Problem 1 (30 points)

A circular tunnel of radius $a$ is located at a depth $z$ in a hydrostatic *in situ* stress field of magnitude $\gamma z$. The tunnel lining is modeled as a ring of concrete of uniform thickness, $t$. The Young's modulus and Poisson's ratio of the rock are defined as $E_r$ and $\nu_r$, and those of the concrete are defined as $E_c$ and $\nu_c$. There is no separation between the concrete and the rock.

a) Derive the analytical expression for the tangential stress $\sigma_\theta$ on the inner surface of the concrete lining ($r=a-t$). That stress component is compressive but must not exceed the unconfined compressive strength of the concrete defined as $\sigma_c$. Let $SF = \sigma_c/\sigma_\theta$ be the safety factor against concrete failure in compression.

b) Plot the variation of $t/a$ versus the depth $z$ for $z \geq 100$ m and for $SF = 1$ and 1.5. The input data are: $\sigma_c=5,000$ psi, $E_c=3 \times 10^6$ psi, $\nu_c=0.2$, $E_r=1 \times 10^6$ psi (soft rock), $\nu_r=0.25$, and $\gamma=160$ lb/ft$^3$.

Problem 2 (30 points)

The objective of this problem is to determine the distribution of *in situ* stresses in the Earth's crust taking into account the curvature of the Earth.


2.) McCutchen (1982) considers an isotropic spherical shell (representing the Earth's crust) of outer radius $R$ consisting of material with unit weight, $\gamma$, and subject to gravity, $g$. The shell is assumed to be situated on an unyielding massive interior body. Using the equations of equilibrium, the stress-strain relations and the constitutive equations, show that the radial stress, $\sigma_r$, (also equal to the vertical stress), and the tangential stress, $\sigma_\theta$ (assumed the same in all tangential directions and equal to the horizontal stress) are equal to

\[
\sigma_r = \frac{\gamma R}{4} \left[ -4(1-\beta)x + (3-4\beta)A - \frac{4RB}{x^3} \right] \\
\sigma_\theta = \frac{\gamma R}{4} \left[ -2(2-3\beta)x + (3-4\beta)A + \frac{2RB}{x^3} \right]
\]  

(1)

In (1), $x$ is the ratio between the distance $r$ from the center of the sphere and the sphere's outer radius $R$ and is also equal to $1-z/R$ where $z$ is the depth below the ground surface. The constant $\beta$ is equal to $0.5(1-2\nu)/(1-\nu)$. Finally, $A$ and $B$ are two constants of integration that can be determined from the boundary condition $\sigma_r = 0$ at $x=1$ and by assuming that at a distance $r_o$ (or a
depth \( z_o \), corresponding to the crust-mantle interface, the tangential strain is equal to zero.
Substituting these two conditions into (1) gives a horizontal to vertical stress ratio \( K=\sigma_\theta/\sigma_r \) that
varies in a non-linear manner between \( K_o=1-2\beta=\nu/(1-\nu) \) at \( z=z_o \) and infinity at \( z=0 \). **Show** this
variation for the first 2,000 m of the crust assuming that the crust-mantle interface is located at a
depth of 35 km and that the Earth has a radius of 6,371 km. Assume that \( \nu=0.2 \).

Note that the main drawback of the model of McCutchen is that the elastic constant and density
of the rock in the crust do not vary with depth and the model does not account for the effect of
the geothermal gradient.

**Problem 3** (40 points)

We are interested in determining the trajectory of a block of rock following its release from the
top of a slope of height \( h_o \). The initial block velocity is \( v_o \). The geometry of the problem is shown
below. The block is assumed to behave as a particle of mass \( m \). The slope surface is assumed to
be frictionless with a coefficient of restitution, \( e \), that can vary between 0 and 1. For input data of
your choice, write a computer program to determine the trajectory of the rock block for different
values of \( e \) between 0 and 1.