HEMISPHERICAL PROJECTION METHODS

1. INTRODUCTION

Most planar and linear features in geology are three dimensional. Planar features include, for instance, fractures, faults, joints, foliation and bedding planes. Linear features include striations, lines of intersection of fracture planes, fold axes, normal of fracture planes, etc... It is often difficult to visualize how these different features are oriented in space and how they interact with each other. Hemispherical projections are *graphical methods* whereby planes and lines in space can be represented in two dimensions on a piece of paper.

Hemispherical projections are of great value in geology to identify preferred directions of fracturing in a rock mass or in structural geology when analyzing the modes of deformation and fracturing at the micro and macro scales. In engineering, and in particular in rock mechanics, hemispherical projections can also be used to identify rock blocks formed by the intersection of discontinuities and conduct stability analysis of such blocks for surface or underground excavations. Force vectors can also be represented using hemispherical projections. Several authors have discussed the use of hemispherical projections in geology and rock mechanics. They include Phillips (1971), Ragan (1973), Goodman (1976), (1989), Priest (1985) and Goodman and Shi (1989).

Before discussing the different types of projection, recall the basic orientation angles that are used in geology to describe lines and planes in space:

(1) The orientation of a line in three dimensions is defined by two angles:

- *Plunge* ψ - acute angle measured in a vertical plane between the line and the horizontal. It varies between -90° and 90°. Positive values are for lines pointing downward.

- *Trend* β - geographical azimuth measured in a clockwise rotation from North of the vertical plane containing the line of plunge ψ. It varies between 0 and 360°.

The orientation of a line is recorded in terms of β and ψ as a three digit and a two digit number separated by a slash (example 201/65).

(2) The orientation of a plane can be defined by two angles (i) strike and dip angles or (ii) dip direction and dip angles. These angles are defined as follows:

- *Strike* - the compass direction of a line formed by the intersection of a horizontal plane and an inclined geologic plane such as a fault, fracture, joint, etc. Because it is a compass direction, the strike is usually expressed relative to North or South. Hence, strike is expressed as "North (or South) *so many* degrees East" or "North (or South) *so many* degrees West".

- *Dip* - the angle between a horizontal plane and the plane of interest. A thin stream of water poured on an inclined surface always runs down parallel to dip. The inclination of the water line down from the horizontal plane is called the (true) *dip angle*. The true dip angle is always measured perpendicular to the strike line. It is the maximum inclination of the plane to the horizontal and varies between 0 and 90°.

- *Dip Direction* - the angle between North and the direction of the line of true dip of an inclined geologic plane. It is measured clockwise and varies between 0 and 360°.

- *Apparent Dip* - the inclination angle of a line on an inclined geologic plane measured in a direction oblique to the strike direction. It varies between the true dip and 0°.

The orientation of a plane is recorded in terms of dip and dip direction as a three digit and a two digit number separated by a slash. If the strike and dip angles are used to define the orientation of a plane, the direction in which the plane dips must also be defined. As an example, a plane with strike N 80°E and dip 40° to the SE is equivalent to a plane with a dip direction of 170° and a dip angle of 40°. The plane will be reported as 170/40 or N 80°E, 40° SE.

2. FUNDAMENTALS

Two types of hemispherical projections are available: the *equal angle* and the *equal area* projections. The basis of those two hemispherical projections is an imaginary sphere of radius R called the *reference sphere* (Figure 1). The sphere is positioned with its center at the center of the area of projection. A horizontal diametral plane passing through the center O of the sphere is called the *projection plane*. It divides the sphere into an upper and lower hemisphere. The intersection of the projection plane with the reference sphere is called the *reference circle*. Any line or plane to be projected on the projection plane is made to pass through the origin O of the sphere. Let x, y, z be a coordinate system attached to the sphere with the x-axis pointing to the East, the y-axis pointing to the North, and the z-axis pointing upward. Points F and F' at the top and bottom of the sphere are called *focal points*. Point F is used for *lower hemisphere projections* whereas point F' is used for *upper hemisphere projections.*

2.1 Equal Angle Projection of a Line

Consider first a line with plunge ψ and trend β piercing the sphere at two points A and A' called *poles* (Figure 2a). The lower and upper hemisphere projections of line AA' are shown in Figures 2b and 2d, respectively. First, consider the vertical plane in Figure 2a containing points F, F', A, A' and the origin O. That vertical plane is reproduced in Figures 2b and 2d.

