 r \ln r)\theta\sin\theta + (L_1 r + M_1 r \ln r)\theta\cos\theta \\
& + \sum_{n=2}^{\infty} [A_n r^n + B_n r^{-n} + C_n r^{2-n} + D_n r^{2+n}]\sin n\theta \\
& + \sum_{n=2}^{\infty} [E_n r^n + F_n r^{-n} + G_n r^{2-n} + H_n r^{2+n}]\cos n\theta
\end{aligned}
\tag{24}
$$

Various terms in equation (24) can be used to obtain solutions to specific problems in polar coordinates in the absence of body forces. The various constants appearing in equation (24) are determined from the boundary conditions of the problem. Not all constants are always required; the number depends on the boundary conditions.

4. EXAMPLES

4.1 Hollow Cylinder under Pressure

General Solution

Consider a hollow cylinder of inner radius R_1 and outer radius R_2. The internal pressure is p_1 and the outer pressure is p_2. The boundary conditions are expressed as follows

$$
\begin{aligned}
& at \;\; r = R_1 \qquad \sigma_r = p_1 \qquad \tau_{r\theta} = 0 \\
& at \;\; r = R_2 \qquad \sigma_r = p_2 \qquad \tau_{r\theta} = 0
\end{aligned}
\tag{25}
$$

The problem is axisymmetric and $\Phi(r,\theta)$ depends on r only. Its expression is

$$\Phi(r,\theta) \; = \; Ar^2 + C\ln r \tag{26}$$

Substituting the stress function into equation (23) gives

$$\sigma_r = 2A + \frac{C}{r^2} \;\; ; \quad \sigma_\theta = 2A - \frac{C}{r^2} \;\; ; \quad \tau_{r\theta} = 0 \tag{27}$$

Both A and C can be determined using the boundary conditions (25). This gives,

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$$\sigma_r = \frac{R_2^2 p_2 - R_1^2 p_1}{R_2^2 - R_1^2} + \frac{R_2^2 R_1^2}{r^2} \frac{(p_1 - p_2)}{R_2^2 - R_1^2}$$

$$\sigma_\theta = \frac{R_2^2 p_2 - R_1^2 p_1}{R_2^2 - R_1^2} - \frac{R_2^2 R_1^2}{r^2} \frac{(p_1 - p_2)}{R_2^2 - R_1^2} \tag{28}$$

Both stresses are principal stresses. Their sum is a constant independent of r and θ, i.e.

$$\sigma_r + \sigma_\theta = 2\frac{R_2^2 p_2 - R_1^2 p_1}{R_2^2 - R_1^2} \tag{29}$$

Under plane strain condition, the hollow cylinder experiences a uniform stress $\sigma_z = \nu(\sigma_r + \sigma_\theta)$. Under plane stress condition, the cylinder experiences a uniform strain $\epsilon_z = -\nu(\sigma_r + \sigma_\theta)/E$.

Under plane strain, the strains ϵ_{rr} and $\epsilon_{\theta\theta}$ are equal to

$$\epsilon_{rr} = -\frac{\partial u_r}{\partial r} = \frac{1-\nu^2}{E}(\sigma_r - \frac{\nu}{1-\nu}\sigma_\theta)$$

$$\epsilon_{\theta\theta} = -\frac{u_r}{r} = \frac{1-\nu^2}{E}(\sigma_\theta - \frac{\nu}{1-\nu}\sigma_r) \tag{30}$$

Substituting (28) into (30), the radial displacement is equal to

$$u_r = \frac{R_2^2 R_1^2 (p_1 - p_2)}{R_2^2 - R_1^2} \frac{(1+\nu)}{Er} - \frac{R_2^2 p_2 - R_1^2 p_1}{R_2^2 - R_1^2} r \frac{(1-\nu-2\nu^2)}{E} \tag{31}$$

