DEFORMABILITY PROPERTIES OF ROCKS AND ROCK MASSES

1. INTRODUCTION

2. DEFORMABILITY OF INTACT ROCK

2.1 Rock under Uniaxial Compression
2.2 Dynamic Elastic Constants
2.3 Hooke's Law
2.4 Rock Anisotropy
2.5 Laboratory Testing

3. ROCK MASS DEFORMABILITY

3.1 Characterizing Rock Mass Deformability
3.2 Measuring Rock Mass Deformability

4. REFERENCES

Recommended Readings:

1) Chapter 6 in *Introduction to Rock Mechanics,* by R.E. Goodman, Wiley, 1989.

2) Bieniawski, Z.T. and Bernede, M.J. (coordinators) (1979) Suggested methods for determining the uniaxial compressive strength and deformability of rock materials, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 16, No.2, pp. 135-140.

3) Amadei, B. (1996) Importance of anisotropy when estimating and measuring *in situ* stresses in rock, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 33, No.3, pp. 293-325.

4) Luehring, R.W. (1988) Methods for determining in situ deformation of rock masses, U.S. Bureau of Reclamation Report REC-ERC-87-14.

1. INTRODUCTION

Engineers need to know how rocks and rock masses deform when subject to the various loads associated with engineering structures. Deformation can take the form of settlements of surface structures, surface subsidence or closing of the walls of underground openings. The overall stability of concrete dams depends, in part, on the relative deformability of the foundation rock with respect to the concrete. Rock deformability can be instant or time-deferred. Rock mass deformability depends on the deformability of both intact rock and the discontinuities.

In this chapter, we will first talk about the deformability of intact rock; isotropic and anisotropic formulations will be presented. Then, we will investigate the contribution of discontinuities in the deformability of rock masses. The problem of scale effect will also be addressed.

2. DEFORMABILITY OF INTACT ROCK

2.1 Rock under Uniaxial Compression

Consider a rock specimen subjected to uniaxial compression. Let ϵ_a and ϵ_l be the axial and lateral (diametral) strains measured during the test and σ be the applied axial stress. Figure 1 shows a typical set of stress-strain response curves. The plot can be divided into four regions:

- region OA corresponds to the closing of microcracks and a general adjustment of the system consisting of the rock and testing machine. This region is usually concave upwards,

- region AB is more or less linear and corresponds to the elastic response of the rock,

- region BC is associated with strain hardening and is concave downwards,

- region CD corresponds to strain softening.

At any stress level in region AB, the rock will follow a path essentially parallel to AB upon unloading and reloading. In that domain, the rock sample can be modeled as a spring. If the rock is isotropic, two elastic constants can be introduced to describe its deformability: the *Young's modulus*, E, and the *Poisson' s ratio,* ν, with

$$E = d\sigma / d\epsilon_a \tag{1}$$

and

$$\nu = -d\epsilon_l / d\epsilon_a \tag{2}$$

where $d\sigma$, $d\epsilon_l$ and $d\epsilon_a$ are increments of stress, lateral strain and vertical strain, respectively.

Figure 2 shows three methods suggested by the ISRM for determining the Young's modulus from axial stress-strain curves (Bieniawski and Bernede, 1979) and Figure 3 shows an example of calculation of those properties.

Various authors have compiled the values of E and ν for different rock types (Balmer, 1953; Johnson and DeGraff, 1988; Hatheway and Kiersch, 1989; etc..). Examples are given in the following tables.

	Granite	Basalt	Gneiss	Schist	Quart-zite	Marble	Lime-stone	Sand-stone	Shale
Av. E	59.3	62.6	58.6	42.4	70.9	46.3	50.4	15.3	13.7
Max. E	75.5	100.6	81.0	76.9	100.0	72.4	91.6	39.2	21.9
Min. E	26.2	34.9	16.8	5.9	42.4	23.2	7.7	1.9	7.5
Range	49.3	65.7	64.2	71.0	57.6	49.2	83.9	37.3	14.4
No. of samples	24	16	17	18	10	16	29	18	9

Table 1. Typical values of Young's moduli (in GPa) for nine common rock types (after Johnson and Degraff, 1988).

	Granite	Basalt	Gneiss	Schist	Quart-zite	Marble	Lime-stone	Sand-stone	Shale
Av. ν	0.23	0.25	0.21	0.12	0.15	0.23	0.25	0.24	0.08
Max. ν	0.39	0.38	0.40	0.27	0.24	0.40	0.33	0.46	0.18
Min. ν	0.10	0.16	0.08	0.01	0.07	0.10	0.12	0.06	0.03
Range	0.29	0.22	0.32	0.26	0.17	0.30	0.21	0.40	0.15
No. of samples	24	16	17	18	10	16	29	18	9

Table 2. Typical values of Poisson's ratios for nine common rock types (after Johnson and Degraff, 1988).

Linearity in the material behavior in uniaxial compression will also be reflected in its response to pure shear and hydrostatic compression. For instance, if a increment in shear stress $d\tau$ is applied, an increment in (engineering) shear strain, $d\gamma$, will result such that

$$d\tau = G d\gamma \tag{3}$$

where G is called the *shear modulus*. On the other hand, for an increment of hydrostatic load dp, an increment in volumetric strain, $d\epsilon_v$, will result such that

$$dp = K d\epsilon_v \tag{4}$$

where K is called the *bulk modulus*. Note that G and K are related to E and ν as follows

$$G = \frac{E}{2(1+\nu)}; \quad K = \frac{E}{3(1-2\nu)} \tag{5}$$

2.2 Dynamic Elastic Constants

The Young's modulus, Poisson's ratio and shear modulus can be determined by dynamic methods. Dynamic elastic properties are obtained by rapid application of stress to a rock specimen. One method of achieving this is to subject the specimen to ultrasonic compression and shear wave pulse using a so-called *sonic velocity equipment* (see section 5 in Lecture Notes 2). The method is called the *seismic pulse method*. Using the theory of elastic wave propagation, the dynamic Young's modulus, E_d, Poisson's ratio ν_d, and shear modulus G_d are calculated from the measured shear and longitudinal wave velocities V_s and V_p (about 1.5 V_s) as follows

$$G_d = \left(\frac{\gamma}{g}\right)V_s^2 \quad ; \quad \nu_d = \frac{1-2(V_s^2/V_p^2)}{2-2(V_s^2/V_p^2)}$$

$$E_d = 2(1+\nu_d)G_d \tag{6}$$

where γ is the unit weight of the rock and g is the acceleration due to gravity. An example of calculation of dynamic elastic properties taken from Johnson and Degraff (1988) is given below for a specimen of Pikes Peak granite:

