

CVEN 5768 INTRODUCTION TO ROCK MECHANICS
Mid-Term Exam (Due by 5:00 p.m. Friday March 21, 2008).

Problem 1 (30 points)

The objective of this problem is to determine the distribution of *in situ* stresses in the Earth's crust taking into account the curvature of the Earth.

- 1.) Read the paper by McCutchen (1982) entitled "Some Elements of a Theory for In-Situ Stress", *Int. J. Rock Mech. Min. Sci.*, Vol. 19, pp 201-203, 1982. The paper will be distributed in class on Wednesday March 14, 2007 or you can get it directly from the engineering library.
- 2.) McCutchen (1982) considers an isotropic spherical shell (representing the Earth's crust) of outer radius R consisting of material with unit weight, γ , and subject to gravity, g. The shell is assumed to be situated on an unyielding massive interior body. Using the equations of equilibrium, the stress-strain relations and the constitutive equations, **show** that the radial stress, σ_r , (also equal to the vertical stress), and the tangential stress, σ_θ (assumed the same in all tangential directions and equal to the horizontal stress) are equal to

$$\sigma_r = \frac{\gamma R}{4} \left[-4(1-\beta)x + (3-4\beta)A - \frac{4\beta B}{x^3} \right]$$
$$\sigma_\theta = \frac{\gamma R}{4} \left[-2(2-3\beta)x + (3-4\beta)A + \frac{2\beta B}{x^3} \right]$$
(1)

In (1), x is the ratio between the distance r from the center of the sphere and the sphere's outer radius R and is also equal to $1-z/R$ where z is the depth below the ground surface. The constant β is equal to $0.5(1-2\nu)/(1-\nu)$. Finally, A and B are two constants of integration that can be determined from the boundary condition $\sigma_r=0$ at $x=1$ and by assuming that at a distance r_0 (or a depth z_0), corresponding to the crust-mantle interface, the tangential strain is equal to zero. Substituting these two conditions into (1) gives a horizontal to vertical stress ratio $K=\sigma_\theta/\sigma_r$ that varies in a non-linear manner between $K_0=1-2\beta=\nu/(1-\nu)$ at $z=z_0$ and infinity at $z=0$. **Show** this variation for the first 2,000 m of the crust assuming that the crust-mantle interface is located at a depth of 35 km and that the Earth has a radius of 6,371 km. Assume that $\nu=0.2$.

Note that the main drawback of the model of McCutchen is that the elastic constant and density of the rock in the crust do not vary with depth and the model does not account for the effect of the geothermal gradient.

Problem 2 (40 points)

We are interested in determining the trajectory of a block of rock following its release from the top of a slope of height h_0 . The initial block velocity is v_0 . The geometry of the problem is shown below. The block is assumed to behave as a particle of mass m . The slope surface is assumed to be frictionless with a coefficient of restitution, e , that can vary between 0 and 1. For input data of your choice, write a computer program to determine the trajectory of the rock block for different values of e between 0 and 1.