Special Cases

1) Consider now the special case when the outer radius R_2 goes to infinity. The problem corresponds to a tunnel, shaft, or borehole subjected to a uniform (hydrostatic) *in situ* stress field at infinity and an internal pressure (water pressure, lining, etc..). In that case, equations (28) and (31) reduce to

$$\sigma_r = p_2 + \frac{R_1^2}{r^2}(p_1 - p_2) \quad ; \quad \sigma_\theta = p_2 - \frac{R_1^2}{r^2}(p_1 - p_2) \tag{32}$$

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and

$$u_r = \frac{R_1^2}{r}(p_1 - p_2)\frac{(1+\nu)}{E} - p_2 r\frac{(1-\nu-2\nu^2)}{E} \tag{33}$$

The tangential stress, σ_θ, varies between $2p_2$-p_1 at the wall of the cavity to p_2 at infinity. The radial stress, σ_r, varies between p_1 and p_2.

2) If the external pressure, p_2, vanishes, equations (32) and (33) reduce further to

$$\sigma_r = p_1\frac{R_1^2}{r^2} \; ; \quad \sigma_\theta = -p_1\frac{R_1^2}{r^2} \tag{34}$$

and

$$u_r = p_1\frac{R_1^2}{r}\frac{(1+\nu)}{E} = p_1\frac{R_1^2}{r}\frac{1}{2G} \tag{35}$$

Equation (34) can be used to assess the stress distribution around a pressurized borehole, tunnel, or shaft. By measuring the radial displacement along the wall of a circular hole, the shear modulus, G, can be determined by substituting $r=R_1$ into equation (35), e.g.

$$u_{r=R_1} = p_1 R_1 \frac{1}{2G} \tag{36}$$

This equation is used to determine the modulus of deformation of a rock mass from the results of dilatometer tests by assuming a value for its Poisson's ratio.

3) Hollow cylinder with outside pressure p_2 only and $p_1 = 0$. Equations (28) and (31) become

$$\begin{aligned} \sigma_r &= \frac{R_2^2 p_2}{R_2^2 - R_1^2}\left(1 - \frac{R_1^2}{r^2}\right) \\ \sigma_\theta &= \frac{R_2^2 p_2}{R_2^2 - R_1^2}\left(1 + \frac{R_1^2}{r^2}\right) \end{aligned} \tag{37}$$

and

$$u_r = -p_2\frac{R_2^2 R_1^2}{R_2^2 - R_1^2}\frac{(1+\nu)}{Er} - p_2\frac{R_2^2 r}{R_2^2 - R_1^2}\frac{(1-\nu-2\nu^2)}{E} \tag{38}$$

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By measuring the inward radial displacement of a hollow core of rock under an outside radial pressure, the shear modulus, G, can be determined by substituting r=R_1 into equation (38), e.g.

$$u_{r=R_1} = -p_2 \frac{R_2^2 R_1}{R_2^2 - R_1^2} \frac{(1-\nu)}{G} = -2p_2 \frac{R_2^2 R_1}{R_2^2 - R_1^2} \frac{(1-\nu^2)}{E} \tag{39}$$

4) All the above solutions were derived for plane strain condition. They can be converted to plane stress by replacing ν by ν/(1+ν). The shear modulus G does not change. Equation (36) does not change and Equation (39) becomes

$$u_{r=R_1} = -2p_2 \frac{R_2^2 R_1}{R_2^2 - R_1^2} \frac{1}{E} \tag{40}$$

The longitudinal strain accompanying the inward radial deformation given in (40) is equal to

$$\epsilon_z = -\frac{\nu}{E}(\sigma_r + \sigma_\theta) = -2\frac{\nu}{E} \frac{p_2 R_2^2}{R_2^2 - R_1^2} \tag{41}$$

The tangential strain on the inner wall of the hollow core is equal to

$$\epsilon_\theta = -\frac{u_{r=R_1}}{R_1} = \frac{2}{E} \frac{p_2 R_2^2}{R_2^2 - R_1^2} = -\frac{\epsilon_z}{\nu} \tag{42}$$

Thus, the Young's modulus, E, and Poisson's ratio, ν, can be determined by measuring the longitudinal and tangential strains on the inner wall of a cylinder of elastic material under outer radial pressure.