Core length, L = 0.123 m.
Unit weight, γ = 25.93 kN/m^3.
Acceleration due to gravity, g = 9.81m/s^2.
P-wave travel time through core, t_p = 2.880 x 10^{-5} sec.
S-wave travel time through core, t_s = 5.426 x 10^{-5} sec.
P-wave velocity, $V_p = L/t_p$ = 4,270.8 m/sec
S-wave velocity, $V_s = L/t_s$ = 2,266.9 m/sec
Dynamic Poisson's ratio, ν_d = 0.304.
Dynamic Young's modulus, E_d = 35.44 GPa

In general, the dynamic Young's and shear moduli are larger than their static values (there are however instances where the opposite is true). Eissa and Kazi (1988) studied the relation existing between the static modulus E (secant or tangent) and the dynamic modulus, E_d. Analysis of 342 observations gave the following empirical equation

$$E = 0.74E_d - 0.82 \tag{7}$$

with a coefficient of correlation of 0.84. Another empirical equation, i.e

$$\log_{10}E = 0.02 + 0.77\log_{10}(\rho E_d) \tag{8}$$

gave a coefficient of correlation of 0.96. In both equations (7) and (8), the moduli are in GPa and in equation (8), ρ is the rock density in g/cm^3.

Kazi et al. (1983) proposed an empirical equation relating the uniaxial compressive strength of intact rocks to their dynamic modulus. A statistical analysis of more than 200 tests reported in the literature on seven different rock types yielded the following empirical equation

$$\log_{10}\sigma = 0.608 + 0.314(E_d/\rho)^{1/2} \tag{9}$$

2.3 Hooke's Law

Isotropic Formulation

In three dimensions, the deformability of a linearly elastic continuum is described by *Hooke's law*. Assuming isotropy, the stresses and strains in an arbitrary x,y,z coordinate system are related as follows

$$\begin{aligned}
\epsilon_x &= \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) \\
\epsilon_y &= \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) \\
\epsilon_z &= \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y))
\end{aligned} \tag{10}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz}; \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}; \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

Note that *two* and only two elastic constants, namely the Young's modulus, E, and the Poisson's ratio, ν, are needed to describe the deformability of a medium in its linear elastic range if the medium is assumed to be isotropic. For continua that are anisotropic (i.e. with directional deformability properties), more than two elastic constants are required, as discussed below.

Anisotropic Formulation

The deformability of anisotropic linearly elastic media is, in general, described by more than two elastic constants. In an arbitrary x,y,z coordinate system, the components of stress and strain are related as follows

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \tag{11}$$

or in a more compact form

$$[\epsilon]_{xyz} = [A][\sigma]_{xyz} \tag{12}$$

which is known as the *Generalized Hooke's* law. Matrix [A] is a (6x6) compliance matrix which is symmetric with $a_{ij}=a_{ji}$ (i,j=1-6).The inverse form of equation (12) can be expressed as follows

$$[\sigma]_{xyz} = [C][\epsilon]_{xyz} \tag{13}$$

where [C] is the matrix of elastic constants. Since matrices [A] and [C] are symmetric, they have at most 21 independent components. That number can be further reduced to 13, 9, 5 and finally to 2 (in the isotropic case), if the medium has any symmetry in the x,y,z coordinate system.

Coordinate Transformation of the Generalized Hooke's Law

Consider two rectangular coordinate systems x,y,z and x',y',z'. The orientation of the x'-, y'-, z'-axes is defined in terms of the direction cosines of unit vectors $\mathbf{e}'_1$, $\mathbf{e}'_2$ and $\mathbf{e}'_3$ in the x,y,z coordinate system (see equation (10), Lecture Notes 3). The constitutive equation of the medium in the x,y,z coordinate system is given by equation (12). In the x',y',z' coordinate system, it is expressed as

$$[\epsilon]_{x'y'z'} = [A'][\sigma]_{x'y'z'} \tag{14}$$

The compliance matrices [A] and [A'] are related. Combining equations (12) and (14), and equations (13), (52) and (53) in Lecture Notes 3, it can be shown that

$$[A] = [T_\sigma]^t[A'][T_\sigma] \tag{15}$$

and

$$[A'] = [T_\epsilon][A][T_\epsilon]^t \tag{16}$$

Note that in general, matrices [A] and [A'] are different unless the material is isotropic, in which case the compliance components are independent of the coordinate system (since the material is isotropic).

2.4 Rock Anisotropy

Types of Anisotropic Rocks

Many rocks exposed near the Earth's surface show well defined fabric elements in the form of bedding, stratification, layering, foliation, fissuring or jointing. In general, these rocks have properties (physical, dynamic, thermal, mechanical, hydraulic) that vary with direction and are said to be inherently *anisotropic*. Anisotropy can be found at different scales in a rock mass ranging from intact specimens to the entire rock mass.

Anisotropy is a characteristic of intact foliated *metamorphic* rocks (slates, gneisses, phyllites, schists). In these rocks, the fabric can be expressed in different ways. Closely-spaced fractures called cleavages are found, for instance, in slates and phyllites. These rocks tend to split into planes due to parallel orientation of microscopic grains of mica, chlorite or other platy minerals. In schists, the fabric is created by the parallel to sub-parallel arrangement of large platy minerals such as mica, chlorite and talc. Foliation can also be expressed by alternating layers of different mineral composition such as in gneisses. Non-foliated metamorphic rocks such as marble also show some anisotropy due to preferred orientation of calcite grains. Anisotropy is also the characteristic of intact laminated, stratified or bedded *sedimentary* rocks such as shales, sandstones, siltstones, limestones, coal, etc... Here, the anisotropy results from complex physical and chemical processes associated with transportation, deposition, compaction, cementation, etc...It is noteworthy that rocks which have undergone several formation processes may present more than one directions of planar anisotropy such as foliation and bedding planes in slates. These directions are not necessarily parallel. Also, linear features such as lineations can be superposed on the planar features, which makes the characterization of the rock anisotropy more complicated.

The rocks mentioned above show clear evidence of anisotropy and were classified as Class B anisotropic rocks by Barla (1974). On the other hand, Class A anisotropic rocks are those rocks that exhibit anisotropic properties despite apparent isotropy. Some intact granitic rocks belong to that group. This section deals exclusively with class B rocks having intact anisotropy.