In Figure 2b, a straight line is drawn from the upper focal point F to point A. The intersection of that line with the plane of projection gives a point, a, which is by definition the *lower hemisphere projection* of line OA. Likewise, point, a', is the lower hemisphere stereographic projection of line OA' but falls outside the reference circle. Note that point, a', is usually not considered in geology textbooks but is quite important in the engineering applications of hemispherical projections when distinction must be made between the part of a line that points upward and the other part that points downward. Using simple trigonometry for triangles OaF and Oa'F, it can be shown that

$$Oa = R\tan(\frac{\pi}{4} - \frac{\Psi}{2}) \quad ; \quad Oa' = R\tan(\frac{\pi}{4} + \frac{\Psi}{2}) \tag{1}$$

According to Figure 2c, the x and y coordinates of points, a, and, a', are equal to

$$x_a = R\tan(\frac{\pi}{4} - \frac{\psi}{2})\sin\beta \quad ; \quad y_a = R\tan(\frac{\pi}{4} - \frac{\psi}{2})\cos\beta \tag{2}$$

and

$$x_{a'} = -R\tan(\frac{\pi}{4} + \frac{\psi}{2})\sin\beta \quad ; \quad y_{a'} = -R\tan(\frac{\pi}{4} + \frac{\psi}{2})\cos\beta \tag{3}$$

Figure 2d shows the *upper hemisphere projection* of OA and OA'. The lower focal point F' instead of the upper focal point F is now used for that projection. Equation (1) is replaced by the following

$$Oa = R\tan(\frac{\pi}{4} + \frac{\Psi}{2}) \quad ; \quad Oa' = R\tan(\frac{\pi}{4} - \frac{\Psi}{2}) \tag{4}$$

The x and y coordinates of points, a, and, a', are again given by equations (2) and (3) with $\psi/2$ replaced by $-\psi/2$. Note that point, a, is now outside the reference circle and point, a', is in the reference circle. For both upper and lower hemisphere projections, $Oa.Oa' = R^2$.

Note that equations (1)-(4) allow us to position exactly the points corresponding to the hemispherical (upper or lower hemisphere) projection of OA and OA' without the use of any net. Remember that the hemispherical projection of *a line in space is a point on the projection plane*. A vertical line projects at the center of the reference circle. A horizontal line projects on the circumference of the reference circle and its position depends on the trend of the line.

2.1 Equal Angle Projection of a Cone

Consider now a small *cone* centered at O and subtending an angle 2ϕ as shown in Figure 3a. The axis, ON, of the cone has a plunge ψ and a trend β. The intersection of the cone with the sphere is a small circle. A vertical section through the cone containing the axis ON forms a wedge also subtending the same angle 2ϕ (Figure 3b). Let OA_1 and OA_2 be the two lines defining that wedge. The lower hemisphere projection of those lines and the axis ON will give three points a_1, a_2 and n. It can be shown that a_1a_2 is the diameter of a circle that represents the lower hemispherical projection of the cone on the projection plane (Figure 3c). Using equations (1)-(3), the radius, r, of the circle and the coordinates x_c, y_c of its center, c, are equal to

$$r = \frac{R}{2}[\tan(\frac{\pi}{4} - \frac{(\psi - \phi)}{2}) - \tan(\frac{\pi}{4} - \frac{(\psi + \phi)}{2})]$$
$$x_c = Oc.\sin\beta \quad ; \quad y_c = Oc.\cos\beta \tag{5}$$

with

$$Oc = \frac{R}{2}[\tan(\frac{\pi}{4} - \frac{(\psi - \phi)}{2}) + \tan(\frac{\pi}{4} - \frac{(\psi + \phi)}{2})] \tag{6}$$

Note that point, c, does not coincide with point, n. The lower hemispherical projection of the cone with axis ON' shown in Figure 3a will fall outside the reference circle.

A construction similar to that shown in Figures 3b and 3c can be carried out for the upper hemispherical projection of the cone with axis ON'. In that case, the cone with axis ON' will project as a circle within the reference circle (Figures 3b and 3c) whereas the cone with axis ON will project outside the reference circle. The upper hemispherical projection of the cone with axis ON' is a circle of radius, r, given by equation (5) and center, c', which is opposite to c with Oc' = Oc.

For the same value of the angle ϕ, equation (5) indicates that the area of the projected circle (= πr^2) is not constant but decreases as ψ increases. Figure 3b also illustrates where the equal angle projection got its name. Indeed, for any value of ψ, the angle A_1FA_2 is always equal to ϕ regardless of the value of the plunge angle ψ.