4.2 Circular Hole Drilled Under a Triaxial Principal Stress Field at Infinity

Consider a cicrcular hole of radius, a, drilled (excavated) in a triaxial *in situ* stress field with components σ_{xo}, σ_{yo}, and σ_{zo} in the x, y and z directions. The medium is isotropic with Young's modulus, E, and Poisson's ratio, ν. The final state of stress is the sum of the initial state of stress (before drilling or excavation of the hole) and the stresses induced by the opening. The stresses are defined in an (r, θ) polar coordinate system.

Initial Stresses

The initial stress components σ_{ro}, $\sigma_{\theta o}$, and $\tau_{r\theta o}$ in the (r, θ) coordinate system are, according to equation (19) equal to

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$$\sigma_{ro} = \sigma_{xo}\cos^2\theta + \sigma_{yo}\sin^2\theta$$

$$\sigma_{\theta o} = \sigma_{xo}\sin^2\theta + \sigma_{yo}\cos^2\theta \tag{43}$$

$$\tau_{r\theta o} = (\sigma_{yo} - \sigma_{xo})\sin\theta\cos\theta$$

Induced Stresses

The stresses induced by the hole can be determined by using the following expression of the stress function $\Phi(r,\theta)$

$$\Phi(r,\theta) \;=\; A\ln r + Br^2 + (Cr^2 + Dr^4 + Er^{-2} + F)\cos 2\theta \tag{44}$$

The induced stress components σ_{rh}, $\sigma_{\theta h}$, and $\tau_{r\theta h}$ are determined by substituting equations (44) into (23). The five coefficients A, B, C, D, E and F are determined from the boundary conditions, e.g. at r = a

$$\sigma_{rh} = -\sigma_{xo}\cos^2\theta - \sigma_{yo}\sin^2\theta$$

$$\tau_{r\theta h} = -(\sigma_{yo} - \sigma_{xo})\sin\theta\cos\theta \tag{45}$$

Also, σ_{rh} and $\tau_{r\theta h}$ must vanish at r = ∞. After algebraic manipulation, this gives

$$\sigma_{rh} = -\frac{a^2}{r^2}\frac{(\sigma_{xo}+\sigma_{yo})}{2} + \left(3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta$$

$$\sigma_{\theta h} = \frac{a^2}{r^2}\frac{(\sigma_{xo}+\sigma_{yo})}{2} - 3\frac{a^4}{r^4}\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta \tag{46}$$

$$\tau_{r\theta h} = \left(3\frac{a^4}{r^4} - 2\frac{a^2}{r^2}\right)\frac{(\sigma_{xo}-\sigma_{yo})}{2}\sin 2\theta$$

If drilling of the opening does not create any longitudinal strain, then ϵ_{zh} vanishes and σ_{zh} is equal to

$$\sigma_{zh} = \nu(\sigma_{rh} + \sigma_{\theta h}) = -4\nu\frac{a^2}{r^2}\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta \tag{47}$$

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Total Stresses

The total stresses are obtained by adding the initial and induced stress components, e.g.

$$\sigma_r = (1-\frac{a^2}{r^2})\frac{(\sigma_{xo}+\sigma_{yo})}{2} + (1+3\frac{a^4}{r^4}-4\frac{a^2}{r^2})\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta$$

$$\sigma_\theta = (1+\frac{a^2}{r^2})\frac{(\sigma_{xo}+\sigma_{yo})}{2} - (1+3\frac{a^4}{r^4})\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta$$

$$\tau_{r\theta} = -(1-3\frac{a^4}{r^4}+2\frac{a^2}{r^2})\frac{(\sigma_{xo}-\sigma_{yo})}{2}\sin 2\theta \qquad (48)$$

$$\sigma_z = \sigma_{zo} - 4\nu\frac{a^2}{r^2}\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta$$

The tangential stress, σ_θ, at the wall of the hole (r=a) varies between $3\sigma_{yo}$-σ_{xo} at θ=0° (x direction) and $3\sigma_{xo}$-σ_{yo} at θ=90° (y direction). The effect of any internal radial pressure, p, acting in the hole can be taken into account by adding

$$\sigma_r = p\frac{a^2}{r^2} \;; \quad \sigma_\theta = -p\frac{a^2}{r^2} \qquad (49)$$

to the stress components in (48).