Constitutive Modeling

The directional character of the deformability properties of anisotropic rocks is usually assessed by field and laboratory testing. Deformability test results on anisotropic rocks are commonly analyzed in terms of the theory of elasticity for anisotropic media by assuming Hooke's law. The latter implies that the rock has at most 21 independent elastic components. However, for most practical cases, anisotropic rocks are often modelled as *orthotropic* or *transversely isotropic* media in a coordinate system attached to their apparent structure or directions of symmetry. Orthotropy (orthorhombic symmetry) implies that three orthogonal planes of elastic symmetry exist at each point in the rock and that these planes have same orientation throughout the rock. Transverse isotropy implies that at each point in the rock there is an axis of symmetry of rotation (n-fold axis of symmetry) and that the rock has isotropic properties in a plane normal to that axis. The plane is called the *plane of transverse isotropy.*

For a rock mass that is *orthotropic* in a local *n,s,t* (or x',y',z') Cartesian coordinate system (Figure 4) attached to clearly defined planes of anisotropy, Hooke's law can be expressed as follows (Lekhnitskii, 1977)

$$\begin{bmatrix} \epsilon_n \\ \epsilon_s \\ \epsilon_t \\ \gamma_{st} \\ \gamma_{nt} \\ \gamma_{ns} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_n} & -\frac{\nu_{sn}}{E_s} & -\frac{\nu_{tn}}{E_t} & 0 & 0 & 0 \\ -\frac{\nu_{ns}}{E_n} & \frac{1}{E_s} & -\frac{\nu_{ts}}{E_t} & 0 & 0 & 0 \\ -\frac{\nu_{nt}}{E_n} & -\frac{\nu_{st}}{E_s} & \frac{1}{E_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{st}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{nt}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ns}} \end{bmatrix} \begin{bmatrix} \sigma_n \\ \sigma_s \\ \sigma_t \\ \tau_{st} \\ \tau_{nt} \\ \tau_{ns} \end{bmatrix} \tag{17}$$

or in a more compact matrix form

$$[\epsilon]_{nst} = [H]\ [\sigma]_{nst} \tag{18}$$

Nine independent elastic constants are needed to describe the deformability of the medium in the n,s,t coordinate system; E_n, E_s and E_t are the Young's moduli in the n,s and t (or 1,2 and 3) directions, respectively; G_{ns}, G_{nt} and G_{st} are the shear moduli in planes parallel to the ns, nt and st planes, respectively; and ν_{ij} (i,j=n,s,t) are the Poisson's ratios that characterize the normal strains in the symmetry directions j when a stress is applied in the symmetry directions i. Because of symmetry of the compliance matrix [H], Poisson's ratios ν_{ij} and ν_{ji} are such that $\nu_{ij}/E_i=\nu_{ji}/E_j$. The orthotropic formulation has been used in the literature to characterize the deformability of rocks such as coal, schists, slates, gneisses, granites and sandstones. For instance, the cleat and bedding planes of coal are often assumed to be planes of elastic symmetry.

Equations (17) and (18) still apply if the rock is *transversely isotropic* in one of the three ns,nt or st planes of Figure 4. In that case, only five independent elastic constants are needed to describe the deformability of the rock in the n,s,t coordinate system. These constants are called E , E', ν, ν' and G' with the following definitions:

- E and E' are Young's moduli in the plane of transverse isotropy and in direction normal to it, respectively,

- ν and ν' are Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel or normal to it, respectively, and,

- G' is the shear modulus in planes normal to the plane of transverse isotropy.

Relationships exist between E, E', ν, ν', G and G' and the coefficients of matrix [H] in equations (17) and (18). For instance, for transverse isotropy in the st plane

$$\begin{gathered}\frac{1}{E_n} = \frac{1}{E'}\ ;\ \frac{1}{E_s} = \frac{1}{E_t} = \frac{1}{E}\ ;\ \frac{1}{G_{ns}} = \frac{1}{G_{nt}} = \frac{1}{G'} \\ \frac{\nu_{ns}}{E_n} = \frac{\nu_{nt}}{E_n} = \frac{\nu'}{E'}\ ;\ \frac{\nu_{st}}{E_s} = \frac{\nu_{ts}}{E_t} = \frac{\nu}{E}\ ;\ \frac{1}{G_{st}} = \frac{2(1+\nu)}{E}\end{gathered} \tag{19}$$

The transverse isotropy formulation has been used to characterize the deformability of rocks such as schists, gneisses, phyllites, siltstones, mudstones, sandstones, shales and basalts. For such rocks, the plane of transverse isotropy is assumed to be parallel to foliation, schistosity or bedding planes. Note that some of the five or nine elastic constants of anisotropic rocks are sometimes assumed to be related. For instance, for transversely isotropic rocks, the modulus G' is often expressed in terms of E, E', ν and ν' through the following empirical equation

$$\frac{1}{G'} = \frac{1}{E} + \frac{1}{E'} + 2\frac{\nu'}{E'} \tag{20}$$

For orthotropic rocks, the shear moduli G_{ns}, G_{nt} and G_{st} are related to the three Young's moduli and Poisson's ratios. These relations were first introduced by St. Venant (1863). In a recent survey of elastic constants of anisotropic rocks, Worotnicki (1993) concluded that most of the published experimental data support the validity of the St. Venant approximation, with however major exceptions.

The five and nine elastic properties of transversely isotropic and orthotropic rocks, respectively cannot be randomly selected. Indeed, some inequalities associated with the thermodynamic constraints that the rock strain energy remains positive definite, must be satisfied (Lempriere, 1968; Pickering, 1970). For instance, for *transverse isotropy*, the five elastic properties E, E', ν, ν' and G' must satisfy the following thermodynamic constraints

$$E \; ; \; E' \; ; \; G' > 0 \tag{21}$$

$$-1 < \nu < 1 \tag{22}$$

$$1 - \nu - 2\nu'^2 \frac{E}{E'} > 0 \tag{23}$$

Equations (22) and (23) reduce to $-1<\nu<0.5$ if the rock is *isotropic* for which $E=E'$, $G=G'$ and $\nu=\nu'$. For orthotropy, the expressions for the thermodynamic constraints on the nine elastic properties are more complex and can be found in Amadei *et al.* (1987).