2.3 Equal Area Projection of a Line

Consider again a line with plunge angle ψ as shown in Figure 4. The line intersects the sphere at points A and A'. Consider the lower hemisphere equal area projection of that line. Point A is now projected by swinging it in a vertical plane through a circular arc centered on point F'. The projection of OA occurs at point, a'', where this circular arc intersects the horizontal plane passing through point F'. The distance F'a'' is equal to

$$F'a'' = 2R.\cos(\frac{\pi}{4} + \frac{\psi}{2}) \tag{7}$$

When ψ is equal to zero, the distance F'a'' is equal to $R\sqrt{2}$. This means that the radius of the resultant projection is larger by a factor of $\sqrt{2}$ than the radius of the reference sphere. Since R is the reference radius, point, a'', is transferred to point, a, a distance Oa from point O by taking Oa = F'a''/$\sqrt{2}$. Thus,

$$Oa = R\sqrt{2}.\cos(\frac{\pi}{4}+\frac{\psi}{2}) \tag{8}$$

The equal area projection is like peeling the skin off the lower reference hemisphere, flattening it out and then shrinking it to a circle of radius R.

Note that the equal area projection of the cones of Figure 3a is not a circle but a fourth-order curve. A property of the equal area projection is that for any given value of ϕ, the area enclosed in the projection of the small circle is constant for all values of ψ. This important property is the reason why the equal angle projection is often used in the statistical analysis of lines in space.

2.4 Equal Angle Projection of a Plane

Consider a plane P with dip angle ψ and dip direction β . The plane intersects the reference sphere along a circle as shown in Figure 5a. The equal angle projection of that circle could be determined by considering an infinite number of lines contained in the circle and by projecting (on the upper or lower hemispheres) each one of those lines using the method proposed above. Another method is to use a theorem that applies to the equal angle projection only. "The equal angle projection of a circle on the reference sphere is a circle on the projection plane". Thus, using that theorem or an infinite number of lines we arrive to the conclusion that the equal angle projection of a plane is a *circle*.

In order to find the position of the center of the circle and its radius consider Figure 5b. This figure shows a vertical section through the reference sphere passing through points F, F',O, the dip vector **OD** of the plane and its opposite vector **OW**. The lower hemisphere projections of lines OD and OW are represented by points, d, and, w. It can be shown that the distance dw is the diameter of the circle representing the equal angle projection of plane P. Using equations (1)-(3), the radius, R', of that circle and the coordinates of its center O' can be determined exactly. They are equal to

$$\begin{gathered} R' = \frac{R}{2}[\tan(\frac{\pi}{4}-\frac{\psi}{2}) + \tan(\frac{\pi}{4}+\frac{\psi}{2})] = R/\cos\psi \\ x_{O'} = -OO'.\sin\beta \quad ; \quad y_{O'} = -OO'.\cos\beta \end{gathered} \tag{9}$$

with

$$OO' = \frac{R}{2}[\tan(\frac{\pi}{4}+\frac{\psi}{2}) - \tan(\frac{\pi}{4}-\frac{\psi}{2})] = R\tan\psi \tag{10}$$

Figure 5c shows the circle on the projection plane. If ψ is equal to zero, the plane is horizontal, R' = R and OO' vanishes and its hemispherical projection coincides with the reference circle. On the other hand, if ψ is equal to 90°, the plane is vertical, R' and OO' are equal to infinity and the projection of a plane is a line parallel to the strike direction.

Figure 5d shows the upper hemisphere projection of plane P on the projection plane. The radius R' and OO' are again given by equations (9) and (10). However, the coordinates of the center are now equal to

$$x_{O'} = OO'.\sin\beta \quad ; \quad y_{O'} = OO'.\cos\beta \tag{11}$$

Note that in classical geology texts, the arc of the circle representing the equal angle projection of a plane contained in the reference circle is considered only and the rest is neglected. For engineering purpose, this is important since the circle helps defining two half spaces: the half space above the plane and the half space below the plane. For the lower hemisphere projection, the half space below the plane is contained in the projection circle whereas the half space above the plane is contained outside the circle. The reverse is true for the upper hemisphere projection.