Total Displacement Components

Substituting the expressions of the stress components into Hooke's law and after integration, the radial and tangential displacement components are equal to:

$$u_r = -\frac{1}{E}[(r+\frac{a^2}{r})\frac{(\sigma_{xo}+\sigma_{yo})}{2} + (r-\frac{a^4}{r^3}+4\frac{a^2}{r})\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta]$$

$$+\frac{\nu}{E}[(r-\frac{a^2}{r})\frac{(\sigma_{xo}+\sigma_{yo})}{2} - (r-\frac{a^4}{r^3})\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta] \qquad (50)$$

$$+\frac{\nu}{E}\sigma_{zo}r + 4\frac{\nu^2}{E}\frac{a^2}{r}\frac{(\sigma_{xo}-\sigma_{yo})}{2}\cos 2\theta$$

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$$
\begin{aligned}
v_\theta = & \frac{1}{E}\left(r + \frac{a^4}{r^3} + 2\frac{a^2}{r}\right)\frac{(\sigma_{xo} - \sigma_{yo})}{2}\sin 2\theta \\
& + \frac{\nu}{E}[(r - 2\frac{a^2}{r} + \frac{a^4}{r^3})\frac{(\sigma_{xo} + \sigma_{yo})}{2}\sin 2\theta \\
& - 4\frac{\nu^2}{E}\frac{a^2}{r}\frac{(\sigma_{xo} - \sigma_{yo})}{2}\sin 2\theta \\
w = & -\frac{z}{E}(\sigma_{zo} - \nu(\sigma_{xo} + \sigma_{yo}))
\end{aligned} \tag{51}
$$

4.3 Circular Hole Drilled Under a Triaxial Stress Field-General Solution (see attached)

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5. USING STRESSES IN ROCK ENGINEERING

Rock masses are initially stressed in their natural state. Whether one is interested in natural geological structures (folds, faults, intrusions, etc..) or man-made structures (tunnels, caverns, mines, surface excavations, etc..), a knowledge of the *in situ* or virgin stress field (along with other rock mass properties) is needed when predicting the response of rock masses to the disturbance associated with those structures. The response can take multiple forms such as deformations of the walls of a surface or underground excavation, stresses and breakouts in the walls of a shaft or borehole, creep of a salt pillar, initiation of a micro-earthquake, shearing of a fault, glacial rebound of a glaciated terrain, etc.. Today, there exists a variety of analytical solutions to many of the geological, geophysical, and geoengineering problems. Computer-based numerical methods for stress, strain, and strength analysis are also available to handle problems with more complex geometries and/or constitutive behavior. Many of the analytical methods and numerical codes use stress (or traction) as a possible boundary condition. Hence, a proper determination, or at least a good estimation of the state of stress *in situ* is needed in order to reach reliable solutions to the problem of interest.

In situ stresses play a critical role in civil engineering projects involving rock either as a construction or foundation material. They enter into the design and stability analysis of various underground structures (tunnels, caverns, shafts, etc..). They are also important when selecting the location of pressure tunnels and shafts and other rock cavities used for hydropower or underground storage of compressed natural gas, liquified natural gas (LNG), liquified petroleum gases (LPG), compressed air, oil or water, for which rock confinement is critical. Finally, *in situ* stresses may control the stability of surface excavations (natural and man-made). The discussion below is limited to the role of in situ stresses in the behavior of underground excavations in rock.

Various stability problems may arise when opening an underground excavation in rock. Hoek and Brown (1980a) defined four major modes of instability: instability due to rock stresses, instability due to adverse structural geology, instability due to weathering and or swelling, and instability due to excessive groundwater pressure or flow. They also noted that there are situations where two or more failure modes could co-exist. The pre-excavation *in situ* stresses and their redistribution following excavation contribute to the first mode of instability.