Degrees of Rock Anisotropy

In general, intact rocks are not too strongly anisotropic compared to other engineering materials such as wood or composites. Typical values of the nine or five elastic constants (static and dynamic) for different types of intact anisotropic rocks can be found in the literature. Comprehensive surveys of elastic properties can be found in Amadei *et al.* (1987), Worotnicki (1993), and Amadei (1996). For instance, Worotnicki (1993) classified anisotropic rocks into four groups:

1. Quartzfeldspathic rocks (*e.g.* granites; quartz and arkose sandstones, granulates and gneisses),

2. Basic/lithic rocks (*e.g.* basic igneous rocks such as basalt; lithic and greywacke sandstones and amphibolates),

3. Pelitic (clay) and pelitic (micas) rocks (e.g. mudstones, slates, phyllites, and schists),

4. Carbonate rocks (*e.g.* limestones, marbles and dolomites),

Based on 200 sets of test results, Worotnicki (1993) concluded that quartzofeldspathic and basic/lithic rocks show low to moderate degrees of anisotropy with a maximum to minimum Young's modulus ratio E_{max}/E_{min} less than 1.3 for about 70% of the rocks analyzed and less than 1.5 in about 80%. This ratio was found not to exceed 3.5 (Figure 5a). Pelitic clay and pelitic micas rocks show the highest degree of anisotropy with E_{max}/E_{min} less than 1.5 for about 33% of the rocks analyzed and less than 2 in about 50%. The modulus ratio was found not to exceed 6 with most cases below 4 (Figure 5b). Finally, carbonate rocks were found to show an intermediate degree of rock anisotropy with E_{max}/E_{min} not exceeding 1.7 (Figure 5c).

2.5 Laboratory Testing

Testing of Isotropic Rocks

Suggested methods for determining the deformability (and strength) of intact rock under uniaxial compression have been proposed by the ISRM (Bieniawski and Bernede, 1979) and the ASTM (D3148-93). These methods are limited to isotropic media, and therefore to the determination of two elastic constants, i.e. the Young's modulus and Poisson's ratio. They involve the testing of right circular cylinders with a length-to-diameter ratio of 2.5-3.0.. Special emphasis has to be placed on the preparation of the test specimens; specimen ends must be flat (to 0.02 mm or 0.0008 in) and perpendicular to the specimen axis (deviation less than 0.001 radian i.e. 0.05 mm in 50 mm); the sides of the specimen must be smooth; no capping material should be used. Stress rates must vary between 0.5 and 1.0 MPa/s. Deformation of the test specimens is measured using strain gages, strain rosettes, Linear Variable Differential Transformers (LVDTs), or optical systems.

Testing of Anisotropic Rocks

There are no standards for laboratory testing of anisotropic rocks. Various testing methods are, however, available and are discussed more extensively in Amadei (1996). In most cases, the rock is treated as transversely isotropic and in rare cases as orthotropic. Since more than two elastic constants need to be determined, several specimens must be cut at different angles with respect to the apparent directions of rock anisotropy.

The laboratory testing methods on anisotropic rock can be divided into static and dynamic. Static methods include uniaxial compression, triaxial compression, multiaxial compression, diametral compression (Brazilian tests), torsion and bending. For all these tests, the specimens are instrumented with strain gages or displacement transducers. The dynamic methods include the resonant bar method and the ultrasonic pulse method. Note that several types of tests (static and/or dynamic) should be conducted on a given rock to determine its anisotropic elastic properties.

The type of loading test and the number of tests required to determine the elastic constants of a given rock depend largely on the degree and type of symmetry assumed for the rock. Consider for instance a *transversely isotropic* rock specimen tested under uniaxial compression (Figure 6). An x,y,z coordinate system is attached to the specimen with the z axis parallel to the plane of transverse isotropy dipping at an angle θ with respect to the xz plane. The rock is transversely isotropic in the st plane of Figure 6 and has five elastic properties E, E', ν, ν' and G' as defined in (19). Strains are measured in the x, y and z directions using strain gages. Using the theory of elasticity for anisotropic media and assuming uniform stresses and strains in the test specimen, the strains ϵ_x, ϵ_y, ϵ_z and γ_{xy} can be related to the applied stress, σ, as follows

$$\epsilon_x = a_{12}\sigma \quad ; \quad \epsilon_y = a_{22}\sigma \quad ; \quad \epsilon_z = a_{23}\sigma \quad ; \quad \gamma_{xy} = a_{26}\sigma \tag{24}$$

with

$$\begin{aligned}
a_{12} &= -\frac{\nu'}{E'}\sin^4\theta - \frac{\nu'}{E'}\cos^4\theta + \frac{\sin^2 2\theta}{4}\left(\frac{1}{E} + \frac{1}{E'} - \frac{1}{G'}\right) \\
a_{22} &= \cos^4\frac{\theta}{E'} + \sin^4\frac{\theta}{E} + \frac{\sin^2 2\theta}{4}\left(\frac{1}{G'} - 2\frac{\nu'}{E'}\right) \\
a_{23} &= -\frac{\nu'}{E'}\cos^2\theta - \frac{\nu}{E}\sin^2\theta \\
a_{26} &= \sin 2\theta\left(\cos^2\theta\left(\frac{1}{E'} + \frac{\nu'}{E'}\right) - \sin^2\theta\left(\frac{1}{E} + \frac{\nu'}{E'}\right)\right) - \frac{\sin 2\theta \cos 2\theta}{2G'}
\end{aligned} \tag{25}$$

By conducting uniaxial compression tests on three specimens cut at different angles with respect to the plane of transverse isotropy, the five elastic properties of the rock can theoretically be determined from the linear portion of the corresponding stress-strain curves. In Figure 7, three specimens of the same rock are tested in uniaxial compression with θ=0°, θ=90° and an inclined angle θ different from 0 or 90°. Using equations (24) and (25), the strains measured on the first specimen (Figure 7a) allow the determination of E' and ν' whereas those on the second specimen (Figure 7b) are used to determine E and ν. The strains measured on the third specimen (Figure

7c) are used to determine the shear modulus G'. A procedure that has been found to work well is to instrument each one of the three specimens in Figure 7 with two to four 45° strain rosettes. All strain measurements are then analyzed simultaneously. Let N be the total number of strain measurements (with $N \geq 5$) for all three specimens in Figure 7. According to equations (24) and (25), each strain measurement is linearly related to the five unknown compliances $1/E$, $1/E'$, ν/E, ν'/E' and $1/G'$. In matrix form, the strain measurements can then be expressed in terms of the five compliances as follows

$$[\epsilon] = [T][C] \tag{26}$$

where $[\epsilon]$ is a (Nx1) matrix of strain measurements, [T] is a (Nx5) matrix and $[C]^t=(1/E \ 1/E' \ \nu/E \ \nu'/E' \ 1/G')$. Equation (26) is then solved for the least square (best fit) estimate of the five compliances terms by multilinear regression analysis. The advantage of this approach is that all strain measurements are taken into account when determining the compliance terms. Furthermore, the method can be extended to more than three specimens.