The projection of the normal ON to a plane can be determined using equations (1)-(4). For the geometry of Figure 5b, the distance On = R.tan(ψ/2). For the lower hemisphere projection (Figure 5c) the coordinates of point, n, are equal to

$$x_n = -On.\sin\beta \quad ; \quad y_n = -On.\cos\beta \tag{12}$$

For the upper hemisphere projection (Figure 5d), On is replaced by -On in equation (12).

2.5 Equal Area Projection of a Plane

The equal area projection of a plane is not a circle and is in general a fourth-order curve with no simple mathematical form.

3. POLAR PROJECTIONS AND POLAR NETS

The equal angle or equal area projection of lines in space can be determined exactly using equations (1) to (4). Another way of doing the same thing, is to use circular grids or nets that have been developed to facilitate the plotting procedure. Figures 6a and 6b show a polar equal angle net and a polar equal area net, respectively.

Polar nets consist of a series of concentric circles corresponding to different values of the plunge angle ψ ranging between 0 and 90° and radial lines oriented at angles ranging between 0 and 360° with 2° increments corresponding to the trend angle β. Note that using equation (5), you can construct your own *equal angle* net by considering a series of cones centered at O with dip angle ψ

= 90°, and subtending an angle 2ϕ ranging between 0 and 180°. For a given value of ϕ, the radius of the circle representing the cone is equal to R.tan(ϕ/2). An alternative is to use equations (1) and (2) and plot the equal angle projection of lines with trend β ranging between 0 and 360° and dip angle ψ ranging between 0 and 90°.

When using the nets of Figures 6a and 6b, a sheet of transparent tracing paper must be overlaid on the net in order to avoid marking the net. The polar nets are ideally suited for plotting the projection of lines in space but is of little value for plotting the projection of planes. In rock mechanics studies, polar nets (and in particular equal area nets) are used for plotting the normals to discontinuities mapped during a rock mechanics survey. Any clustering of data points indicates preferred directions of discontinuity planes.

4. EQUATORIAL PROJECTIONS AND EQUATORIAL NETS

Equatorial projections are used for the projection of both lines and planes. This can be done using the equations presented earlier or by using another set of circular grids called *equatorial nets.*

The equal angle equatorial net (Figure 7a), sometimes called *Wulf net*, consists of two sets of circles: the big circles and the small circles. The big circles are the stereographic projection of planes that all strike North-South and dip to the East or the West with dip angles ranging between 0 and 90° with 2° increments. The small circles are the stereographic projections of cones centered at O with horizontal axes in the North-South direction and half-apex angle ranging between 0 and 90° with 2° increments. They also describes the changing orientation of a given line when it is rotated about a given North-South horizontal axis (Figures 8a and 8b).

The big circles can be constructed by using equations (9) and (10) with β = 90 or 270° and ψ ranging between 0 and 90° with 2° increments. The small circles can be constructed by using equations (5) and (6) with ψ = 0°, β = 0° (North pointing cone) and β = 180° (South pointing cone) and ϕ ranging between 0 and 90° with 2° increments. The small circles have a radius r = R.tan(ϕ) and their center is at a distance equal to ± R/cosϕ from O.

The construction of the equal area net is more complex. The great and small circles of the equal angle net are now replaced by fourth-order curves (see Figure 7b).

Note that the equatorial nets of Figures 7a and 7b contain two straight lines: the North-South and East-West diameters. The former represents the great circle of a vertical plane with a North-South strike. The latter corresponds to a cone with an apex angle of 180°. It is also the projection of a vertical plane with an East-West strike. The center represents the stereographic projection of a vertical line or the normal to the horizontal plane. Any point on an equatorial net represents by definition a line. It is always at the intersection of a great circle (a plane in space containing the line) and a small circle (cone containing the line making a constant apex angle with the North-South horizontal line) (see Figure 9).

When using equatorial nets, a sheet of transparent tracing paper must be overlaid on the nets in order to avoid marking the nets. This makes it possible to plot a plane of any general orientation by rotating the tracing paper until the plane has its strike temporarily on the North-South diameter of the net.

The equatorial nets are ideally suited for plotting the projection of lines or planes. Equal angle nets are used mostly for rock mechanics design whereas equal area nets are used for statistical analysis of discontinuity data.

5. EXAMPLES

Before proceeding with the examples, you should have a copy of an equatorial equal-angle net mounted on a reinforcing backing. A drawing pin is pressed through the back of the net at the center of the net. A sheet of tracing paper is placed on top of the net and is free to rotate around the pin axis. The North (N) point is marked on the tracing paper corresponding to a 0° azimuth. You may also want to mark the South (S), East (E), and (W) directions.