In situ stresses often contribute to the second mode of instability as they can enhance or reduce the role played by the rock fabric and geological structures such as foliations, joints, faults, etc.. Compressive stresses provide confinement and therefore tend to lock or clamp the rock mass, thus reducing the effect of blocks, wedges or slabs that would otherwise become unstable by sliding along discontinuities. Block stability is increased further if the discontinuities are rough and are not (or partially) free to dilate. On the other hand, tensile stresses may open fractures, accelerate the weathering, and create more stability problems. Finally, *in situ* stresses affect water flow since compressive stresses make the rock mass tighter and less likely to create seepage problems upon excavation and thereafter.

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In situ stresses are mostly included in the analytical/numerical approach of the design of underground excavations. In the empirical approach, which is based on rock mass classifications, *in situ* stresses are "subjectively" included in the form of scaling factors. For instance, in the Q-rating of Barton et al., *in situ* stresses are taken into account in the stress reduction factor (SRF). In the Geomechanics Classification of Bieniawski, the RMR rating is adjusted using a coefficient that varies between 0.6 and 1.2 depending if the user judges the stresses to be beneficial or detrimental.

In the analytical/numerical approach to design, stress analyses are usually carried out by considering a region of a rock mass subject to fixed boundary conditions such as tractions (stress) or displacements, or mixed boundary conditions. The stresses measured *in situ* are used as boundary conditions. A variety of analytical solutions (mostly two-dimensional) are available to determine stresses around openings of simple shapes, in homogeneous and continuous rock and subjected to uniform *in situ* stresses at infinity. Figure 1 shows the classical example of stress redistribution around a circular opening in a vertically stressed elastic plate. Numerical methods such as the Finite Element Method, the Boundary Element Methods, and the Discrete (Distinct) Element Methods can be used to handle problems with more complex (two- or three-dimensional) geometries and for rock masses with more complex constitutive behavior. A review of the analytical and numerical methods used in rock mechanics can be found in Obert and Duvall (1967), Jaeger and Cook (1979), Hoek and Brown (1980a), and Pande et al. (1990), among others. Figure 2 shows, as an example, the results of a three-dimensional elastic stress analysis for a power house cavern conducted by Sugawara et al. (1986) using a three-dimensional boundary element method.

Stress analyses are usually conducted to determine stress concentrations, and the type of stress concentration (compression versus tension). Since rock masses are weak in tension, tensile stresses may open existing fractures or create new ones which could result in block stability problems. On the other hand, the magnitude of compressive stresses in the walls of an excavation may be large enough to overstress the rock and mobilize the rock mass strength locally or not. This can result in problems such as rock bursts, spalling, buckling and heaving of rock layers, squeezing and excessive deformation in the form of roof closure, sidewall movement and/or ground subsidence. The location, extent and nature of the overstressed zones depend on many parameters such as rock mass properties (strength, deformability), the rock fabric (anisotropy, joints, etc..), the *in situ* stress field (magnitude and orientation), the geometry and depth of the excavation, and external conditions such as the topography, and the method of excavation. High horizontal stresses are likely to create problems in the roof and floor of an underground opening whereas high vertical stresses are likely to create problems in its sidewalls. In mountainous areas with extreme topography and steep valley walls (such as in Norwegian fjords), tunnels parallel to the valleys have been found to experience rock bursts in the tunnel wall and in part of their roof located the closest to the valley side due to stress concentrations. In very deep excavations such as the deep mines in South Africa and North America, stability problems can occur all around the excavations. There are also situations where floor and roof stability problems along a section of an underground opening are replaced

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by sidewalls problems (or no stability problems whatsoever) along other sections.