Equations (24) and (25) show that shear strains can develop under uniaxial compression as long as the dip angle, θ, of the planes of rock anisotropy differs from 0 or 90° or in other words as long as the applied stress does not coincide with the fabric. In that case, the principal strain directions do not coincide with the principal stress directions as for isotropic media.

Equations (24) and (25) can also be used to calculate the apparent Young's modulus, E_y, and apparent Poisson's ratios ν_{yx} and ν_{yz} of the rock in the x,y,z coordinate system of Figure 6 with

$$E_y = \frac{1}{a_{22}} \quad ; \quad \nu_{yx} = -\frac{a_{12}}{a_{22}} \quad ; \quad \nu_{yz} = -\frac{a_{23}}{a_{22}} \tag{27}$$

These three quantities depend on the angle θ. Examples of variation of such quantities can be found in Amadei (1996).

The five and nine elastic properties of transversely isotropic and orthotropic rocks can also be determined by diametral compression (line or strip load) of thin discs of rocks (Brazilian loading). Strains are measured using strain gage rosettes (usually 45° rosettes) glued at the center of the discs.

Consider the geometry of Figure 8. A disc of rock of diameter, D, and thickness, t, is subject to a diametral load, W, applied over an angular width 2α (assumed here to be small). The rock is assumed to be *orthotropic* with one of its three planes of elastic symmetry (defined as *ns* in Figure 8) parallel to the disc *xy* plane. The n and s axes are inclined at an angle, ψ, with respect to the x and y axes. The applied pressure, p, is equal to $W/(\alpha D t)$. As shown by Amadei *et al.*

(1983), the stress components at the center of the disc are equal to

$$\sigma_x = q_{xx}\frac{W}{\pi Dt} \quad ; \quad \sigma_y = q_{yy}\frac{W}{\pi Dt} \quad ; \quad \tau_{xy} = q_{xy}\frac{W}{\pi Dt} \tag{28}$$

For the geometry of Figure 8, it can be shown that the stress concentration factors q_{xx}, q_{yy} and q_{xy} have complex expressions which depend on the rock compliances $1/E_n$, $1/E_s$, ν_{ns}/E_n and $1/G_{ns}$ and the dip angle ψ. If the rock were isotropic, q_{xx}=-2, q_{yy}=6 and q_{xy}=0. The strains ϵ_x, ϵ_y and γ_{xy} at the disc center are related to the compliances $1/E_n$, $1/E_s$, ν_{ns}/E_n and $1/G_{ns}$ by combining equation (28) with the rock's constitutive equations.

Consider the three diametral compression tests (j=1,3) shown in Figure 9. The rock is again assumed to be *orthotropic*. Each test consists of loading a disc whose middle plane is parallel to one of the three planes of elastic symmetry of the rock (*ns*, *st* or *nt*). In each test, the strains at the center of the disc are measured in three directions ϵ_{aj}, ϵ_{bj} and ϵ_{cj} (j=1,3) using a 45° strain rosette, at an arbitrary load, *W*, and in the linear elastic range of rock behavior. To simplify the presentation, the three discs are assumed to have same geometry, same strain gage orientation, the same loading angle, 2α, and the strains are measured at the same load level *W*. For the geometry of Test 1, the stress concentration factors q_{xx1}, q_{yy1} and q_{xy1} depend on the rock compliances $1/E_n$, $1/E_s$, ν_{ns}/E_n and $1/G_{ns}$. Similarly, for Test 2, the stress concentration factors q_{xx2}, q_{yy2} and q_{xy2} depend on the rock compliances $1/E_s$, $1/E_t$, ν_{st}/E_s and $1/G_{st}$. Finally, for Test 3, the stress concentration factors q_{xx3}, q_{yy3} and q_{xy3} depend on the rock compliances $1/E_n$, $1/E_t$, ν_{nt}/E_n and $1/G_{nt}$. In addition, the nine stress concentration factors depend on the angle 2α and the orientation angles ψ_j (j=1,3). Combining equation (28) and the rock's constitutive equations for each disc in Figure 9, it can be shown that the nine compliances are related to the nine strain gage measurements in matrix form as follows

$$[\epsilon] = \frac{W}{\pi Dt}[T][C] \tag{29}$$

where $[\epsilon]$ is a (9x1) matrix of strain measurements, [T] is a (9x9) matrix and $[C]^t=(1/E_n\ 1/E_s\ 1/E_t\ \ \nu_{ns}/E_n\ \ \nu_{st}/E_s\ \ \nu_{nt}/E_n\ \ 1/G_{ns}\ \ 1/G_{st}\ \ 1/G_{nt})$. The components of matrix [T] depend on the nine stress concentration factors, which themselves depend on the nine compliances. The system of equations (29) is therefore highly non-linear and is furthermore constrained since the nine elastic constants must satisfy some thermodynamic constraints. A solution to this constrained problem can be obtained using the *Generalized Reduced Gradient Method* which is essentially a constrained optimization technique.

The same methodology applies if the rock is *transversely isotropic*. The five elastic properties of the rock can be determined by conducting two instead of three diametral compression tests: one in the plane of transverse isotropy and another perpendicular to the plane of transverse isotropy. If the plane of transverse isotropy coincides, for instance, with plane *st*, then the five elastic constants can be determined by conducting Tests 1 and 2 in Figure 9. The strains measured in Test 2 are then analyzed with q_{xx}=-2, q_{yy}=6 and q_{xy}=0. This gives the Young's modulus $E=E_s=E_t$ and the Poisson's ratio $\nu=\nu_{st}$. On the other hand, the strains measured in Test 1 lead to a system of equations similar to equation (29) with only three equations and three unknowns $E'=E_n$, $\nu'=\nu_{ns}=\nu_{nt}$ and $G'=G_{ns}=G_{nt}$. This system of equations is then solved using the *Generalized Reduced Gradient Method* for the three unknowns taking into account the constraints defined in inequalities (21)-(23).

3. ROCK MASS DEFORMABILITY

3.1 Characterizing Rock Mass Deformability

Rock masses are far from being continua and consist essentially of two constituents: intact rock and discontinuities. The existence of one or several sets of discontinuity planes in a rock mass creates anisotropy in its response to loading and unloading. Also, compared to intact rock, jointed rock shows increased deformability in directions normal to those planes. Furthermore, discontinuities create scale effects. Since laboratory tests on small scale samples are usually found inadequate to predict the deformability of rock masses, field tests are necessary.