To plot the projection of a line, rotate the tracing paper until the trend direction of the line lies temporarily on an EW or NS diameter. In doing so you have placed the trend parallel to a vertical plane. The angle of plunge can then be counted from the perimeter of the net along the EW or NS diameter using the great circles on the net as graduations.

The plotting of the great circle and the normal to a plane is an extension of the plotting procedure for a line.

Using the lower hemisphere stereographic projection:

(1) Plot the projection of line 1 with a plunge of 40° to the N 30° E (30/40);

(2) Plot the projection of line 2 with a plunge of 20° to the N 20° W (340/20);

(3) Plot the projection of plane 1 with orientation N 50° E 20° NW (320/20) and its normal and dip vectors. What is the apparent dip of plane 1 in the NS direction?

(4) Plot the projection of plane 2 with orientation N 60° W 45° SW (210/45) and its normal and dip vectors. What is the apparent dip of plane 2 in the NS direction?

(5) What is the angle between lines 1 and 2?

(6) What is the orientation of the line of intersection, I_{12}, between planes 1 and 2? This can be determined by the line of intersection between the great circles representing the two planes. It can also be determined by first constructing the great circle representing the

plane containing the normals to planes 1 and 2. The normal to that plane gives I_{12}.

(7) Plot the locus of lines at 45° with the normal to plane 1.

6. STATISTICAL ANALYSIS OF ORIENTATION DATA

The discontinuity orientation data obtained from field measurements in boreholes or surface outcrops can be presented graphically on a hemispherical projection. The way to do this is to plot the normals (poles) to all the planes measured in a certain volume of rock of interest. Various symbols can be used to differentiate between the different types of discontinuities (joints, faults, bedding joints., etc..). The size of each symbol can also be used to represent the range of discontinuity sizes. An example of such a plot is shown in Figure 10a.

The objective of statistical analysis is to identify groups or clusters of sub-parallel discontinuities, also called *sets*, that may have major influence upon the engineering behavior of a rock mass. In the absence of such sets, the rock mass can be said to be *randomly fractured*.

Contouring methods are used to determine major fracture sets from the poles on equatorial *equal area nets*. In general, they consist of a three-stage process:

- Each discontinuity normal (pole) is plotted on a lower hemisphere projection;
- A sampling window (usually circular) is placed over the data, to generate a matrix of moving average values, representing the variation in the concentration of discontinuity normals over the projection;
- The moving average values are contoured at some appropriate interval.

The moving average is obtained by finding the sub-sample that appears within a small window placed over the discontinuity normal data points. The window is usually a circle with a radius, r, equal to one-tenth the sphere radius, R. Thus, the area of the window is equal to 1% of the projection area. This method is called the *floating circle counting method*. The process can be done automatically by computers or manually.

If the manual approach is used, two circles that are 2R apart need to be drawn on a strip of paper or cut in a strip of perspex. Before proceeding with the counting process, it is necessary to construct a square grid with a line spacing, r, to provide reference points for the counting circle (Figure 10b). The grid is drawn on tracing paper and fixed on top of the projection containing the data points that have to be counted. A transparent overlay is placed on top with the N, S, W, and E points marked on it. The counting circle is positioned with its center at a grid intersection and the total sub-sample within the circle is calculated. Let, n, be the number of poles in the sub-sample. The sub-sample is expressed as a percentage of the total population, N, and is recorded at the grid point on the overlay with a percentage of 100(n/N). When one of the two counting circles on the strip falls on the circumference of the net, the total number of poles falling within the circle is given by adding the poles in the circle and the circle that is diametrically opposite to it. When the counting process is complete, the transparent overlay can be removed and contoured where the contour lines join points with same percentage. A contour interval of 1% is usually satisfactory for most practical purposes. The inter-contour zones can also be shaded to emphasize the different concentrations.

Figures 11a, 11b and 11c show an example of application of the floating circle contouring method presented by Hoek and Brown (1980).