As an example, Figure 3 shows the results of a two-dimensional stress analysis conducted by Eissa (1980) using the boundary element method. The extent of the potential overstressed zone around a horseshoe opening in a biaxial stress field with a vertical component, p, and a horizontal component, 0.5p, was determined for different values of the ratio between the vertical stress, p, and the intact rock compressive strength, σ_c. The rock mass was assumed to be under plane strain and its strength was defined using the criterion proposed by Hoek and Brown (1980b) with two parameters, m and s. Three rock masses of decreasing strength were considered and are defined as A, B and C. Figure 3 shows clearly an increase in the extent of the overstressed zone around the opening as the rock mass strength decreases. Also, for given values of the parameters m and s, the extent of the overstressed zone increases with the ratio p/σ_c.

The value of the ratio between the vertical stress and the unconfined compressive strength of the intact rock material is critical when assessing the potential for squeezing of weak clay-bearing rocks such as shales, claystones, mudstones, etc.. A good example, described by Morton and Provost (1980), is that of the Stillwater tunnel in the U.S. where squeezing ground was encountered in shale beneath ground cover of the order of 2,500 ft (762 m), resulting in a complete stoppage of a Tunnel Boring Machine. In this case, the vertical stress to unconfined compressive strength ratio was of the order of 0.25.

Stress analysis can be used to determine the extent of the zone where stresses have been disturbed by excavation of an underground opening. Such an analysis is important when predicting the interaction between adjacent openings and the interaction between an opening and adjacent geological structures such as faults. Eissa (1980) analyzed the stress distribution around openings of various shapes and under different levels of horizontal and vertical stresses. He concluded that the zone of influence of an opening depends largely on its height to width ratio and the horizontal to vertical *in situ* stress ratio. For all practical purposes, stress redistribution takes place over a distance from the wall of the opening of at least one to 1.5 times the opening span. This rule of thumb seems to be realistic when compared to actual stress measurements around openings.

Figure 4 shows an illustrative example of stress redistribution measured by Obara et al. (1995) in Japan using the conical-shaped borehole overcoring technique. A total of 18 overcoring stress measurements were carried out at distances ranging between 0.6 and 29.5 m from the wall of a 6 m span gallery excavated in granodiorite at a depth of 520 m. Three steeply dipping sub-parallel faults were found to interact with the gallery; two intersecting the gallery itself (not shown in Figure 4) and one between measurements 6 and 7. Figures 4a and 4b show the distribution of the principal stresses in the vertical plane and in the horizontal plane, respectively. Here, the stress distribution is affected by both the presence of the gallery and the fault(s). The stress measurements at points 17 and 18 (located more than 25 m from the gallery and 20 m from the fault) were taken as *in situ* stress values.

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Stress analysis can also help in the selection and design of support systems. For instance, the extent of overstressed rock can be used to determine the length of rock bolts. *In situ* stresses are an integral part of the ground-support interaction model proposed by Brown et al. (1983) since they form the "loading system". In this model, the extent of the plastic zone around an underground excavation and the displacement of the walls of the opening depend on the magnitude of the *in situ* stresses. As the shape of the ground reaction curve varies with the level of applied stress, so is the amount of support required to reach mechanical equilibrium between the support system and the rock mass.

The distribution and magnitude of *in situ* stresses affect the geometry, shape, dimensioning, excavation sequence and orientation of underground excavations. The main goal when designing underground openings in rock (where stresses are likely to be a problem), is to minimize stress concentrations, create a compressive stress field as evenly distributed as possible ("harmonic" hole concept) in the excavation walls, and avoid tensile stress regions. This can be done by changing the shape and geometry of the openings as well as their orientation with respect to the known principal *in situ* stresses. The theory of elasticity is often used to that effect. However, as noted by Hoek and Brown (1980a), the "harmonic" hole concept applies when virgin stresses are low compared to the rock strength. If the virgin stresses are high enough, the harmonic hole concept could result in uniform but large compressive stresses all around the opening which could create stability problems. In that case, Hoek and Brown (1980a) suggested following a recommendation proposed earlier by Fairhurst (1968) whereby the excavation shape is selected in such a way that the zones of overstressed rock are concentrated in sharp corners, and are limited in extent. According to Broch (1993), the same recommendation should be followed for deep caverns where the *in situ* stresses are so high that rock bursts and spalling can be expected.