Rock mass anisotropy can be found in volcanic formations (basalt) and sedimentary formations consisting of alternating layers or beds of different (isotropic or anisotropic) rock types. Rock masses cut by one or several regularly spaced joint sets are anisotropic in addition to being discontinuous. The rock between the joints can be isotropic or anisotropic. It is not unusual to have several types of planar anisotropy in a rock mass such as joints and foliation planes or joints and bedding planes. If the joints develop parallel to the foliation or bedding planes, they are called foliation joints or bedding joints, respectively.

With few exceptions, it is incorrect to ignore the presence of discontinuities when modeling rock mass response to loading and unloading. Three approaches can be followed to account for the effect of joints on rock mass deformability: an empirical approach, a discrete approach, an equivalent approach.

Empirical Approach

The first approach consists of empirically reducing the deformability properties of rock masses from those measured on intact rock samples in the laboratory. Rock mass modulus, also called *modulus of deformation*, can be estimated in different ways. For instance, based on an extensive literature review, Heuze (1980) concluded that the modulus of deformation of rock masses ranges between 20 and 60 % of the modulus measured on intact rock specimens in the laboratory. Bieniawski and Orr (1976), Bieniawski (1978), and Serafim and Pereira (1983) proposed empirical relationships between the modulus of deformation of rock masses, E_M (in GPa), and their (Geomechanics Classification System) RMR ratings e.g.

$$E_M = 2RMR - 100 \tag{30}$$

if RMR > 50% and

$$E_M = 10^{(RMR-10)/40} \tag{31}$$

if RMR < 50%. Although still the most used, the empirical approach lacks a mechanistic basis.

Discrete Approach

A second approach consists of treating joints as discrete features. This is usually done in numerical methods such as the finite element, boundary element and discrete element methods in which the complex response of joints to normal and shear stresses can be introduced in an explicit manner. The main drawback of this approach is that only rock masses with a limited amount of joints can be analyzed due to computer limitations.

Equivalent Approach

The third approach is to treat jointed rock as an equivalent anisotropic continuum with deformability properties that are directional and also reflect the properties of intact rock and those of the joint sets, i.e. orientation, spacing and normal and shear stiffnesses. The discontinuities are characterized without reference to their specific locations. The rock mass is replaced by an equivalent transversely isotropic continuum if cut by a single joint set or an equivalent orthotropic continuum if cut by two or three orthogonal joint sets. This approach is recommended when the number of discontinuities is large where it is not feasible to account for each joint plane when assessing the overall deformability of a rock mass.

Consider for instance a rock mass cut by three joint sets, each set being normal to one the three axes of the *n,s,t* coordinate system of Figure 4. The intact rock between the joints is assumed to be linearly elastic and isotropic with Young's modulus, E, Poisson's ratio, ν, and shear modulus $G=E/2(1+\nu)$. Each joint set i (i=1,2,3) is characterized by its spacing, S_i, and its normal and shear stiffnesses k_{ni} and k_{si}. As shown by Duncan and Goodman (1968), the regularly jointed rock mass can be replaced by an equivalent orthotropic continuum whose constitutive relation in the *n,s,t* coordinate system is given by equations (17) and (18) with

$$\frac{1}{E_i} = \frac{1}{E} + \frac{1}{k_{ni}S_i} \tag{32}$$

and

$$\frac{1}{G_{ij}} = \frac{1}{G} + \frac{1}{k_{si}S_i} + \frac{1}{k_{sj}S_j} \tag{33}$$

where i,j=1,2,3 or n,s,t respectively. All non-zero off-diagonal terms in equation (17) are now equal to $-\nu/E$. Since k_{ni} and k_{si} have units of stress/length or force/length[3], the quantities $k_{ni}S_i$ and $k_{si}S_i$ in equations (32) and (33) can be seen as normal and shear joint moduli. Note that the constitutive relation for a regularly jointed rock mass converges to that for an intact isotropic medium when the joint spacings or joint stiffnesses in equations (32) and (33) approach infinity.

Although convenient, the equivalent continuum formulation has certain limitations that limit its range of application. For instance, Duncan and Goodman (1968) made three main assumptions in the equivalent continuum approach. First, the normal and shear joint stiffnesses are constant and independent of the stress level acting across the joints. Second, the joint response remains in the elastic domain (pre-slip condition). Third, the joints are assumed to have negligible thicknesses and not to create any Poisson's effect upon loading of the rock mass. In other words, intact rock and joints are assumed to undergo equal strains in directions parallel to the contact planes. This assumption makes all non-zero off-diagonal terms in equation (17) equal to the same value, $-\nu/E$. It also results in reducing the number of elastic constants necessary to describe the deformability of the rock mass when cut by a single joint set to four, namely, E, ν, k_nS and k_sS. For a rock mass cut by three joint sets the number of elastic constants is equal to eight: E, ν, $k_{ni}S_i$ and $k_{si}S_i$ with i=1,2,3.

The equivalent approach has also been used in the past for modeling the deformability of thinly layered, laminated or stratified rock masses that are clearly heterogeneous in addition to being anisotropic. Since it is not possible to account for each layer on an individual basis, one approach has been proposed whereby the rock mass is replaced by an equivalent homogeneous transversely isotropic continuum (Salamon, 1968; Wardle and Gerrard, 1972). Again several conditions need to be satisfied when using such an equivalent model. First, the rock mass consists of isotropic and/or transversely isotropic layers whose thickness and elastic properties vary randomly with depth. Second, the rock is a continuum and remains a continuum when subjected to stresses. Third, the elastic properties of the equivalent continuum are derived by examining the behavior of two cubes both having the same edge dimension L. One cube is a representative sample of the rock mass whereas the other cube is cut from the equivalent continuum and is subject to homogeneous stress and strain distributions. The representative sample of the rock mass contains a large number of layers with known thicknesses and deformability properties and its volume is sufficiently small to make negligible in the equivalent continuum the variations of stresses and strains across it.

