CVEN 5768 INTRODUCTION TO ROCK MECHANICS
Mid-Term Exam (Due by 5:00 p.m. Friday March 24, 2006).

Problem 1

A circular tunnel of radius a is located at a depth z in a hydrostatic *in situ* stress field of magnitude γz. The tunnel lining is modeled as a ring of concrete of uniform thickness, t. The Young's modulus and Poisson's ratio of the rock are defined as E_r and ν_r, and those of the concrete are defined as E_c and ν_c. There is no separation between the concrete and the rock.

a) Derive the analytical expression for the tangential stress σ_θ on the inner surface of the concrete lining (r=a-t). That stress component is compressive but must not exceed the unconfined compressive strength of the concrete defined as σ_c. Let SF = σ_c/σ_θ be the safety factor against concrete failure in compression.

b) Plot the variation of t/a versus the depth z for z $\geq$100 m and for SF = 1 and 1.5. The input data are: σ_c=5,000 psi, E_c=3 x 10^6 psi, ν_c =0.2, E_r=1 x 10^6 psi (soft rock), ν_r=0.25, and γ=160 lb/ft^3.

Problem 2

The objective of this problem is to determine the distribution of *in situ* stresses in the Earth's crust taking into account the curvature of the Earth.

1.) Read the paper by McCutchen (1982) entitled "Some Elements of a Theory for In-Situ Stress", *Int. J. Rock Mech. Min. Sci.*, Vol. 19, pp 201-203, 1982. The paper will be distributed in class on Monday March 20 or you can get it directly from the engineering library.

2.) McCutchen (1982) considers an isotropic spherical shell (representing the Earth's crust) of outer radius R consisting of material with unit weight, γ, and subject to gravity, g. The shell is assumed to be situated on an unyielding massive interior body. Using the equations of equilibrium, the stress-strain relations and the constitutive equations, **show** that the radial stress, σ_r, (also equal to the vertical stress), and the tangential stress, σ_θ (assumed the same in all tangential directions and equal to the horizontal stress) are equal to

$$\begin{aligned}\sigma_r &= \frac{\gamma R}{4}\left[-4(1-\beta)x+(3-4\beta)A-\frac{4\beta B}{x^3}\right] \\ \sigma_\theta &= \frac{\gamma R}{4}\left[-2(2-3\beta)x+(3-4\beta)A+\frac{2\beta B}{x^3}\right]\end{aligned} \qquad (1)$$

In (1), x is the ratio between the distance r from the center of the sphere and the sphere's outer radius R and is also equal to 1-z/R where z is the depth below the ground surface. The constant β is equal to 0.5(1- 2ν)/(1-ν). Finally, A and B are two constants of integration that can be determined from the boundary condition σ_r=0 at x=1 and by assuming that at a distance r_o (or a depth z_o), corresponding to the crust-mantle interface, the tangential strain is equal to zero. Substituting these two conditions into (1) gives a horizontal to vertical stress ratio $K=\sigma_\theta/\sigma_r$ that varies in a non-linear manner between $K_o=1-2\beta=\nu/(1-\nu)$ at $z=z_o$ and infinity at $z=0$. **Show** this variation for the first 2,000 m of the crust assuming that the crust-mantle interface is located at a depth of 35 km and that the Earth has a radius of 6,371 km. Assume that $\nu=0.2$.

Note that the main drawback of the model of McCutchen is that the elastic constant and density of the rock in the crust do not vary with depth and the model does not account for the effect of the geothermal gradient.