Stress concentrations based on the two-dimensional theory of elasticity are available for single openings in a uniform *in situ* stress field, or in a stress field increasing linearly with depth. Stress concentration factors are also available for parallel openings of various shapes and for different values of the opening span to pillar width ratio (Hoek and Brown, 1980a; Eissa, 1980). Figure 5 shows a summary of tangential stress concentrations in the roof and spring-line of single openings in a uniform *in situ* stress field with vertical component p and horizontal component kp. The rock is assumed to be homogeneous, continuous, and linearly elastic. This figure indicates that within the context of linear elasticity, openings with a major horizontal axis are better suited in stress fields with horizontal stresses higher than the vertical stress. On the other hand, their major axis should be vertical if the vertical stress is larger. The ratio between the major and minor axes of the openings should be of the same order as the horizontal to vertical *in situ* stress ratio. A general characteristic of stress concentrations around openings in isotropic media is that they are independent on the elastic properties of the medium. The reader should be aware that this is no longer true if the medium is anisotropic, in which case the stress concentrations depend not only on the rock elastic properties but also on the orientation of the planes of rock anisotropy with respect to the opening (Amadei, 1983). In general, stress concentrations in layered rocks can be quite different from those in isotropic rocks.

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Stress concentrations around single or multiple underground openings can also be (qualitatively) inferred using the stream flow analogy proposed by Hoek and Brown (1980a) for elastic models. The idea is that the applied stress field is analogous to an undisturbed stream flow. In the wall of an excavation (equivalent to a pier in the stream flow), zones of tension may develop and are equivalent to a separation of the stream lines. On the other hand, the zones of compression correspond to a crowding of the stream lines. The stream flow analogy can be useful when trying to understand the overall stress pattern around multiple excavations, and stress shadow effects that occur when excavations are in the near vicinity to each other.

The orientation of *in situ* stresses with respect to an excavation can have a large effect on the excavation stability. The excavation layout should be optimized in order to minimize the impact of *in situ* stresses. Geological structures or any other constraints (topography, water, etc..) need also to be taken into account in the optimization process.

The stability of rock caverns is very much controlled by their orientation with respect to the *in situ* stress field, along with other factors (Broch, 1993). In general, aligning caverns in rocks with their long axis perpendicular to the largest horizontal *in situ* stress component should be avoided. Excellent case studies showing the decisive role of *in situ* stresses on cavern orientation selection can be found in the literature. Figures 6a and 6b show strength/stress contours around a proposed powerhouse cavern in the Niagara Falls area (Haimson et al., 1986). In this example, the rock is horizontally stratified and the maximum and minimum horizontal *in situ* stresses are the major and intermediate principal stresses. The stresses around the cavern were determined using the finite element analysis and the strength was defined using the empirical Hoek and Brown (1980b) failure criterion. Figures 6a and 6b differ in the orientation of the cavern with respect to the horizontal stresses. In Figure 6a, the long axis of the opening is perpendicular to the maximum horizontal stress σ_H=9.2 MPa. This results in large overstressed zones in the wall and floor of the opening. On the other hand, in Figure 6b, the long axis of the opening is parallel to the maximum horizontal stress σ_H=9.2 MPa and perpendicular to the minimum horizontal stress σ_h=6.0 MPa. For that orientation, the size of the predicted overstressed zone is reduced considerably.

It is usually accepted that stress related instability problems increase with depth and that structurally controlled problems are more likely to occur at shallow depths. There are many exceptions to that trend in particular in regions where high horizontal *in situ* stresses exist at shallow depths. This is the case near lake Ontario (southern Ontario and upper New York state) where stresses of the order of 5-15 MPa have been measured at shallow depths and where stress relief phenomena such as heave of canal and quarry floors, natural pop-ups, rock squeeze, rock bursts, tunnel wall spalling, cracking of tunnel concrete linings, and/or movement of the walls of unsupported excavations (tunnels, shafts, canals) have been observed (Franklin and Hungr, 1978). Rock squeeze applies additional loads to surface and underground structures which may present considerable difficulties in maintaining operations. It is noteworthy that many of the problems observed near Lake Ontario in Canada and in the U.S. have also been observed in highly horizontally stressed sedimentary Post Permian rocks in the Sydney basin in Australia.