Consider, for instance, m horizontal layers forming a representative sample of the rock mass. If the thickness of the *jth* layer is h_j, its relative thickness is $\Phi_j=h_j/L$. The deformability of each layer is defined by five elastic constants E_j, E_j', ν_j, ν_j' and G_j'. As shown by Salamon (1968), the five elastic properties of the equivalent homogeneous continuum E, E', ν, ν' and G' are equal to

$$\frac{1}{E}=\frac{\sum \frac{\Phi_j E_j}{1-\nu_j^2}}{\sum \frac{\Phi_j E_j}{1+\nu_j}\sum \frac{\Phi_j E_j}{1-\nu_j}}$$

$$\frac{\nu}{E}=\frac{\sum \frac{\Phi_j E_j \nu_j}{1-\nu_j^2}}{\sum \frac{\Phi_j E_j}{1+\nu_j}\sum \frac{\Phi_j E_j}{1-\nu_j}} \qquad \frac{\nu'}{E'}=\frac{\sum \Phi_j \frac{E_j}{E_j'}\frac{\nu_j'}{1-\nu_j}}{\sum \frac{\Phi_j E_j}{1-\nu_j}} \tag{34}$$

$$\frac{1}{E'}=\sum \Phi_j\left(\frac{1}{E_j'}-2\nu_j'^2\frac{E_j}{E_j'}.\frac{1}{1-\nu_j}\right)+2\frac{\left(\sum \Phi_j\frac{E_j}{E_j'}.\frac{\nu_j'}{1-\nu_j}\right)^2}{\frac{\sum \Phi_j E_j}{1-\nu_j}}$$

$$\frac{1}{G}=\frac{1}{\sum \Phi_j G_j} \qquad \frac{1}{G'}=\sum \frac{\Phi_j}{G_j'}$$

in which the summation from j=1,m is implied. The shear modulus, G, is also equal to $E/(2(1+\nu))$.

Degrees of Rock Mass Anisotropy

If rock mass anisotropy is induced by joints, the ratio of anisotropy can be much larger than for intact rock and depends on the stress level acting across the joint planes. As an illustrative example consider a rock mass cut by a single joint set. Using again the model of Duncan and Goodman (1968), the ratio E/E' is equal to

$$\frac{E}{E'} = 1+\frac{E}{k_n S} \tag{35}$$

Laboratory tests on rock joints have shown that the joint normal stiffness, k_n, depends on the normal stress, σ_n, acting across the joint planes. Using, for instance, the expression for the tangential normal stiffness, k_n, proposed by Bandis *et al*. (1983), equation (35) becomes

$$\frac{E}{E'} = 1 + \frac{E}{k_{ni} S} . \left(\frac{k_{ni} V_m}{\sigma_n + k_{ni} V_m} \right)^2 \tag{36}$$

where k_{ni} is the initial joint normal stiffness and V_m is the maximum joint closure. According to equation (36), at zero normal stress ($\sigma_n=0$), the ratio E/E' is equal to $1+E/(k_{ni}\ S)$ which can be large for joints with small values of the spacing and/or initial stiffness. As more compression is applied across the joint surfaces, the rock joints become stiffer and the ratio E/E' approaches unity as less anisotropy is induced by the joints.

3.2 Measuring Rock Mass Deformability

Because of scale effect, field tests are required to determine rock mass deformability. The tests can be static or dynamic. In order to assess the degree of rock mass anisotropy, field tests are conducted in different directions with respect to the apparent rock mass fabric.

The analysis of field tests is usually done assuming that the rock mass behaves linear elastically when subjected to the applied loads. Also, because of the complexity of the tests and the more sophisticated nature of the equations that are necessary in the analysis of the test results, very often, assumptions are made about the rock deformability in order to reduce the number of elastic constants that need to be determined. Sometimes, test results in anisotropic ground are analyzed using equations derived from the theory of linear elasticity for isotropic media. The Young's modulus is determined for different loading directions.

It should be kept in mind that, in general, field tests involve larger rock volumes than laboratory tests. Compared to laboratory tests, the stress distributions in those volumes are more complex and the measured elastic properties are more likely to be average properties and to be affected by the level of applied load (in particular if the rock mass is cut by joint sets). Also, discrepancies can arise between rock mass properties determined with one field testing method and another because different rock volumes are involved.

Static field tests include: plate loading tests, borehole expansion tests (dilatometer, NX-borehole jack, modified borehole jack), gallery and radial jacking tests, and flat jack tests. Case studies and reviews of their advantages and limitations can be found in Goodman et al. (1970), Rocha et al. (1970), Bieniawski and Van Heerden (1975), Bieniawski (1978), De La Cruz (1978), Goodman (1989), Heuze and Amadei (1985), Luehring (1988), and Amadei (1996). Several papers dealing with the determination of the modulus of deformation of rock masses can also be found in the proceedings of a special ASTM symposium held in Denver in 1969 (ASTM Publ. 477, 1970).

Testing procedures have been proposed by the ISRM (Coulson, 1979) and the ASTM (ASTM standards D 4395-84, 4971-89, 4506-90, 4394-84, 4729-87).

In the dynamic field tests, wave velocity is measured by swinging a sledgehammer against an outcrop or by using an explosive source. Directional geophones placed at the rock surface or in boreholes measure the time it takes for the longitudinal and transverse waves to propagate from their source. A technique called "petite sismique", introduced in France in the late 1960s, can be used to determine the dynamic modulus of deformation of the rock mass (Bieniawski, 1978). The effect of rock fractures upon the velocity of propagation of seismic waves can be assessed using the following time-average formula

$$\frac{L}{V_p\ (rock\ mass)} = = \frac{nw}{V_p\ (fracture\ filler)} + \frac{(L-nw)}{V_p\ (intact\ rock)} \tag{37}$$

where L is the direct path length in m, n is the number of fractures, and w is the average width of the fractures. Equation (37) is written for longitudinal waves. A similar equation can be written for shear waves with V_p replaced by V_s. Equation (37) indicates that larger the number and width of the fractures and the lower the velocity of the infill, then the lower will be the measured rock mass velocity. As a numerical example, consider the propagation of longitudinal waves through limestone (V_p=4,000 m/s for intact rock) over a length L=20 m. The limestone is cut by n=10 fractures of width, w=0.05 m. If the fractures are water filled (V_p=1,450 m/s for water), the longitudinal velocity for the rock mass is reduced to 3,831 m/s. If the fractures are dry (V_p=330 m/s for air), the velocity is reduced further to 3,129 m/s.

Various authors have tried to relate rock mass quality, the RQD, and the velocity of propagation of seismic waves. A correlation has been suggested between the RQD and a so-called *velocity index* defined as the square of the ratio of the compressional wave velocity measured *in situ* (V_F) to that measured in the laboratory (V_L) (see Table 3). Correlations have also been proposed between the velocity index and the modulus of deformation (Coon and Merritt, 1970).

Rock Quality Classification	RQD (%)	Fracture Frequency (1/m)	V_F/V_L	$(V_F/V_L)^2$
Very poor	0-25	15	0-0.4	0-0.2
Poor	25-50	15-18	0.4-0.6	0.2-0.4
Fair	50-75	8-5	0.6-0.8	0.4-0.6
Good	75-90	5-1	0.8-0.9	0.6-0.8
Excellent	90-100	1	0.9-1.0	0.8-1.0

Table 3. Seismic evaluation of rock mass quality (after McDowell, 1993).