Beside the floating circle contouring method, another statistical technique consists of determining the mean joint orientation of a cluster of poles of discontinuity. Consider a group of N discontinuities, each one being defined by a unit vector parallel to its normal. Let **R** be the resultant of all unit vectors in the group. If all the discontinuity planes are parallel, their normals must also be parallel, and the magnitude of **R** denoted R must be equal to N. As the discontinuity directions become dispersed, R becomes less than N. An index of dispersion is equal to

$$K = \frac{N}{N-R} \tag{13}$$

The coefficient K approaches infinity as the dispersion decreases. The value of K can be very helpful in finding the probability of occurrence of a discontinuity normal in a given direction that belongs to the group of N discontinuities. According to the *Hemispherical Normal Distribution*, Fisher (see Goodman, 1976) has shown that the probability P that a normal will make an angle of ψ degrees or less with the direction of **R** is such that

$$\cos\psi = 1 + \left(\frac{1}{K}\right)\log(1-P) \tag{14}$$

The standard deviation of that distribution is equal to $1/\sqrt{K}$. A *chi* square test must be carried out to check to which degree the hemispherical distribution fits the data.

7. BLOCK REPRESENTATION

The stereographic projection is a valuable tool to visualize the blocky nature of hard rock masses. Blocks are essentially volumes of rock limited by planar features. Since each plane divides the 3D space into two half-spaces, a block can be seen as the intersection of half spaces. In the stereographic projection (equal angle projection), a plane is represented by a circle and the top and bottom half spaces are represented by the area inside the circle or outside depending on the lower or upper projection is used:

- When using the UH or upper hemisphere projection (lower-focal point), the half space above a plane (denoted U) is contained in the projection circle whereas the half space below the plane (denoted L) is contained outside the circle. This is shown in Figure 12a for a plane P_i.

- When using the LH or lower hemisphere projection (upper-focal point), the half space below a plane (denoted L) is contained in the projection circle whereas the half space above the plane (denoted U) is contained outside the circle.

If a rock mass is cut by i = 1, N discontinuity planes, each plane can be represented by a circle and the upper and lower half spaces U_i and L_i can be identified. Each region on the stereonet will consist of a combination of U_i's and L_i's. For instance, block 1 in Figure 12b is formed by the intersection of four discontinuity planes and its "stereographic" label is $U_1U_2U_3U_4$. Note that the stereographic projection of a block (i.e a 3D object) is a surface on the projection plane.

Block theory is a branch of rock mechanics dealing with the characterization of blocks, their mechanical behavior and their stability. The theory was pioneered by Goodman (1976, 1989) and Goodman and Shi (1985). An important concept in block theory is that of "key block", i.e. a block that is the most critical in the stability of an excavation (surface or underground). They may be more than one key blocks at a given time in a rock mass. Being able to identify those blocks and understand their stability is paramount in block theory. Figure 13 shows the different types of blocks in the block theory.

8. ANALYSIS OF BOREHOLE DATA

The problem here is how to determine the orientation of a given fracture plane intersecting two or more boreholes. Consider the geometry of Figure 14a showing a fracture plane of unknown orientation intersecting a borehole core of diameter, D. The angle ϕ that the normal to the fracture plane makes with the core axis can be determined by measuring the distances h_1 and h_2 along the borehole axis from an arbitrary reference datum (a-a). The two distances are elevations of the two extremities of the longitudinal axis of the ellipse representing the trace of the plane on the core surface. The angle ϕ is equal to

$$\phi = \tan^{-1}\left(\frac{h_2 - h_1}{D}\right) \tag{15}$$

Unless the core is oriented, it is not possible to determine the complete orientation of the fracture plane except when the angle ϕ is zero which corresponds to a borehole with axis perpendicular to the fracture plane. We will assume that we do not have down-the-hole core orientation devices and that the angle ϕ is not zero. A second borehole inclined with respect to the first is also assumed to be available.

For a given value of the angle ϕ and a given borehole axis, the locus of all possible normals to the fracture plane is a cone subtending an angle 2ϕ (see Figure 14b). On an equal LH stereographic projection, this will be represented as a circle with radius and center given by equations (5) and (6). The borehole axis with trend β and plunge ψ will be represented by a point, L, in the circle with coordinates given by equation (2).

If we know the orientation angles (β_1, ψ_1) of borehole axis #1, the orientation angles (β_2, ψ_2) of borehole axis #2, and the angles ϕ_1 and ϕ_2 measured on two sets of core samples, it is then possible to construct two circles representing the cones with apex angles $2\phi_1$ and $2\phi_2$. The two circles will intersect at two points corresponding to two lines in space, each one being a possible normal to the fracture plane passing by both boreholes. Additional information (qualitative information) may help in deciding which of the two solutions is the correct one. A third borehole intersecting the same fracture will indicate which orientation is correct since three points define a plane.

9. REFERENCES

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