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Other examples where high horizontal *in situ* stresses have played a significant role in the stability of civil (and mining) engineering excavations have been reported for various underground works in Norway and Sweden.

In high horizontal stress fields, stress induced instability problems are likely to occur in the roof and floor of excavations in the form of spalling or squeezing due to high stress concentrations. As shown in Figure 5, for the simple case of a circular opening, the maximum tangential stress in the roof and floor of the opening is about three times the horizontal stress if the vertical stress is insignificant. If the opening is close to the surface, the stress concentrations in the roof and floor are no longer equal and depend on the rock cover. For a rock cover equal to 10% of the tunnel diameter, the stress concentration in the roof is about seven times the horizontal stress. Hanssen and Myrvang (1986) reported several examples of N-S running tunnels in the Kobbelv area in northern Norway experiencing heavy spalling in their roofs and floors as they were driven perpendicular to the E-W trending major *in situ* stress component.

If the rock is bedded, layered or stratified, the effect of high horizontal *in situ* stresses will be enhanced and will be expressed in the form of buckling and heaving of roof and floor layers, respectively. Slip along layers may also create large inward and horizontal displacements of the tunnel sidewalls. Several case studies showing this phenomenon were described by Franklin and Hungr (1978) for some tunnels in sedimentary rocks in southern Ontario. Guertin and Flanagan (1979) reported several cases of tunnel problems following excavation of tunnels at shallow depths (1-7 m) in a highly stressed (up to 14 MPa) undeformed dolostone in Rochester, New York. Horizontal displacements as large as 40 mm were measured at the tunnel springline, whereas the crown experienced a heave of 0.4 mm. The displacements were accompanied with invert and crown spalling (due to high tangential compressive stresses), and cracking at the spring line (due to tangential tensile stresses). All these case studies revealed, among other things, that in stratified rock masses, the roof and floor stability problems may not necessarily be the same due to differences in rock layer stiffness and strength.

In general, the presence of high horizontal stresses at any depth should be clearly defined and properly taken into account in the planning, design and construction of all major rock engineering projects. Failure to anticipate the existence of high stresses may result in structural damage and expensive remedial work as most design methods based on gravitational loading only are invalidated in that case (Franklin and Hungr, 1978). High horizontal stresses may also create problems during excavation. Myrvang (1993) noted several case studies showing the difficulties associated with Tunnel Boring Machine boring in a rock with violent surface spalling.

Different strategies can be followed to minimize the impact of high horizontal *in situ* stresses. Franklin and Hungr (1978) recommended delaying tunnel lining following excavation. They also suggested maximizing the distance between the location of rock excavation and the nearest rigid support by over-excavation. Linings and supports should either be flexible enough or should be protected by a deformable interface capable of accommodating up to 10 cm of

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movement of the excavation walls. Monitoring of displacements and rock pressures are also highly recommended. It has also been found that spalling and overbreak in the crown of tunnels can be successfully controlled by using rock bolting with or without shotcrete. In mining, the impact of high ground stresses can be minimized by playing with mining sequence and controlled yielding of pillars and closing of excavations.

It must be kept in mind that high horizontal stresses may be beneficial in making the rock tighter, self supporting and less inclined to create seepage problems. Pathways for contaminant transports are also smaller. In some cases, high horizontal stresses may permit the use of large (mostly unsupported) roof spans in underground caverns, a recent example being the 61 m span underground olympic ice hockey hall in Lillehammer, Norway (Myrvang, 1993).

It is noteworthy that the effect of *in situ* stresses on the stability of underground openings may vary with time. Stress changes around underground openings may be associated with rock creep, nearby excavations, sequences of excavation, pumping, poor drainage, earthquake loading, blasting, etc..

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