4. REFERENCES

Amadei, B. (1996) Importance of anisotropy when estimating and measuring *in situ* stresses in rock, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 33, No.3, pp. 293-325.

Amadei B., Rogers J.D. and Goodman R.E. (1983) Elastic constants and tensile strength of anisotropic rocks. *Proc. 5th ISRM Congress*, Aachen, A189-A196.

Amadei B., Savage W.Z. and Swolfs H.S. (1987) Gravitational stresses in anisotropic rock masses, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr*., Vol. 24, pp. 5-14.

ASTM D3148-93. Standard test method for elastic moduli of intact core specimens in uniaxial compression, ASTM Volume 04.08.

ASTM (1969) Determination of the in situ modulus of deformation of rock, *ASTM Spec. Tech. Publ.* 477.

Balmer, G.G. (1953) Physical properties of some typical foundation rocks, *U.S. Bureau of Reclamation Report No. SP-39.*

Bandis S.C., Lumsden A.C. and Barton N. (1983) Fundamentals of rock joint deformation. *Int. J. Rock Mech. Min. Sci & Geomech. Abstr.*, Vol. 20, pp. 249-268.

Barla G. (1974) Rock anisotropy: Theory and laboratory testing, in *Rock Mechanics*. Müller, L. Editor, Udine, Italy, pp. 131-169.

Bieniawski, Z.T. (1978) Determining rock mass deformability: Experience from case histories, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 15, No.5, pp. 237-248.

Bieniawski, Z.T. (1993) Classification of rock masses for engineering: the RMR system and future trends", in *Comprehensive Rock Engineering* (J.A. Hudson ed.), Pergamon, Vol. 3, pp. 553-73.

Bieniawski, Z.T. and Van Heerden, W.L. (1975) The significance of in situ tests on large rock specimens, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 12, No.4, pp. 101-113.

Bieniawski, Z.T. and Orr, C.M. (1976) Rapid site appraisal for dam foundations by the geomechanics classification, *Proc. 12th Int. Cong. on Large Dams*, Mexico, pp. 483-501.

Bieniawski, Z.T. and Bernede, M.J. (coordinators) (1979) Suggested methods for determining the uniaxial compressive strength and deformability of rock materials, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 16, No.2, pp. 135-140.

Coon, R.F. and Merritt, A.H. (1970) Predicting in situ modulus of deformation using rock quality indexes, in *Determination of the In Situ Modulus of Deformation of Rock, ASTM Spec. Tech.*

Publ. 477, pp. 154-173.

Coulson, J.H. (1979) (Coordinator) Suggested methods for determining in situ deformability of rock, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 16, No.3, pp. 195-214.

De La Cruz, R.V. (1978) Modified borehole jack method for elastic property determination in rocks, *Rock Mechanics*, Vol. 10, pp. 221-239.

Duncan, J.M. and Goodman, R.E. (1968) Finite element analyses of slopes in jointed rock, Final Report to U.S. Army Corps of Engineers, Vicksburg, Mississippi, Report S-68-3.

Eissa, E.A. and Kazi, A. (1988) Relation between static and dynamic Young's moduli of rocks, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 25, No.6, pp. 479-482.

Goodman, R.E. (1989) *Introduction to Rock Mechanics*, Wiley & Sons, 2nd. Edition.

Goodman, R..E, Van, T.K. and Heuze, F.E. (1970) Measurement of rock deformability in boreholes, *Proc. 10th. U.S Symp. on Rock Mech.*, Univ. of Texas, Austin, pp.523-555.

Hatheway, A.W. and Kiersch, G.A. (1989) Engineering properties of rocks, in *Physical Properties of Rocks and Minerals* (R.S. Carmichael, ed.), CRC Publ., pp. 671-715.

Heuze, F.E. (1980) Scale effects in the determination of rock mass strength and deformability, *Rock Mechanics*, Vol. 12, pp.167-192.

Heuze, F.E and Amadei, B. (1985) The NX-borehole jack: A lesson in trial and error, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 22, No.2, pp. 105-112.

Johnson, R.B. and DeGraff, J.V. (1988) *Principles of Engineering Geology*, Wiley.

Kazi, A., Sen, Z. and Sadagah, B-E. H. (1983) Relationship between sonic velocity and uniaxial compressive strength of rocks, *Proc. 24th US Symp. Rock Mech.*, College Station, pp. 409-419.

Lekhnitskii S.G. (1977) *Theory of Elasticity of an Anisotropic Body*, Mir Publishers, Moscow.

Lempriere B.M. (1968) Poisson's ratios in orthotropic materials. *J. Am. Inst. Aeronaut. Astronaut.*, Vol. 6, pp. 2226-2227.

Luehring, R.W. (1988) Methods for determining in situ deformation of rock masses, U.S. Bureau of Reclamation Report REC-ERC-87-14.

McDowell, P.W. (1993) Seismic investigation for rock engineering, in *Comprehensive Rock Engineering* (J.A. Hudson ed.), Pergamon, Vol. 3, pp. 619-634.

Niandou, H. et al. (1997) Laboratory investigation of the mechanical behavior of Tournemire shale, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 34, No.1, pp. 3-16.

Pickering D.J. (1970) Anisotropic elastic parameters for soils. *Geotechnique,* Vol. 20, pp. 271-276.

Rocha, M., et al. (1970) Characterization of the deformability of rock masses by dilatometer tests, *Proc. 2nd. Cong. Int. Soc. Rock Mech*. (ISRM), Belgrade, 2-32.

Salamon, M.D.G. (1968) Elastic moduli of a stratified rock mass, *Int. J. Rock Mech. Min. Sci.*, Vol. 5, pp. 519-527.

Serafim, J.L. and Pereira, J.P. (1983) Considerations of the geomechanics classification of Bieniawski, *Proc. Int. Symp. on Engineering Geology and Underground Construction*, LNEC, Lisbon, Portugal, vol .1, pp. II.33-II.42.

Saint-Venant B. (1863) Sur la distribution des élasticités autour de chaque point d'un solide ou d'un milieu de contexture quelconque. (in French). *J. de Mathématiques Pures et Appliquées*, Ser. 2, Vols. 7-8, pp. 353-430 and 257-261.

Wardle, L.J. and Gerrard, C.M. (1972) The equivalent anisotropic properties of layered rock and soil masses. *Rock Mechanics*, Vol.4, pp.155-175 (1972)

Worotnicki G. (1993) CSIRO triaxial stress measurement cell, in *Comprehensive Rock Engineering,* Pergamon Press, Oxford (J.A. Hudson, Editor), Chapter 13, Vol. 3, 329-394.