Problem 1:

Consider a circular opening of radius $R_1$ subjected to a hydrostatic stress $p_o=\gamma z$ acting at infinity. The internal pressure in the tunnel is $p_i$. The rock is assumed to be isotropic and has a linearly elastic - perfectly plastic behavior. Failure of the rock is assumed to be described by a Mohr-Coulomb criterion with

$$\sigma_1 = C_o + q \sigma_3$$  \hspace{1cm} (2)$$

with $q = \tan^2(\pi/4 + \phi/2)$ and $C_o = 2S_o\tan(\pi/4 + \phi/2)$. In equation (1), $\phi$ and $S_o$ are the internal friction angle and shear strength (cohesion) of the rock, respectively.

For certain values of the applied stresses and the rock strength properties, failure of the rock may occur in a region $R_1 \leq r \leq R$ in which equation (2) is satisfied. Outside this region ($r > R$), the rock behaves elastically.

Consider the following equation of equilibrium

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_o}{r} = 0$$  \hspace{1cm} (3)$$

Also, assume that $\sigma_o > \sigma_r$.

(a) Determine the expressions for the stresses in the plastic and elastic zones. Determine the expression of $R/R_1$ in terms of $p_o/p_i$ and $C_o/p_i$.

(b) Assume $p_i = 5$ psi, $R_1 = 20$ ft, $\gamma = 160$ pcf, $\phi = 40^\circ$ and $S_o = 1,000$ psi. Show the variation of $R/R_1$ with depth $z$. Select a depth $z$ for which $R/R_1$ is much larger than unity. For that depth show the stress distribution around the opening. How do these stresses differ from those obtained assuming elastic response only ?

(Hint: Read section 16.5 in “Fundamentals of Rock Mechanics” by J.C. Jaeger and N.G.W. Cook)
Problem 2

A block weighing 200 MN rests on a plane P, striking N 10 E and dipping 60 NW (60/280). The available friction angle is believed to be 30 degrees.

1) Find the minimum force for stabilizing the block with a safety factor of 2.5 using rock bolts.

2) Find the force for stabilizing the block with a safety factor of 2.5 if the bolts are installed 10 degrees below horizontal to the N 88 E.

3) What water force acting on plane P could cause failure after the rock bolts are installed as in case b)?